ELEMENTS OF HYDRAULICS AND HYDRAULIC MACHINERY

IN M.K.S. UNITS

[FOR FINAL YEAR OF DIPLOMA COURSE IN MECHANICAL AND ELECTRICAL ENGINEERING AND DEGREE AND A.M.I.E. COURSES]

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First Edition : August 1964
Second Edition : August 1966
Third Edition : August 1967

RUPEES
15·00

Printed & Published by
Shri J. C. Shah at Acharya Printing Press,
Panigate – Baroda
for
Acharya Book Depot,
Opp. Gandhi Nagargruh – Baroda.

Preface to Third Edition

We are called upon to take out the third edition of this book in less than an year's time. This speaks for the favour shown to the book. We have seized this opportunity to overhaul the book thoroughly to tune it up for present demands. A new portion is added on Jet propulsion in chapter 9. This is done with a view to make the book useful for the Degree and A.M.I E. examinations.

A large number of worked out problems are added in each chapter to expose special intricate features of the topics of study Some of the diagrams have also been redrawn.

The authors are thankful to the vast community of student readers and teachers who have been continuously encouraging us to serve them more and more. Our thanks are alo due to Shri Jayantilal Shah of Acharya Book Depot, who never fails to take the pains of improving the get up of the book.

Baroda,
15th August 1967

R. C. Patel
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Preface to Second Edition

It is indeed a matter of gratification to the authors that they are required to bring out the third edition of this volume in such a short span of time. This indicates the usefulness and popularity of this work.

This edition is enlarged to enhance the usefulness of the book by making it more lucid in expression and addition of subject matter and illustrated problems at some places. The entire work is now in M.K.S. system of units.

The authors take this opportunity of thanking all those who have been good enough to draw their attention to some of the slips that had inadvertently crept in the first edition and those who offered valuable and useful suggestions and comments with a view to make the book really one of the best of its kind.

The authors will be very highly pleased to receive suggestions from the readers.

Authors acknowledge with thanks the help rendered by servashri A. A. Patel, B.E. (Mech.), M. M. Patel, B.E. (Mech.), B. C. Patel, D.M.E., K. H. Patel, B.E. (Mech.) in checking the calculations and reading the proofs and A. S. Karkhanis for redrawing the sketches. Our sincere thanks are also due to the publishers.

Baroda,
2nd August, 1966

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A. D. Pandya
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Preface to First Edition

It gives us great pleasure to put before the large community of Engineering students our humble work on Elements of Hydraulics and Hydraulic Machinery. We have felt, during our long association with the profession of teaching, that there is no suitable book on the subject of Hydraulics to be studied by the students of Final Year of the Diploma Courses in Mechanical & Electrical Engineering. In the absence of a suitable book, students are required to refer to books in which many more things, than necessary for them, are given and many intermediate stages of explanation omitted The result is that the students are not able to study the subject matter precisely as they are expected to do.

In this book on Elements of Hydraulics and Hydraulic Machinery we have presented the subject matter in a very much simplified manner. Each small topic of theory is illustrated by numerical problems to bring home the basic engineering principle involved. Tutorials are also constructed and arranged in chronological order with a view to provide scope for independent work to inspire self-confidence in the students.

Well drawn diagrams have been sparingly incorporated to assist the understanding of the theory and data of the problems.

Since it has been decided to switch over to metric system, the whole work incorporates both the systems side by side, and we feel that this should help to a great extent in the present transition stage.

We take the opportunity to extend our deep sense of thankfulness to Shri A. S Karkhanis for nicely drawing the sketches of the book.

We sincerely hope that present manual will be accorded the same warm and enthusiastic welcome that is being extended to our previous books on Elements of Mechanical Engineering and Elements of Heat Engines. We request the readers to send in their valuable suggestions to make the book more useful.

R. C. Patel
A. D. Pandya
B. M Patel

Baroda,
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Contents

Flow Over Notches and Weirs

1
HYDROSTATICS

Introduction

The science of *Fluid Mechanics* deals with the behaviour of fluid at rest or in motion. Mechanics of fluids at rest is known as *Hydrostatics*, and that of fluids in motion and subjected to changes of pressure, frictional resistance, flow through various types of ducts, orifices, impact of jets, and production of power is known as *Hydrodynamics*.

In fact the term fluid applies to liquids and gases. The fluid mechanics of liquids is known as *Hydraulics*. In Hydraulics liquid is usually considered as an incompressible fluid.

Properties of Fluid

The term fluid is used for all the substances that offer no resistance to change of shape. Thus, fluid includes liquid, vapour and gases.

Liquid offers great resistance to compression while vapours and gases can be easily compressed. The value of bulk elastic modulus of water under compression is 21,100 kg/sq. cm.

Liquids are not much affected by change of temperature while vapour and gases are greatly affected by change of temperature.

Liquids can resist a slight tension due to molecular attraction. This will induce shear stress between two adjacent layers. This shear resistance is termed as *viscosity*.

The density of pure water is taken as 1,000 kg/cu. m. The ratio of the density of liquid to the density of water is known

as the *specific gravity* of the liquid. Liquids cannot exist as liquids at zero pressure or in absolute vacuum. In reality, all known liquids vaporize at pressure slightly above absolute zero, depending on the temperature. Water vaporizes at a pressure of 0·24 kg./sq. cm. at about 20°C. As the vaporization starts, gases which have remained dissolved in water are given off. This phenomenon is known as *separation*. Thus, existance of low pressure causes great inconvenience in problems of Hydraulics. For this reason care must be taken to prevent the pressure of water getting below 2·5 m height of water.

Pressure of Fluid

Fluid pressure, P is defined as the normal force exerted by the fluid on the wetted surface. The intensity of pressure, p is defined as the pressure per unit area (kg/sq. cm). Referring to fig. 1–1, the pressure, p of liquid against the boundary suface at A is assumed inclined at an angle θ to the normal (perpendicular) to the surface. This pressure has two components: $p \sin \theta$, (tangential) and $p \cos \theta$, (normal); parallel and normal to the surface respectively. Since the liquid is at rest, it cannot have a tangential component or shear force which only comes in play when fluid is in motion. Thus, there cannot be any tangential component. Hence, it can be concluded that the pressure of liquid at rest must be normal to the surface.

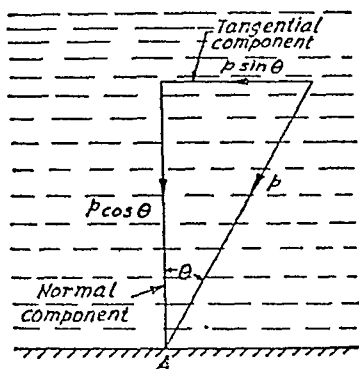


Fig. 1–1

Pascal showed that in fluid at rest, the pressure is transmitted equally in all the directions. If a fluid is contained in a vessel and is subjected to uniform pressure throughout, a slight increase in the intensity of pressure at one point will be immediately

transmitted to all parts of the vessel. Referring to fig. 1-2, suppose a curved surface to be under an uniform pressure, p. The direction of p at any point will be normal to the surface. Consider a small elementary area, a on the surface, the inclination of the area being θ with the horizontal.

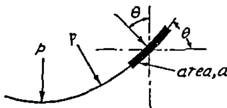


Fig. 1-2

The force on the area, $a = p \times a$ .. .. .. (1·1)

Vertical component of this force is $p{\cdot}a \cos\theta$. But, $a \cos\theta$ is the horizontal projection of area, a. Therefore, vertical component of force on $a = p \times$ (horizontal projection of area a). Thus, it can be concluded that, if a surface of any shape is subjected to uniform pressure, the force acting on it in any given direction is given by the intensity of pressure multiplied by the projected area normal to the given direction.

Problem - 1 : *A drum of a boiler is 2 m in diameter and 5 m long. If the pressure of steam inside the boiler is 10 kg/sq. cm, estimate the force tending to burst the drum along the longitudinal section.*

Bursting force $= 10^4\, p \times$ horizontal projection of area
$= 10{,}000 \times 10 \times 2 \times 5$
$= 10{,}00{,}000$ kg or 1,000 tonnes.

Problem - 2 : *A hemispherical dome (fig. 1-3) of radius 1 metre contains a fluid under pressure of 10 kg/sq. cm (gauge). Find the total force tending to lift the dome.*

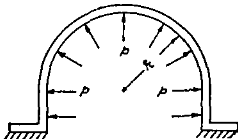


Fig. 1-3

Total vertical force
$= p \times$ horizontal projected area
$= p \times \pi r^2$
$= 10 \times \pi \times 100^2$
$= 3{,}14{,}200$ kg or 314·2 metric tons (tonnes)

Principle of Hydraulic Press

The principle that the fluid transmits pressure equally in all directions is utilised in a number of machines and the hydraulic press is a simple example of it. It is a machine by which the pressing of cotton bales in ginning factories is done and it can be used to lift large weights by applying a much smaller force. It consists of two cylinders A & B, one smaller and the other large, both containing liquid and connected by a pipe D near their bottom. The larger cylinder B contains a ram C, on which heavy weight is supported and the smaller cylinder A, contains a plunger E. The plunger is operated by the lever JKG which is free to turn on the fulcrum at J under the effect of the effort P, applied to its other end (fig. 1-4).

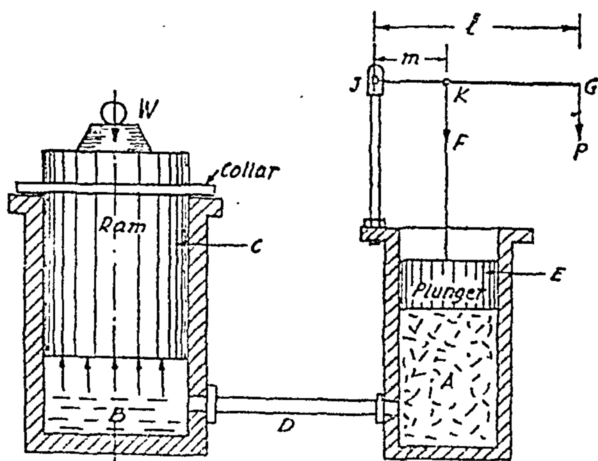


Fig. 1-4 Hydraulic press

Let, p – pressure of the liquid,

a – area of cross-section of the plunger, and

A – area of cross-section of the ram.

As the pressure in a liquid is the same throughout,

$$F = p \times a \text{ and } W = p \times A$$

$$\because \quad \frac{F}{a} = p = \frac{W}{A}$$

$$\therefore \quad \frac{W}{F} = \frac{A}{a}\,.$$

$$\therefore \quad F = W \times \frac{a}{A}\,. \quad \ldots \quad \ldots \quad \ldots (1\cdot2)$$

Now, the term $\frac{W}{F}$ is known as the *mechanical advantage*, hence we can say that mechanical advantage is equal to the ratio of the areas of the ram and plunger. Also for frictionless machines, mechanical advantage is equal to the velocity ratio. So in the case of the hydraulic press, the ratio of the areas of the ram and plunger denotes the velocity ratio.

Let, l - length of the lever JG,

m - distance JK *i. e.* distance from the hinge upto the point where the plunger is attached,

P - effort applied at the end of the lever, and

F - force on the plunger.

Taking moments about the hinge point J, we have

$$P \times l = F \times m$$

$$\therefore \quad F = P \times \frac{l}{m} \quad \ldots \quad \ldots \quad \ldots (1\cdot3)$$

The ratio $\frac{l}{m}$ is known as the leverage.

From eqns. (1·2) and (1·3) we have,

$$W \times \frac{a}{A} = P \times \frac{l}{m}$$

$$\therefore \quad \frac{W}{P} = \frac{A}{a} \times \frac{l}{m}$$

= ratio of the areas × leverage of the lever

Let, V - volume of water displaced by the plunger in one stroke,

L - length of the stroke of the plunger,

S - displacement of the ram,

n - number of strokes of the plunger to move the ram through a distance, S.

$$\therefore \quad V = aL$$

The volume of water required to displace the ram through a height S, is $A \times S$.

$$\therefore \quad \text{number of strokes, } n = \frac{AS}{V} = \frac{AS}{aL} \qquad \ldots \quad \ldots (1{\cdot}4)$$

Problem-3 : *In a hydraulic press the area of the ram is 200 sq. cm and its weight is 9 kg, while the area of the plunger is 1 sq.cm and its weight is 1 kg. Determine the weight that can be lifted by the ram if a force of 3 kg is applied to the plunger.*

The total force that creates the pressure on water is the force on the plunger and its weight.

$\therefore$ the pressure at each point of the liquid under the plunger is

$$p = \frac{3+1}{1} = 4 \text{ kg/cm}^2 \text{ (cm}^2 = \text{sq.cm)}$$

This pressure is transmitted to each square centimetre of the ram which is 200 sq.cm.

Hence, the total upward force on the ram $= p \times A = 4 \times 200 =$ 800 kg. The weight of the ram opposes the upward force on the ram, hence the weight that can be lifted by the ram will be less than this upward force by an amount equal to its own weight.

$\therefore$ the weight lifted by the ram $= 800 - 9 = \underline{791 \text{ kg}}$

Problem-4 : *A load of 20 tons (metric) is to be lifted by means of a hydraulic press. The maximum effort applied at the end of a lever is 25 kg. If diameter of the ram is 150 mm, determine the diameter of plunger. The leverage of the lever is 30: 1. Determine also the number of strokes to be performed in order to lift the load through 500 mm, if the stroke length of the plunger is 250 mm. Disregard the effect of friction.*

Force on the plunger, F = effort $\times$ leverage

$= 25 \times 30 = 750$ kg

Force on the plunger is given by eqn. (1·2) as

$$F = \frac{Wa}{A}$$

$$i.e.\ 750 = \frac{20 \times 1{,}000 \times \frac{\pi}{4}\left(\frac{d}{1{,}000}\right)^2}{\frac{\pi}{4}\left(\frac{D}{1{,}000}\right)^2}$$

(Here 1 metric ton = 1,000 kg and d and D are diameters in mm of plunger and ram respectively)

$$\therefore\ 750 = 20{,}000\left(\frac{d}{D}\right)^2$$

$$\therefore\ \frac{D}{d} = \sqrt{\frac{20{,}000}{750}} = 5{\cdot}15$$

$$\therefore\ d = \frac{D}{5{\cdot}15} = \frac{150}{5{\cdot}15} = \underline{29{\cdot}2\text{ mm}}$$

Using eqn. (1·4), number of strokes is given by

$$n = \frac{AS}{aL} = \left(\frac{D}{d}\right)^2 \times \frac{S}{L}$$

$$= 5{\cdot}15^2 \times \frac{0{\cdot}50}{0{\cdot}25} = 53{\cdot}5 \text{ say } \underline{54} \text{ strokes}$$

Problem – 5 : *In a hydraulic press the diameters of the plunger and the ram are 2·5 cm and 30 cm respectively. The leverage of the handle is 15 to 1. A body weighing 10 tonnes is placed on the ram. The efficiency of the machine is 70%. The stroke of the plunger is 15 cm. The plunger makes 200 strokes while the load is lifted. Determine the effort applied at the end of the lever and the height through which the load is lifted.*

Area of the ram, $A = \frac{\pi}{4}\ 30^2 = 706$ sq cm

Area of the plunger, $a = \frac{\pi}{4}\ 2{\cdot}5^2 = 4{\cdot}9$ sq.cm

$$\therefore\ \frac{A}{a} = \frac{706}{4{\cdot}9} = 144; \text{ and leverage} = 15$$

Velocity ratio, $V = \frac{A}{a} \times \text{leverage} = 144 \times 15 = 2{,}160$

Efficiency, $\eta = \frac{W}{PV}$

$$\text{or} \quad 0{\cdot}7 = \frac{10 \times 1{,}000}{P \times 2{,}160}$$

$$\therefore \quad P = \underline{6{\cdot}61 \text{ kg}}$$

$$\text{Number of strokes}, n = \frac{A}{a} \times \frac{S}{L}$$

$$i.e.\ 200 = 144 \times \frac{S}{15}$$

$$\therefore \quad S = \frac{200 \times 15}{144} = \underline{20{\cdot}8 \text{ cm}}$$

Problem- 6 : *A hydraulic press has a plunger of 1·5 cm diameter and its stroke is 25 cm. The plunger makes 360 strokes in 12 minutes. The press is raising a load of 1 tonne. The force on the plunger is 10 kg. Determine the diameter of the ram and H. P. required to drive the plunger. Neglecting all losses, and assuming the motion of the load continuous, determine the lift of the load.*

$$\text{Since,} \quad \frac{W}{P} = \frac{A}{a}$$

$$\therefore \quad \frac{W}{p} = \left(\frac{D}{d}\right)^2$$

$$\therefore \quad \frac{D}{d} = \sqrt{\frac{W}{p}} = \sqrt{\frac{1000}{10}} = 10$$

$$\therefore \quad D = 10d = 10 \times 1{\cdot}5$$

$$= \underline{15 \text{ cm}}$$

W.D./min. by the plunger = force×stroke length×number of strokes.

$$= 10 \times \frac{25}{100} \times \frac{360}{12} = 75 \text{ kg–m.}$$

$$\therefore \quad \text{H.P. required to drive the plunger} = \frac{75}{4{,}500} = 0{\cdot}0166$$

$$\text{No. of strokes, } n = \frac{AS}{aL}$$

$$\therefore \quad S = \frac{naL}{A} = 360 \times \left(\frac{1{\cdot}5}{15}\right)^2 \times 25 = \underline{90 \text{ cm}}$$

Pressure Head of Liquid

A liquid is subjected to pressure due to its own weight, and this pressure increases as the depth of the liquid increases or vice versa. This statement will be easily understood from the fact that people find it difficult to breathe air at high altitude. This is due to the fact that pressure of air at high altitude is less than that at the sea-level.

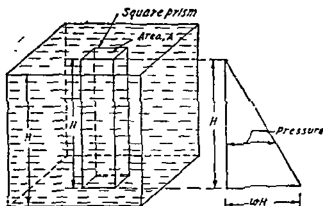


Fig 1-5

Let us consider a vessel containing the liquid of depth H metre as shown in fig. 1-5.

Let, A – area of cross-section of the liquid prism, and
w – weight in kg of 1 cu. m of the liquid.

The height of the square prism considered is also H m. The total pressure of liquid on the bottom surface of the prism is the weight of the liquid contained in that prism.

$\therefore$ total pressure $= wAH$ kg

Let, p be pressure of the liquid (per unit area).

$\therefore$ total pressure $= p \times A$

$\therefore$ $p \times A = wAH$

$\therefore$ $p = wH$ .. (1·5)

$$\text{or} \quad 0.7 = \frac{10 \times 1{,}000}{P \times 2{,}160}$$

$$\therefore \quad P = \underline{6.61 \text{ kg}}$$

$$\text{Number of strokes, } n = \frac{A}{a} \times \frac{S}{L}$$

$$i.e. \; 200 = 144 \times \frac{S}{15}$$

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Problem– 6 : *A hydraulic press has a plunger of 1·5 cm diameter and its stroke is 25 cm. The plunger makes 360 strokes in 12 minutes. The press is raising a load of 1 tonne. The force on the plunger is 10 kg. Determine the diameter of the ram and H. P. required to drive the plunger. Neglecting all losses, and assuming the motion of the load continuous, determine the lift of the load.*

$$\text{Since,} \quad \frac{W}{P} = \frac{A}{a}$$

$$\therefore \quad \frac{W}{p} = \left(\frac{D}{d}\right)^2$$

$$\therefore \quad \frac{D}{d} = \sqrt{\frac{W}{p}} = \sqrt{\frac{1000}{10}} = 10$$

$$\therefore \quad D = 10d = 10 \times 1.5$$

$$= \underline{15 \text{ cm}}$$

W.D /min. by the plunger = force×stroke length×number of strokes.

$$= 10 \times \frac{25}{100} \times \frac{360}{12} = 75 \text{ kg–m.}$$

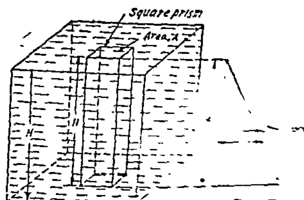
$$\therefore \quad \text{H.P. required to drive the plunger} = \frac{75}{4{,}500} = 0.0166$$

$$\text{No. of strokes, } n = \frac{AS}{aL}$$

$$\therefore \quad S = \frac{naL}{A} = 360 \times \left(\frac{1.5}{15}\right)^2 \times 25 = \underline{90 \text{ cm}}$$

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A liquid is subjected to pressure due to its own weight, and this pressure increases as the depth of the liquid increases or vice versa. This statement will be easily understood from the fact that people find it difficult to breathe air at high altitude. This is due to the fact that pressure of air at high altitude is less than that at the sea-level.



As $p = wH$, the intensity of pressure in a liquid due to its depth will vary directly with depth, since w is constant.

As the pressure at any point in a liquid depends on the free surface above that point, it is sometimes convenient to express the liquid pressure in terms of the height of the free surface which would cause that pressure, *i.e.* from eqn. (1·5),

$$H = \frac{p}{w} \qquad \text{.. (1·6)}$$

The intensity of pressure at any point in the vertical direction is due to the weight of the liquid above that point. The liquid transmits pressure equally in all directions, and hence the pressure in vertical direction causes an equal horizontal pressure on the sides of the vessel. Thus the intensity of pressure on the sides of the vessel at a depth of H metre below the free surface of the liquid is equal to wH; and it will decrease uniformly to zero at the free surface. This is represented by the pressure diagram to the right of fig. 1–5.

If P is the total force of liquid on the side of vessel, then

p = average intensity of pressure × area of the side

$$\text{Average intensity of pressure} = \frac{\begin{pmatrix}\text{intensity of}\\ \text{pressure at top}\end{pmatrix} + \begin{pmatrix}\text{intensity of pre-}\\ \text{ssure at bottom}\end{pmatrix}}{2}$$

$$= \frac{0 + wH}{2} = \frac{wH}{2}$$

$$\therefore \quad p = \frac{wH}{2} \times \text{area of side}$$

Problem – 7 : *A rectangular tank 5·5 m long, 3 m wide and 1·5 m deep is full of water. Find the intensity of pressure on the bottom of the tank and the total force on the side having 3 m width. Determine the depth of water so that the total force on this side is reduced to half the total force on the same surface when the tank is full.*

Intensity of pressure at the base, $p = wH$

$= 1{,}000 \times 1{\cdot}5$

$= 1{,}500$ kg/sq.m

or $0{\cdot}15$ kg/sq cm

Average pressure intensity on the side surface $= \frac{wH}{2}$

$\therefore$ total force on 3 m wide surface, P = average pressure × area

or $P = \frac{1{,}000 \times 1{\cdot}5}{2} \times 3 \times 1{\cdot}5$

$= 3{,}375$ kg

Let y be the depth of water when the force on this surface is reduced to the half of 3,375 kg.

Area of the surface $= 3 \times y$ sq.m.

Average intensity of pressure on this surface is $\frac{wy}{2}$ kg/sq.m

$\therefore$ total force on this wetted surface is

$$P_1 = \frac{wy}{2} \times 3 \times y = 1{\cdot}5\, wy^2 \text{ kg}$$

But, P_1 should be $\frac{1}{2} \times 3{,}375 = 1{,}687{\cdot}5$ kg.

$\therefore$ $1{\cdot}5\, wy^2 = 1{,}687{\cdot}5$

$\therefore$ $y^2 = 1{\cdot}125$

$\therefore$ depth of water, $y = \sqrt{1{\cdot}125}$ m $= 1{\cdot}06$ m

Problem-8 : *A rectangular tank 6 m × 3 m × 2 m deep is full of water. Find the intensity of pressure on the base of the tank and the total force on its smaller vertical side. Determine the depth of uncovered side surface from the top of the tank so that the total force on this is half of total force on the whole surface when the tank was completely filled.*

Intensity of pressure on the base,

$p = wH = 1{,}000 \times 2 = 2{,}000$ kg/sq.m. *i.e.* $0{\cdot}2$ kg/sq cm.

Average pressure intensity on the smaller side surface is

$$p_{avg} = \frac{1,000 \times 2}{2} = 1,000 \text{ kg/sq.m}$$

The total force on the smaller side surface is

$P = p_{avg} \times \text{area} = 1,000 \times (3 \times 2) = 6,000$ kg

Further let y be the uncovered depth of the side surface.

$\therefore$ area of wetted surface $= 3 \times (2 - y)$ sq.m

The average intensity of pressure $= \dfrac{w\,(2-y)}{2}$

$$= \frac{1,000 \times (2-y)}{2} \text{ kg/sq.m}$$

and total force on the wetted surface is

$$P_1 = \frac{1,000\,(2-y)}{2} \times 3 \times (2-y) = 1,500\,(2-y)^2 \text{ kg}$$

But, as P_1 is to be half of P,

$\therefore \quad \frac{1}{2} \times 6,000 = 1,500\,(2-y)^2$

$\therefore \quad 2 - y = \sqrt{2} = 1{\cdot}414$

$\therefore \quad y = 2 - 1{\cdot}414 = \underline{0{\cdot}586 \text{ m}}$

Pressure Gauges

The study of the problems of Hydraulics involves the measurement of the important parameters, pressure being one of them and the correctness of the results depends on the exactness of measuring instruments. There are many devices called pressure gauges or manometers available for this measurement of pressure. A few more common of them are described here.

Piezometer tube : The pressure or head of the liquid in a pipe or vessel can conveniently be measured by use of the piezometer tube (fig. 1–6) which is a simple hollow glass tube open at both ends and curved through a right angle at one end, which is inserted in the vessel. The liquid rises in this tube and the height h (equivalent to the static head of the pressure) of the column of liquid indicates the pressure. The tube can be calibrated to directly read the head.

height of the centre of pipe above liquid surface in the open limb is h'. The surface of contact between the heavy liquid and water is known as the *common surface*. The pressure at $x - x$ in the left limb must be equal to the pressure at $x - x$ in the right limb, *i.e.*

pressure in left limb = height of manometer liquid above atmospheric pressure = sy.

and pressure in right limb = height of water above atmospheric pressure = $y + h' + H$.

Since these pressures are equal,

$$sy = y + h' + H$$

$$\therefore \quad H = \{ y(s-1) - h' \} \qquad \qquad ..(1{\cdot}7)$$

If mercury is used in the manometer, $s = 13{\cdot}6$ and

$$H = (12{\cdot}6y - h') \text{ height of water} \qquad \qquad ..(1{\cdot}7a)$$

If measurements is in cm, head of water in pipe is

$$H = \frac{12{\cdot}6y}{100} - \frac{h'}{100} \text{ metres} \qquad \qquad (1{\cdot}7b)$$

The U-tube manometer can as well be used to measure the vacuum in vessel or pipes such as the suction pipes of the reciprocating and centrifugal pumps. In this case (fig. 1–8) one (left) limb of the U-tube is connected by a rubber tubing to the suction pipe and the other (right) limb is left open to atmosphere. Because the absolute pressure in the pipe is less than atmospheric pressure, the atmospheric pressure acting on the liquid in the right limb forces some liquid to rise in the left limb. Let the equilibrium be considered at level $x - x$.

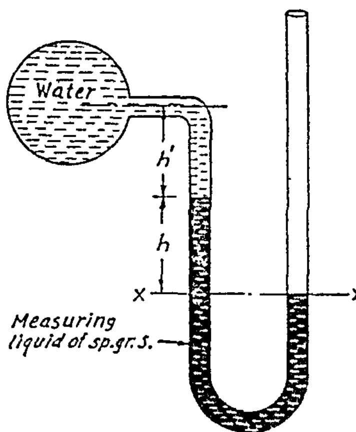


Fig. 1–8 U-tube vacuum gauge containing mercury in the tube

Pressure in right limb = H_{atm} absolute

Pressure in left limb = $(sh + h' + H)$ absolute

Here H is the actual absolute pressure head in the pipe. Equating the two,

$$H_{atm} = sh + h' + H$$

$$\therefore \quad H = H_{atm} - (sh + h') \qquad \ldots (1\cdot8)$$

Since vacuum is the difference between the atmospheric pressure and actual (absolute) pressure inside the pipe,

$$\text{vacuum head, } h_{vac} = H_{atm} - H$$
$$= H_{atm} - \{H_{atm} - (sh + h')\}$$
$$= sh + h' \qquad \ldots (1\cdot9)$$

If incidentally, the pump is at the ground level and the U-tube is at a higher level than the suction pipe, such that the vertical distance between the top of left limb and suction pipe is h'', then by analogy it will be seen that the vacuum head is given by

$$h_{vac} = sh + h' - h'' \qquad \ldots (1\cdot9a)$$

Inverted U-tube manometer: Sometimes it becomes necessary to know the pressure difference over certain length of the pipe instead of the actual pressure at the ends of the length as in case of the venturimeter. This can either be done by taking the tappings from the two ends connecting them by rubber tubing to the two limbs of the U-tube or by the use of the inverted U-tube manometer shown in fig 1-9. The upper part of the U-tube contains air, and water from the two sections of the pipe enters into the

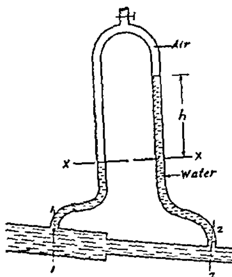


Fig. 1-9 Inverted U-tube manometer

two limbs of the tube. The height of water columns can be adjusted to convenient levels by letting out air through the valve at the top. Since the air trapped in the upper part of the tube is under constant pressure, the difference of pressures between sections 1–1 and 2–2 of the pipe is given by the difference h in the height of the two columns of water. Hence,

$$\text{pressure difference} = h \text{ millimetres of water} \quad .. \ (1\cdot10)$$

Differential gauge: If the pressure difference is very small, the pressure gauge using air does not give very accurate readings. For such purpose, the oil gauge with enlarged ends of the U–tube (fig. 1–10) is used The sensitivity of the gauge is increased by using two liquids whose specific gravity vary by a small margin. In this gauge water and oil are kept in the limbs, the free surface of each liquid being in the enlarged ends.

Let, A – area of enlarged end,
a – area of tube,
s – specific gravity of oil used, and
1 – 1 – common surface when there is equal pressure in the two limbs.

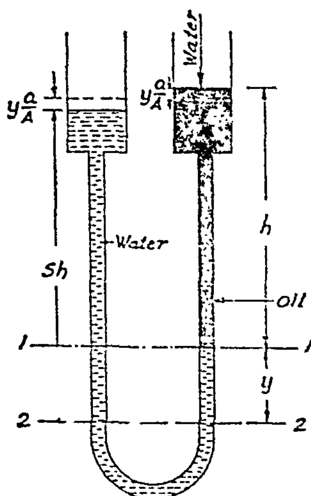


Fig. 1–10 Differential pressure gauge

When the gauge is not connected to any vessel, the two limbs of the ube are subjected to atmospheric pressure and at that time h is the height of column of oil above the common surface. Then, the height of water column above 1 – 1 in the left limb will be sh. If now the oil surface in right limb is subjected to gas pressure (H_m of water), the common surface is lowered by height y, to level 2–2. The level of oil in the enlarged portion will consequently be decreased by $y\left(\frac{a}{A}\right)$ and level of water in the enlarged portion

will go up by the same amount. Let us now equate the pressures in the two limbs at the common surface 2-2.

$$\text{Height of oil above } 2-2 = h + y - y \times \frac{a}{A}$$

$$= h + y\left(1 - \frac{a}{A}\right)$$

$$\text{Height of water above } 2\text{-}2 = sh + y + y \times \frac{a}{A}$$

$$= sh + y\left(1 + \frac{a}{A}\right)$$

Then, as pressures in both limbs at 2-2 are equal,

$$sh + y\left(1 + \frac{a}{A}\right) = s\left[h + y\left(1 - \frac{a}{A}\right)\right] + H$$

$$\text{or } y\left(1 + \frac{a}{A}\right) = sy\left(1 - \frac{a}{A}\right)\Big] + H$$

$$\text{i.e. } H = y\left[1 + \frac{a}{A} - s\left(1 - \frac{a}{A}\right)\right] \quad \text{.. (1·11)}$$

Bourdon tube pressure gauge: This is entirely a mechanical device to record the pressure of fluids which is above the atmospheric.

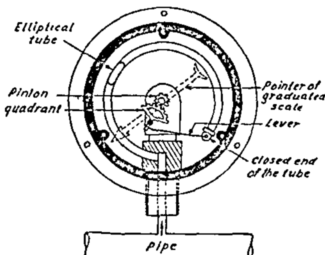


Fig. 1-11 Bourdon pressure gauge

The interior mechanism of the pressure gauge is shown in fig. 1-11. The essential component of the gauge is the elliptical Bourdon

spring tube which is of a special quality of bronze and is solid drawn. Its one end is closed by a plug and the other end is firmly connected to a hollow block which is attached to the casing by means of screw. The fluid whose pressure is to be measured enters the elliptical tube. The pressure of the fluid in the tube makes the cross section more nearly circular and this change of cross-section makes the free end of the tube move outwards. The movement is proportional to the difference between the external and internal pressures on the tube. Since the outside pressure is atmospheric, the movement of the tube is a measure of the pressure of the fluid, and it is above atmospheric pressure. Hence to convert gauge pressure into absolute pressure we have to add atmospheric pressure to the gauge pressure.

The movement of the tube end is magnified by the mechanism which consists of the levers, toothed quadrant and the pinion. The pointer of the graduated scale is attached to the pinion. The pressure gauge is generally constructed to indicate the pressure upto double the working pressure for a given machine so that pointer is vertical at the working pressure.

Problem - 9 : *The left limb of the mercury U–tube manometer is open to atmosphere and the right limb is connected to a pipe carrying water under pressure. The centre of the pipe is at the level of the free surface of mercury. Find the pressure of water in the pipe in absolute value, if the difference of levels of mercury in the limbs is 76 mm. Atmospheric pressure is 10·363 m of water.*

At the common surface, which passes through the surface where mercury and water meet in the right limb, the pressures, above the atmosphere, are equal. Let these be considered in millimetres of water. Then,

$$\text{pressure in left limb} = \text{pressure in right limb}$$

$$\text{or } 76 \times 13{\cdot}6 = H + 76$$

$$\therefore \quad H = (13{\cdot}6 - 1)\, 76 = 957 \text{ mm of water.}$$

$$\text{or the absolute pressure head} = 10{\cdot}363 + \frac{957}{1{,}000} = 11{\cdot}32 \text{ m of water.}$$

Problem-10 : *A mercury U-tube manometer is connected to a tank which stores water under pressure. The outlet point of the tank is 2 m above the free surface of mercury in the U-tube. When a rubber tube connects the tank to the U-tube, the difference in levels of mercury was observed to be 20 cm. Calculate the pressure of water in the tank in kg/sq.cm gauge value. Atmospheric pressure is 10·363 m of water.*

From eqn. (1·7 c), the pressure head of water in the tank is

$$H = \frac{(13 \cdot 6 - 1)\,y}{100} - \frac{h'}{100}$$

$$= \frac{12 \cdot 6 \times 20}{100} - \frac{2 \times 100}{100} = 0 \cdot 52 \text{ m of water}$$

(Here $h' = 2$ m $= 2 \times 100$ cm of water)

Hence pressure, $p = wH = 1{,}000 \times 0 \cdot 52 = 520$ kg/sq. m

$$= \frac{520}{(100)^2} = \underline{0 \cdot 052 \text{ kg/sq. cm}}$$

Problem-11 : *The vacuum head in the suction pipe of a centrifugal pump is measured by the U-tube vacuum gauge. The end of the tube is 1·5 m above the centre of the pipe at which the tapping for water is provided. When water is allowed to come to the U-tube, the difference in mercury levels in the U-tube is 25 cm and there is 10 cm high water column above the mercury column. Determine the vacuum head of the pump*

From eqn. (1·9a),

vacuum head, $h_{vac} = sh + h' - h''$

$$= \frac{13 \cdot 6 \times 25}{100} + \frac{10}{100} - 1 \cdot 5$$

$$= \underline{2 \text{ m of water}}$$

Problem-12 : *A pressure gauge is made up of two cylindrical bulbs of diameter 25 mm and they are connected by a U-tube with vertical limbs of diameter 6 mm. Water and oil of specific gravity of 0·75 are used in it, which are used to measure the pressure of gas*

flowing in a 30 mm diameter main which is connected to the limb containing oil. After the connections are made, the fall in oil level was found to be 50 mm in the U-tube. The original height of oil was 250 mm. Determine the pressure of gas in the main in kg/sq.cm.

Ratio of area of bulb to area of the U tube is

$$\frac{A}{a} = \left(\frac{25}{6}\right)^2 = 17{\cdot}35$$

The pressure in the main is given by eqn. (1·11) as

$$H = y\left[1 + \frac{a}{A} - s\left(1 - \frac{a}{A}\right)\right]$$

$$= \frac{50}{10}\left[1 + \frac{1}{17{\cdot}35} - 0{\cdot}75\left(1 - \frac{1}{17{\cdot}35}\right)\right]$$

$= 1{\cdot}754$ cm of water

$= 0{\cdot}01754$ m of water

Let p be pressure of water, in kg/sq.cm.

$$\therefore \quad p = \frac{wH}{10{,}000}$$

$$= \frac{1{,}000 \times 0{\cdot}01754}{1{,}0000}$$

$= \underline{0{\cdot}001754 \text{ kg/sq. cm.}}$

Resultant Thrust and Centre of Pressure of Immersed Surfaces

(a) Plane vertically immersed surface: A vertical surface immersed below the free surface of water is shown in fig. 1–12.

Let A – area of vertically immersed surface,

$\bar{x}$ – depth of the centre of gravity of the immersed surface,

$\bar{h}$ – depth of centre of pressure,

P – total pressure (force) on the immersed surface,

G – centroid (centre of gravity) of the immersed surface,

I_0 – moment of inertia of the immersed surface about an axis of intersection.

Now consider an elementary strip of width dx and length Z, at a depth x below the free surface of water.

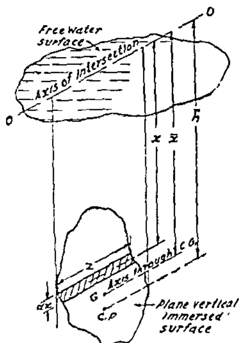


Fig. 1-12

Area of the elementary strip is

$$da = Z\,dx$$

Intensity of pressure of water on the elementary area is,

$$p = wx$$

This will be uniform and equal over the elementary area.

Total pressure on the elementary area is

dP = area of the strip × intensity of pressure

$= da \times p$

$= da \times wx \qquad \ldots \ldots \ldots \ldots \ldots \ldots (1)$

The total pressure on the immersed surface can be obtained by integrating the expression (1) above.

i.e. $\int dP = \int w\, da \cdot x$

$\therefore \quad P = w \int da \cdot x$

Now, according to the condition of equilibrium of a static body, the moment of a resultant force should be equal to the resultant moment of all the forces about the same point.

$$\therefore \quad wI_0 = P \cdot \bar{h}$$

$$\therefore \quad \bar{h} = \frac{wI_0}{P} \qquad \ldots \quad \ldots \quad \ldots \text{(iv)}$$

But, $I_0 = I_{GG} + A(\bar{x})^2$

$$\therefore \quad \bar{h} = \frac{w(I_{GG} + A(\bar{x})^2)}{P}$$

But, $P = wA\bar{x}$

$$\therefore \quad \bar{h} = \frac{w(I_{GG} + A\bar{x})^2}{wA\bar{x}} = \frac{I_{GG}}{A\bar{x}} + \bar{x} \qquad \ldots (1\cdot 13)$$

(b) Plane immersed inclined surface : Such a surface is shown in fig. 1-13.

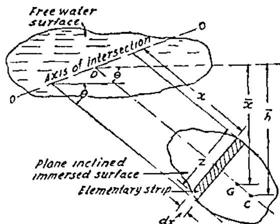


Fig. 1-13

Let, A – area of immersed inclined surface,

x – vertical depth of c g. from the free surface of water,

$\bar{h}$ - vertical depth of the centre of pressure for the immersed surface,

P – total pressure on the immersed surface,

G – centroid (centre of gravity) of the immersed surface,
C – centre of pressure of the immersed surface,
I_o – moment of inertia of the surface about an axis of intersection, 0–0, and
θ – angle of inclination of the immersed surface.

Consider an elementary strip of width dx and length Z on the immersed surface at a distance x which is measured at an inclination θ with the free surface of the water level. The vertical depth of the strip below the free surface of water will be $x\sin\theta$ and hence the intensity of pressure of water over the whole elementary strip will be $wx\sin\theta$.

The total pressure on the elementary strip is

$$dP = \text{area of the strip} \times \text{intensity of pressure}$$
$$= Z\,dx \times w \cdot x \sin\theta$$
$$= da \times w \cdot x \sin\theta \qquad \text{where } da = Zdx.$$

The total pressure on the immersed inclined surface is

$$\int_0^P dP = w \sin\theta \int da \cdot x$$

Now $\int da \cdot x$ is the first moment of the area about D.

$$\therefore \quad \int da \cdot x = \frac{A\bar{x}}{\sin\theta}$$

$$\therefore \quad P = w \sin\theta \times \frac{A\bar{x}}{\sin\theta}$$
$$= wA\bar{x} \qquad \ldots \quad \ldots \quad \ldots (v)$$

Further, to determine the centre of pressure we take the moments about D.

Moment of the force on the strip is

$$dM = dPx$$
$$= w da x \sin\theta \, x$$

Hence, the resultant moment which is the sum of moments of forces on the strip is

$$M = w \sin\theta \int da \cdot x^2$$

Let I_D be moment of inertia of the surface about D.

Then, $I_D = \int da \cdot x^2$

$\therefore$ total resultant moment $= w \sin\theta \cdot I_D$

Now, this will be equal to the moment of the resultant force which is equal to $\frac{P\bar{h}}{\sin\theta}$.

$$\therefore \quad \frac{P\bar{h}}{\sin\theta} = w \cdot \sin\theta \cdot I_D$$

$$\therefore \quad \bar{h} = \frac{w I_D \sin^2\theta}{P}$$

But, $P = wA\bar{x}$

$$\therefore \quad \bar{h} = \frac{I_D \sin^2\theta}{A\bar{x}}$$

$$\text{and } I_D = I_{GG} + \frac{A(\bar{x})^2}{\sin^2\theta}$$

$$\therefore \quad \bar{h} = \frac{I_{GG}}{A\bar{x}} \sin^2\theta + \bar{x} \qquad \text{.. (1·14)}$$

Moment of Inertia of Plane Surfaces

In finding the total pressure and centre of pressure, we always need to know the position of c. g. (G) of the plane immersed surface and I_{GG}, the moment of inertia of the plane immersed surface about a horizontal axis passing through the c. g. and lying in the plane of the immersed surface. So it will be advisable to know these particulars for certain known common sections, which are given below :

(a) Rectangular
(b) Parallelogramic
(c) Circular
(d) Semi-circular
(e) Triangular
(f) Trapezoidal.

It is, however, advisable to know the general method for arriving at the position of the c. g. (G) and the magnitude of I_{GG} of

plane area of any shape. The I_{GG} and the position of G of the above-said common plane surfaces are given below :

(a) *Rectangular plane surface :*

$I_{GG} = \frac{1}{12} BD^3$, where B is the breadth of the surface parallel to the axis about which I_{GG} is taken and D is the depth of surface at right angle to the axis.

$\bar{x} = \frac{D}{2}$, where it $\bar{x}$ the perpendicular distance of G from the edge which is breadth of the surface parallel to the axis.

(b) *Parallelogramic plane surface :*

$I_{GG} = \frac{1}{12} BD^3$, where B is the length of the side parallel to the axis about which I_{GG} is taken and D is the perpendicular distance between the sides parallel to the above axis.

$\bar{x} = \frac{D}{2}$, where $\bar{x}$ is the perpendicular distance from the side parallel to the above axis.

(c) *Circular plane surface :*

$I_{GG} = \frac{\pi}{64} D^4$, where D is the diameter of the plane circular section.

$\bar{x} = \frac{D}{2}$, thus it may be noted that the c. g. is at the centre of the circle.

(d) *Semi-circular plane surface :*

$I_{GG} = 0{\cdot}11r^4$, where 2 r is the length of the horizontal base of the semi-circular area, so that $r = \frac{D}{2}$ is the height of the section.

$\bar{x} = 0{\cdot}424r$. The c. g. lies on a line which is the perpendicular bisector of the base at a distance $\bar{x}$ from the base.

plane area of any shape. The I_{GG} and the position of G of the above-said common plane surfaces are given below :

(a) *Rectangular plane surface :*

$I_{GG} = \frac{1}{12} BD^3$, where B is the breadth of the surface parallel to the axis about which I_{GG} is taken and D is the depth of surface at right angle to the axis.

$\bar{x} = \frac{D}{2}$, where it $\bar{x}$ the perpendicular distance of G from the edge which is breadth of the surface parallel to the axis.

(b) *Parallelogramic plane surface :*

$I_{GG} = \frac{1}{12} BD^3$, where B is the length of the side parallel to the axis about which I_{GG} is taken and D is the perpendicular distance between the sides parallel to the above axis.

$\bar{x} = \frac{D}{2}$, where $\bar{x}$ is the perpendicular distance from the side parallel to the above axis.

(c) *Circular plane surface :*

$I_{GG} = \frac{\pi}{64} D^4$, where D is the diameter of the plane circular section.

$\bar{x} = \frac{D}{2}$, thus it may be noted that the c. g. is at the centre of the circle.

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$I_{GG} = 0{\cdot}11r^4$, where 2 r is the length of the horizontal base of the semi-circular area, so that $r = \frac{D}{2}$ is the height of the section.

$\bar{x} = 0{\cdot}424r$. The c. g. lies on a line which is the perpendicular bisector of the base at a distance $\bar{x}$ from the base.

(e) *Triangular plane surface* :

$I_{GG} = \frac{BD^3}{36}$ where B is the base of the triangle parallel to the axis about which I_{GG} is to be taken and D is the height of the triangle at right angle to the base.

$\bar{x} = \frac{2D}{3}$ from the vertex or $\bar{x} = \frac{D}{3}$ from the base parallel to the axis.

(f) *Trapezoidal plane surface* :

Let 'a' and 'b' be the length of the parallel sides which are parallel to axis about which I_{GG} is to taken, and 'd' be the perpendicular distance between the above parallel sides. Let side 'a' be on the top and let it be less than side 'b'.

$$I_{GG} = \left(\frac{a^2 + 4ab + b^2}{36\,(a+b)}\right) \cdot d^3$$

and $\bar{x} = \left(\frac{2a+b}{a+b}\right) \times \frac{d}{3}$ so that c. g. lies on a line which is the perpendicular bisector of the parallel sides.

Problem-13 . *A circular plate 3 m diameter is placed vertically in water so that the centre of the plate is 2 m below the free surface of water. Find the depth of the centre of pressure and the total pressure on the plate*

The moment of inertia of the plate about its neutral axis is

$$I_{GG} = \frac{\pi D^4}{64} = \frac{\pi \times 3^4}{4} = 3{\cdot}98 \text{ m}^4$$

Area of the plate, $A = \frac{\pi \times 3^2}{4} = 7{\cdot}06$ sq.m

The depth of centre of plate below water surface is

$$\bar{x} = 2 \text{ m}$$

Now, $\bar{h} = \frac{I_{GG}}{A\bar{x}} + \bar{x} = \frac{3{\cdot}98}{7{\cdot}06 \times 2} + 2 = 2{\cdot}282$ m

The total force on the plate is

$P = wA\bar{x} = 1{,}000 \times 7{\cdot}06 \times 2 = \underline{14{,}120 \text{ kg}}$

Problem-14 : *A right angled triangular plate having base width of 3 m and height 3 m is immersed vertically in water as shown in fig. 1-14. Determine the total thrust and depth of the centre of pressure, if the base of the plate lies 1 m below the free surface of water and parallel to it.*

Area of the plate is

$A = \frac{1}{2} \times 3 \times 3 = 4{\cdot}5$ sq.m

and $\overline{x} = 1 + 3/3 = 2$ m

Total thrust, $P = wA\overline{x}$

$= 1{,}000 \times 4{\cdot}5 \times 2$

$= 9{,}000$ kg

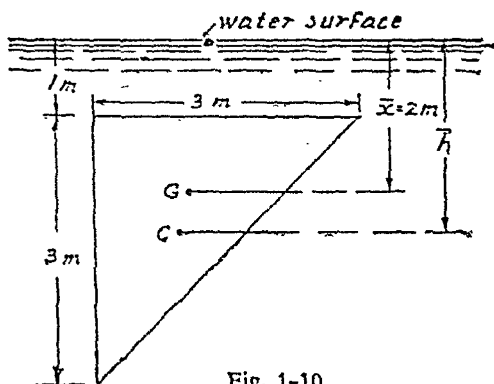


Fig. 1-10

Moment of inertia,

$$I_{GG} = \frac{bh^3}{36} = \frac{3 \times 3^3}{36} = 2{\cdot}25 \text{ m}^4$$

$$\text{Hence, } \overline{h} = \frac{I_{GG}}{A\overline{x}} + \overline{x} = \frac{2{\cdot}25}{4{\cdot}5 \times 2} + 2 = 2{\cdot}25 \text{ m}$$

Problem-15 : *A rectangular opening in a vertical reservoir wall is 1 m wide and 2 m deep and its top edge is 6 m below the free surface of water in the reservoir. If a rectangular gate closes this opening, find the total pressure on the gate and its point of application.*

Area of the gate, $A = 1 \times 2 = 2$ sq.m

C.G. of the gate below the water level, $\overline{x} = 6 + 2/2 = 7$ m

Total pressure of water on the gate is given by

$$P = wA\overline{x} = 1{,}000 \times 2 \times 7 = 14{,}000 \text{ kg}$$

The moment of inertia of the gate about an axis through its C.G is

$$I_{GG} = \tfrac{1}{12} BD^3 = \tfrac{1}{12} \times 1 \times 2^3 = 2/3$$

The depth of centre of pressure is given by

$$\overline{h} = \frac{I_{GG}}{A\overline{x}} + \overline{x} = \frac{2}{3 \times 2 \times 7} + 7 = 7{\cdot}05 \text{ m}$$

Problem-16 : *A rectangular plate is submerged vertically in water with its upper edge parallel to, and at a depth d, below the free surface of water. The lower edge is at a distance D, from the upper edge. Prove that the depth of centre of pressure is given by the expression,*

$$\frac{\frac{2}{3}(D^2 + 3dD + 4d^2)}{2d + D}$$

Let S be the length of the side parallel to the water surface.

$$I_{GG} = \frac{1}{12}SD^3;\ \bar{x} = d + \frac{D}{2};\ \text{area, } A = S \times D$$

$$\text{Now, } \bar{h} = \frac{I_{GG}}{A\bar{x}} + \bar{x} = \frac{\frac{1}{12}SD^3}{S \times D\left(d + \frac{D}{2}\right)} + \left(d + \frac{D}{2}\right)$$

$$= \frac{\frac{1}{12}D^2}{\left(\frac{2d + D}{2}\right)} + \frac{(2d + D)}{2}$$

$$= \frac{\frac{1}{12}D^2 + \left(\frac{4d^2 + 4dD + D^2}{4}\right)}{\left(\frac{2d + D}{2}\right)}$$

$$= \frac{\frac{1}{3}D^2 + dD + d^2}{\frac{2d + D}{2}}$$

$$= \frac{\frac{2}{3}(D^2 + 3dD + 3d^2)}{2d + D}$$

Problem-17 : *A vertical sided 1·5 m high tank is square in plan whose each side is 1 m long. The tank contains oil of sp. gr. 0·8 to a depth of 50 cm floating on 1 m depth of water. Determine, (a) the total pressure on one side of the tank, and (b) the height of the centre of pressure above the base.*

Density of the oil, w_1 = 0·8 × 1,000 = 800 kg/cu.m.

Problem-14 : *A right angled triangular plate having base width of 3 m and height 3 m is immersed vertically in water as shown in fig. 1-14. Determine the total thrust and depth of the centre of pressure, if the base of the plate lies 1 m below the free surface of water and parallel to it.*

Area of the plate is

$A = \frac{1}{2} \times 3 \times 3 = 4{\cdot}5$ sq.m

and $\overline{x} = 1 + 3/3 = 2$ m

Total thrust, $P = wA\overline{x}$

$= 1{,}000 \times 4{\cdot}5 \times 2$

$= \underline{9{,}000 \text{ kg}}$

Moment of inertia,

$I_{GG} = \frac{bh^3}{36} = \frac{3 \times 3^3}{36} = 2{\cdot}25 \text{ m}^4$

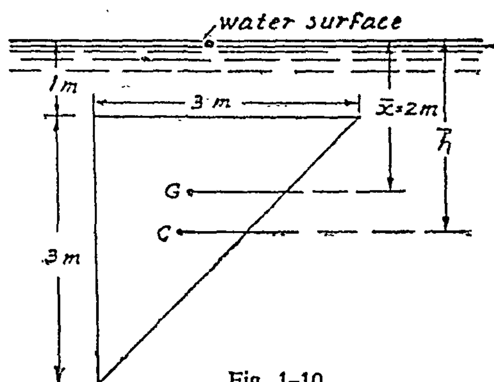


Fig. 1-10

Hence, $\overline{h} = \frac{I_{GG}}{A\overline{x}} + \overline{x} = \frac{2{\cdot}25}{4{\cdot}5 \times 2} + 2 = 2{\cdot}25$ m

Problem-15 : *A rectangular opening in a vertical reservoir wall is 1 m wide and 2 m deep and its top edge is 6 m below the free surface of water in the reservoir. If a rectangular gate closes this opening, find the total pressure on the gate and its point of application.*

Area of the gate, $A = 1 \times 2 = 2$ sq.m

C.G. of the gate below the water level, $\overline{x} = 6 + 2/2 = 7$ m

Total pressure of water on the gate is given by

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The moment of inertia of the gate about an axis through its C.G is

$$I_{GG} = \frac{1}{12} BD^3 = \frac{1}{12} \times 1 \times 2^3 = 2/3$$

The depth of centre of pressure is given by

$$\overline{h} = \frac{I_{GG}}{A\overline{x}} + \overline{x} = \frac{2}{3 \times 2 \times 7} + 7 = \underline{7{\cdot}05 \text{ m}}$$

Problem-16 : *A rectangular plate is submerged vertically in water with its upper edge parallel to, and at a depth d, below the free surface of water. The lower edge is at a distance D, from the upper edge. Prove that the depth of centre of pressure is given by the expression,*

$$\frac{\frac{2}{3}(D^2 + 3dD + 4d^2)}{2d + D}$$

Let S be the length of the side parallel to the water surface.

$$I_{GG} = \frac{1}{12} SD^3, \ \bar{x} = d + \frac{D}{2}; \text{ area, } A = S \times D$$

$$\text{Now, } \bar{h} = \frac{I_{GG}}{A\bar{x}} + \bar{x} = \frac{\frac{1}{12} SD^3}{S \times D\left(d + \frac{D}{2}\right)} + \left(d + \frac{D}{2}\right)$$

$$= \frac{\frac{1}{12} D^2}{\left(\frac{2d + D}{2}\right)} + \frac{(2d + D)}{2}$$

$$= \frac{\frac{1}{12} D^2 + \left(\frac{4d^2 + 4dD + D^2}{4}\right)}{\left(\frac{2d + D}{2}\right)}$$

$$= \frac{\frac{1}{3} D^2 + dD + d^2}{\frac{2d + D}{2}}$$

$$= \frac{\frac{2}{3}(D^2 + 3dD + 3d^2)}{2d + D}$$

Problem-17 : *A vertical sided 1·5 m high tank is square in plan whose each side is 1 m long. The tank contains oil of sp. gr. 0 8 to a depth of 50 cm floating on 1 m depth of water. Determine, (a) the total pressure on one side of the tank, and (b) the height of the centre of pressure above the base.*

Density of the oil, $w_1 = 0{\cdot}8 \times 1{,}000 = 800$ kg/cu.m.

The pressure distribution due to two liquids will be as shown in fig. 1-15.

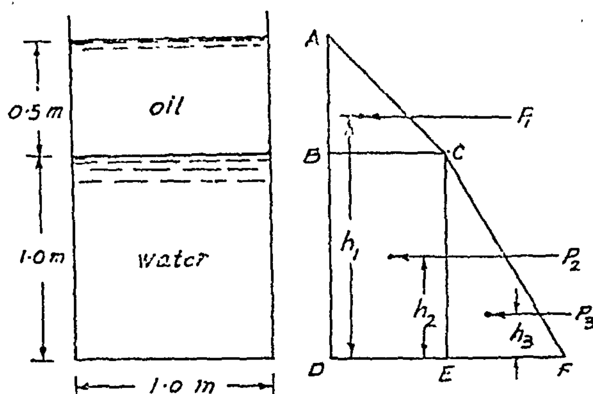


Fig. 1-15

The intensity of pressure due to oil at common surface is shown by BC. It is $p_1 = 800 \times 0{\cdot}5 = 400$ kg/sq. m.

Pressure intensity near the base is due to oil and water which is equal to DE + EF where DE = p_2 = 400 kg/sq. m. and EF = p_3 = 1,000 kg/sq. m.

$$\therefore \quad P_1 = \frac{p_1}{2} \times B \times H_1 = \frac{400}{2} \times 1 \times 0{\cdot}5 = 100 \text{ kg}$$

The constant oil pressure p_2 = 400 kg/sq. cm acts on the lower 1 m height.

and $P_2 = p_2 \times B \times H_2 = 400 \times 1 \times 1 = 400$ kg

The total pressure due to water,

$$P_3 = \frac{p_3}{2} \times B \times H_3$$

$$= \frac{1000}{2} \; 1 \times 1$$

$$= 500 \text{ kg.}$$

(a) The total pressure on one side of the tank is

$$P = P_1 + P_2 + P_3$$

$$= 100 + 400 + 500 = 1{,}000 \text{ kg}$$

(b) Now, the heights of points of action of these forces are

$h_1 = 1 + \frac{1}{3} \times 0.5 = 1.166$ m, $h_2 = \frac{1}{2} = 0.5$ m,

$h_3 = \frac{1}{3} \times 1 = 0.333$ m

Taking moments about the base, we have,

$$P\bar{h} = P_1h_1 + P_2h_2 + P_3h_3$$

$$\therefore \bar{h} = \frac{P_1h_1 + P_2h_2 + P_3h_3}{P_1 + P_2 + P_3} = \frac{(100 \times 1.166) + (400 \times 0.5) + (500 \times 0.333)}{1,000}$$

$$= 0.4831 \text{ m}$$

Problem - 18 : *A vertical wall 120 cm thick and 6 m high supports water on one side to a height of h m from bottom. The resultant force cuts the base of the wall at a distance of 20 cm from the middle of its thickness as shown in fig. 1-16. If the masonry weighs 1,900 kg/cu. m, determine the height h.*

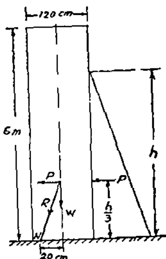


Fig. 1-16

Let P, W and R be the total pressure of water per metre length of wall, weight of wall per metre run and resultant of the water pressure and weight of wall respectively.

Wetted area of the wall,

$$A = h \times 1 = h \text{ sq. m}$$

Distance of C. G of the wetted area below the free surface of water is $\bar{x} = \frac{h}{2}$.

Total pressure of water,

$$P = wA\bar{x}$$

$$= 1,000 \times h \times \frac{h}{2} = 500h^2 \text{ kg}$$

This acts at a distance $\frac{h}{3}$ from the base.

The total weight of the wall, $W = \rho \times b \times d \times 1$

$$= 1,900 \times \frac{120}{100} \times 6 \times 1$$

$$= 13,680 \text{ kg}$$

Taking moments of the forces about N, the point where the resultant force cuts the base,

$$P \times \frac{h}{3} = W \times \frac{20}{100}$$

$$i.e. \quad 500h^2 \times \frac{h}{3} = 13{,}680 \times 0{\cdot}2$$

$$\therefore \quad h^3 = 16{\cdot}4$$

$$\therefore \quad h = 2{\cdot}54 \text{ m}$$

Problem-19 : *A hollow triangular box, the ends of which are equilateral triangles of 1 m sides, is submerged in water in such a way that one of its rectangular faces lies in the surface of water. Find the net total pressure and the position of the centre of pressure on one of the triangular ends, (a) when the inside of the box is at atmospheric pressure, and (b) when the inside of the box is at a pressure of 1 kg/sq.cm above the atmospheric pressure.*

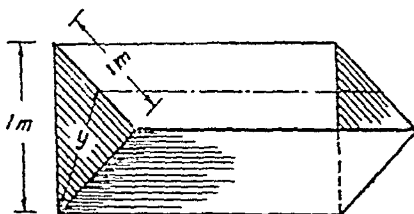


Fig. 1-17

(a) Let y be height of the altitude in the triangular end.

Then, $y = 1 \times \sin 60° = 1 \times 0{\cdot}866 = 0{\cdot}866$ m

Area of the end, $A = \frac{1}{2} \times 1 \times 0{\cdot}866 = 0{\cdot}433$ sq.m

and $\bar{x} = \frac{0{\cdot}866}{3} = 0{\cdot}288$ m

Total pressure, $P = wA\bar{x} = 1{,}000 \times 0{\cdot}433 \times 0{\cdot}288 = 125$ kg

and height of centre of pressure, $\bar{h} = \frac{I_{GG}}{A\bar{x}} + \bar{x}$

$$= \frac{\frac{1 \times (0{\cdot}866)^3}{36}}{0{\cdot}433 \times 0{\cdot}288} + 0{\cdot}288 = 0{\cdot}433 \text{ m}$$

(b) Pressure due to water on the outside of the box is 125 kg acting at a distance of 0·433 m below the free surface of water. Total pressure due to inside pressure which is constant throughout is Q and it will act at the C. G. of the trianglular end.

$$\therefore \quad Q = \text{area} \times \text{intensity of pressure}$$
$$= 0{\cdot}433 \times 1 \times 10{,}000 = 4{,}330 \text{ kg}$$

This will act at a vertical distance $\overline{y} = \frac{1}{3}$ m from the surface lying in water level.

$\therefore$ net total pressure, $R = 4{,}330 - 125 = 4{,}205$ kg

Taking moments about water surface,

$$Q \times \overline{y} + P\overline{h}_1 = R \times \overline{h}$$

$$\therefore \quad 4{,}330 \times \tfrac{1}{3} + 125 \times 0{\cdot}433 = 4{,}205 \times \overline{h}$$

$$\therefore \quad \overline{h} = \underline{0{\cdot}281 \text{ m}}$$

Problem-20. *A dock gate is to be reinforced with three equal horizontal beams. If water acts on one side only to a depth of 7 m, find the positions of the beams, measured below water level so that each beam will carry equal load and determine the load on the beam per metre run.*

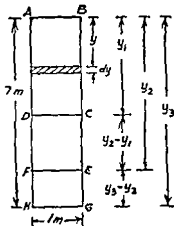


Fig. 1-18

As the beams have to carry equal load, the whole height of the gate is divided in 3 parts so that the total pressure on each part is the same and the beams will then be placed at the centre of pressure of each portion. The portions will become smaller as we go deeper below the free surface of water.

Consider an elementary strip of thickness dy of the gate 1 metre wide (fig. 1-18) at a depth y below the free surface of

water. Area of the strip is $dy \times 1 = dy$ sq. m and c. g. of the strip is at a depth y. The thrust on this element is $wdy \cdot y$ kg. Hence the total thrust on area ABCD is

$$P_1 = \int_0^{y_1} wdy \cdot y = \tfrac{1}{2} wy_1^2$$

Similarly, thrust on area DCFE is

$$P_2 = \int_{y_1}^{y_2} wdy \cdot y = \tfrac{1}{2} w\,(y_2^2 - y_1^2)$$

and thrust on area FEGH is

$$P_3 = \int_{y_2}^{y_3} wdy \cdot y = \tfrac{1}{2}w\,(y_3^2 - y_2^2) = \tfrac{1}{2}w\,(7^2 - y_2^2)$$

As the load on each portion is to be the same,

$$P_1 = P_2 = P_3$$

or $$\frac{wy_1^2}{2} = \frac{w}{2}(y_2^2 - y_1^2) = \frac{w}{2}(7^2 - y_2^2)$$

Solving this equation, we have

$$y_1 = 4{\cdot}05 \text{ m and } y_2 = 5{\cdot}7 \text{ m}$$

Now we proceed to find the position of the centre of pressure for each section. For this,

moment of thrust on element about the water surface $= wy^2dy$ and moment of the thrust on rectangle ABCD about water surface is

$$w \int_0^{y_1} y^2 dy = \tfrac{1}{3} wy_1^3,$$

and $$\bar{h}_1 = \frac{\frac{wy_1^3}{3}}{\frac{wy_1^2}{2}} = \frac{2y_1}{3} = \frac{2 \times 4{\cdot}05}{3} = \underline{2{\cdot}7 \text{ m}}$$

Similarly,

$$\bar{h}_2 = \frac{\frac{w}{3}(y_2^3 - y_1^3)}{\frac{1}{2}w\,(y_2^2 - y_1^2)} = \frac{2\,(5{\cdot}7^3 - 4{\cdot}05^3)}{3\,(5{\cdot}7^2 - 4{\cdot}05^2)} = \underline{4{\cdot}95 \text{ m}}$$

and $$\bar{h}_3 = \frac{2}{3}\frac{(7^3 - 5{\cdot}7^3)}{(7^2 - 5{\cdot}7^2)} = \underline{6{\cdot}4 \text{ m}}$$

Thrust on each beam = $wA\bar{x}$

$$= 1{,}000 \times 1 \times 4{\cdot}05 \times \frac{4{\cdot}05}{2} = 8{,}200 \text{ kg}$$

Problem-21 : *A circular opening A of 1 m diameter size in the vertical wall of a reservoir is closed by a disc valve B. The disc is hinged at point C and a balance weight W is just sufficient to hold the valve closed (against wind pressure) when the reservoir is empty. How much additional weight should be placed on the arm 1·1 m long from the hinge in order that valve shall remain closed until the water level is 1 m above the centre of the valve.*

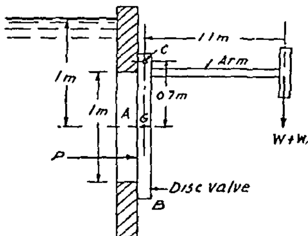


Fig. 1-19

Force on the gate, $P = wA\bar{x}$

where, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (1)^2 = \frac{\pi}{4}$ sq. m and $\bar{x} = 1$ m

$\therefore \quad P = 1{,}000 \times \frac{\pi}{4} (1)^2 \times 1 = 784$ kg

The height of centre of pressure is

$$\bar{h} = \frac{I_{GG}}{A\bar{x}} + \bar{x} = \frac{\frac{\pi}{64} (1)^4}{\frac{\pi}{4} (1)^2 \times 1} + 1 = 0{\cdot}0625 + 1 = 1{\cdot}0625 \text{ m}$$

The distance of the centre of pressure from the hinge is

$$(1.0625 - 1) + 0.7 = 0.7625 \text{ m}$$

Taking moments about the hinge, we have

$$P \times 0.7625 = W_1 \times 1.1$$

$$\therefore \text{ additional weight, } W_1 = \frac{P \times 0.7625}{1.1}$$

$$= \frac{784 \times 0.7625}{1.1} = \underline{535 \text{ kg}}$$

Problem - 22 : *Find (a) the total pressure and (b) the position of the centre of pressure, on one side of the immersed rectangular plate, 2 m long and 1 m wide, when the plane of the plate makes an angle of 30° with the surface of the water level and 1 m edge of the plate is parallel to and at a depth of 1.5 m below the free surface of water level.*

Area of the plate = 2 ×1 = 2 sq.m.

Depth of C. G. of the plate from free surface of water level is, $\bar{x} = 1.5 + \frac{2 \times \sin 30^\circ}{2} = 1.5 + 0.5 = 2 \text{ m}$

∴ (a) Total pressure on the plate, $P = WA\bar{x}$

$$= 1{,}000 \times 2 \times 2 = \underline{4{,}000 \text{ kg}}$$

Depth of centre of pressure below the free surface of water level is, $h = \frac{I_{GG} \times \sin^2\theta}{A\bar{x}} + \bar{x}$

$$= \frac{\left\{\frac{1}{12} \times 1 \times (2)^3\right\} \times \sin^2 30^\circ}{2 \times 2} + 2 = \frac{8 \times (0.5)^2}{12 \times 4} + 2 = \underline{2.041 \text{m}}$$

Problem - 23 : *A circular plate 3 m diameter is immersed in water, its greatest and least depths below the surface being 2 m and 1 m respectively ; find (a) the total pressure on one face of the plate ; and (b) the position of the centre of pressure.*

Area of the plate, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (3)^2 = \frac{9\pi}{4}$ sq.m

Inclination of the plate with the water surface,

$$\sin\theta = \frac{\text{greatest depth} - \text{least depth}}{\text{diameter of the plate}} = \frac{2-1}{3} = \frac{1}{3}$$

Depth of C. G. of the plate below the free surface of water level, $\bar{x}$ = least depth + $\sin\theta \times \frac{\text{diameter}}{2}$

$$= 1 + \frac{1}{3} \times \frac{3}{2} = 1{\cdot}5 \text{ m}$$

Total pressure on the plate, $P = WA\bar{x}$

$$= 1{,}000 \times \frac{9\pi}{4} \times 1{\cdot}5 = \underline{10{,}600 \text{ kg}}$$

Depth of centre of pressure below the free surface of water level is, $\bar{h} = \frac{I_{GG} \sin^2\theta}{A\bar{x}} + \bar{x}$

$$= \frac{\frac{\pi}{64}(3)^4 \times \left(\frac{1}{3}\right)^2}{\frac{9\pi}{4} \times 1{\cdot}5} + 1{\cdot}5 = \underline{1{\cdot}542 \text{ m}}$$

Problem - 24 : *A tank containing water has one side in the form of a plane wall inclined at 45° to the water surface. In this wall there is a door, 1 m wide and 2 m long hinged along its 1 m long horizontal top edge which is at a vertical depth of 1·2 m below the free surface of water level in the tank. Determine the magnitude of the horizontal force applied at the bottom edge of the door to keep it closed.*

Inclination of door, $\theta = 45°$

Area of the plate = 1 × 2 = 2 sq m

Depth of C. G, $\bar{x} = 1{\cdot}2 + \frac{2\sin 45°}{2}$

$$= 1{\cdot}2 + 0{\cdot}707 = 1{\cdot}907 \text{ m}$$

Total pressure, $P = wA\bar{x} = 1{,}000 \times 2 \times 1{\cdot}907 = 3{,}814$ kg

Depth of C. P. below the water level, $h = \frac{I_{GG} \sin^2\theta}{A\bar{x}} + \bar{x}$

$$\therefore \bar{h} = \frac{\left\{\frac{1}{12} \times 1 \times (2)^3\right\} \times (0{\cdot}707)^2}{2 \times 1{\cdot}907} + 1{\cdot}907 = 1{\cdot}996 \text{ m}$$

Distance of C.P. from the hinged edge is say $Z = \frac{\overline{h}}{\sin 45^\circ} - \frac{1.2}{\sin 45^\circ}$

$$\therefore \quad Z = \frac{\overline{h} - 1.2}{\sin 45^\circ} = \frac{1.996 - 1.2}{\sin 45^\circ}$$

Taking moment about the upper hinge of door,

$$F \times 2 \sin 45^\circ = P \times Z$$

$$\therefore \quad F = \frac{P \times Z}{2 \sin 45^\circ} = \frac{3{,}814 \times (1.996 - 1.2)}{\sin 45^\circ \times 2 \times \sin 45^\circ}$$

$$= \frac{3{,}814 \times 0.796}{0.707 \times 2 \times 0.707} = \underline{3{,}060 \text{ kg}}$$

Problem-25 : *A rectangular gate 3 m × 2 m size is hinged at the base and is inclined at an angle of 60° with the horizontal. The upper end of the gate is kept in position by a weight of 4,000 kg acting at an angle of 90° as shown in fig. 1–20. Find the level of water when the gate begins to fall. The weight of the gate is 100 kg.*

Let x m be the height of water from the bottom.

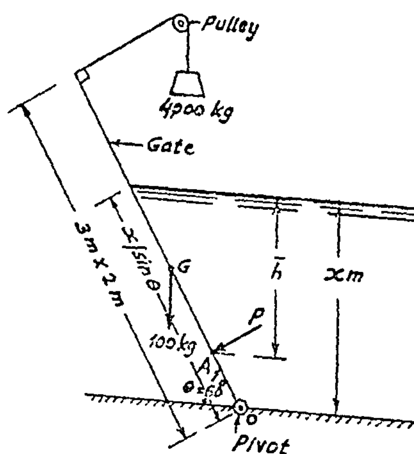


Fig. 1–20

Total pressure on the gate, $P = wA\overline{x}$

A is the area of wetted surface of the gate.

$$\therefore \quad A = \frac{x}{\sin\theta} \times 2 = \frac{2x}{\sin 60^\circ} \text{ sq.m}$$

$$\bar{x} = \frac{x}{2}$$

$$\therefore \quad P = 1{,}000 \times \frac{2x}{\sin 60^\circ} \times \frac{x}{2} = \frac{x^2 \times 1{,}000}{0{\cdot}866}$$

Position of centre of pressure, $\bar{h} = \dfrac{I_{GG}\sin^2\theta}{A\bar{x}} + \bar{x}$

$$\therefore \quad \bar{h} = \frac{\frac{1}{12} \times 2\left(\frac{x}{\sin\theta}\right)^3 \times (0{\cdot}866)^2}{2 \times \left(\frac{x}{\sin\theta}\right) \times \frac{x}{2}} + \frac{x}{2}$$

$$= \frac{x}{6} + \frac{x}{2} = \frac{2x}{3}$$

Distance of C.P. from the hinge, $OA = \dfrac{x - \dfrac{2x}{3}}{\sin 60} = \dfrac{x}{3 \times 0{\cdot}866} = 0{\cdot}385x$

Taking moments about the hinge 0, we have

$$\frac{1{,}000\, x^2}{0{\cdot}866} \times 0{\cdot}385x + 100 \times \text{OG} \cos\theta = 4{,}000 \times 3$$

$$440\, x^3 + 100 \times 1{\cdot}5 \times \tfrac{1}{2} = 12{,}000$$

or $x^3 = \dfrac{12{,}000 - 75}{440} = \dfrac{11{,}925}{440} = 27{\cdot}30$

$$\therefore \quad x = 3{\cdot}02 \text{ m}$$

Problem-26 *A gate is provided in a tank as shown in the figure. It is 4 m square and is hinged at its top edge. The tank is filled with oil of sp. gr 0·7 to a depth of 1 m above the top edge of the gate. The space above the oil surface is subjected to a pressure of + 0·07 kg/sq. cm. as shown in fig. 1-21. Find the vertical force 'F' to be applied at the lower edge of the gate so as to open it. The weight of the gate is 300 kg.*

In this example, surface of the gate is below the atmospheric pressure.

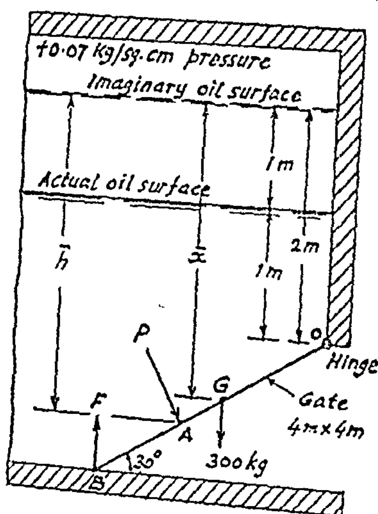


Fig. 1-21

Converting + 0·07 kg/sq. cm. pressure to metre of oil it gives

$$\frac{0{\cdot}07 \times 10{,}000}{0{\cdot}7 \times 1{,}000} = 1 \text{ m of oil.}$$

As the pressure is positive, this will increase the head and the net head over the gate is 1 + 1 = 2 m above the top edge of the gate as shown by imaginary oil surface. All other calculations are now made with reference to this imaginary oil surface.

Area of the wetted surface of the gate = 4 × 4 = 16 sq. m.

depth of C.G. below the imaginary oil surface, $\bar{x} = 2 + OG \sin\theta$

$$= 2 + 2 \times \sin 30° = 3 \text{ m}$$

$\therefore \quad P = wA\bar{x} = 1{,}000 \times 16 \times 3 = 48{,}000$ kg

The depth of centre of pressure below the imaginary water surface, $\bar{h} = \dfrac{I_{GG} \sin^2\theta}{A\bar{x}} + \bar{x}$

$$= \frac{\frac{1}{12} \times 4\,(4)^3 \times (\frac{1}{2})^2}{4 \times 4 \times 3} + 3 = 3{\cdot}11 \text{ m}$$

The vertical distance of C.P. below the top of the gate is

$3{\cdot}11 - 2 = 1{\cdot}11$ m

Taking moment above the hinge 0, we have

$F \times OB \cos 30^\circ = P \times OA + 300 \times OG$ But $OA = \frac{1{\cdot}11}{\sin 30^\circ} = 2{\cdot}22$ m

$\therefore \quad F \times 4 \times 0{\cdot}866 = 48{,}000 \times 2{\cdot}22 + 300 \times 2$

$$F = \frac{1{,}06{,}560 - 600}{3{\cdot}464} = \underline{31{,}300 \text{ kg}}$$

Problem - 27 : *A plate of the composite section which is 2·7 m square with a triangular shape on one side of 2 m height. If this plate is immersed vertically in water so that the vertex A of the triangular section lies in the water surface as shown in fig. 1–22, find the total pressure and its point of application.*

In such a case it will be advisable to divide the given section into the known geometrical areas for finding the total pressure

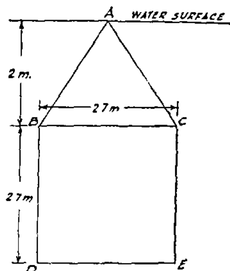


Fig 1-22

and centre of pressure of each area. The total pressure for the composite section is the sum of pressures on each area and its centre of pressure can be obtained by taking moments of pressure on each area about any point.

(a) Triangular area ABC :

Total pressure on the area ABC, $P_1 = w\, A_1\, \bar{x}_1$

Area of the triangle, $A_1 = \frac{1}{2} \times 2{\cdot}7 \times 2$; and $\bar{x}_1 = \frac{2}{3} \times 2 = \frac{4}{3}$ m

$$P_1 = 1{,}000 \times \tfrac{1}{2} \times 2{\cdot}7 \times 2 \times \tfrac{2}{3} \times 2 = 3{,}600 \text{ kg}$$

$$\bar{h}_1 = \frac{I_{GG1}}{A\,\bar{x}_1} + \bar{x}_1 = \frac{\frac{1}{36} \times 2{\cdot}7 \times 2^3}{\frac{1}{2} \times 2{\cdot}7 \times 2 \times \frac{4}{3}} + \frac{4}{3}$$

$$= \tfrac{1}{6} + \tfrac{4}{3} = 1{\cdot}5 \text{ m}$$

(b) Square area DBCE :

Area of the section, $A_2 = 2{\cdot}7 \times 2{\cdot}7$ sq.m

depth of the c.g. from the water surface, $\bar{x}_2 = 2 + \dfrac{2{\cdot}7}{2} = 3{\cdot}35$ m

$$\therefore \quad P_2 = w\, A_2\, \bar{x}_2 = 1{,}000 \times 2{\cdot}7^2 \times 3{\cdot}35 = 24{,}420 \text{ kg}$$

$$\bar{h}_2 = \frac{I_{GG2}}{A_2\,\bar{x}_2} + \bar{x}_2 = \frac{\frac{1}{12} \times 2{\cdot}7 \times 2{\cdot}7 \times 2{\cdot}7 \times 2{\cdot}7}{2{\cdot}7 \times 2{\cdot}7 \times 3{\cdot}35} + 3{\cdot}35 = 3{\cdot}531 \text{ m}$$

The total pressure on the composite section, $P = P_1 + P_2$

$= 3{,}600 + 24{,}420$

$= 28{,}020$ kg

Taking moments about the vertex A which lies in the water surface, we have

$$P\,\bar{h} = P_1\bar{h}_1 + P_2\bar{h}_2$$

$$\therefore \quad \bar{h} = \frac{P_1\bar{h}_1 + P_2\bar{h}_2}{P} = \frac{(3{,}600 \times 1{\cdot}5) + (24{,}420 \times 3{\cdot}531)}{28{,}020}$$

$$= \frac{91{,}720}{28{,}020} = 3{\cdot}273 \text{ m}$$

Problem-28 : *A circular plate of 4 m diameter is immersed vertically in water so that its upper edge is 1 m below the water. The plate is having a triangular hole which has a base of 80 cm and height of 60 cm in such a position that its vertex coincides with the centre of the plate and the base is below the centre of the plate parallel to the water surface as shown in fig. 1–23.*

Find the total pressure and centre of pressure.

Let area of the whole plate be A_1; and area of the triangular hole be A_2.

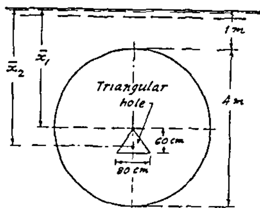


Fig. 1-23

$$A_1 = \frac{\pi}{4}(4)^2 = 12{\cdot}568 \text{ sq.m}, \quad \bar{x}_1 = 1 + 2 = 3 \text{ m}$$

$$A_2 = \frac{1}{2} \times \frac{80}{100} \times \frac{60}{100} = 0{\cdot}24 \text{ sq m}, \quad \bar{x}_2 = 1 + 2 + \frac{2}{3} \times \frac{60}{100} = 3{\cdot}4\text{m}$$

The total pressure on the circular portion is,

$$P_1 = wA_1\bar{x}_1 = 1{,}000 \times 12{\cdot}568 \times 3 = 37{,}704 \text{ kg}$$

The total pressure on the triangular portion is,

$$P_2 = wA_2\bar{x}_2 = 1{,}000 \times 0{\cdot}24 \times 5{\cdot}4 = 816 \text{ kg}$$

The total pressure on the composite section is,

$$P = P_1 - P_2 = 37{,}704 - 816 = \underline{36{,}888} \text{ kg}$$

$$\bar{h}_1 = \frac{I_{GG1}}{A_1\bar{x}_1} + \bar{x}_1 = \frac{\frac{\pi}{64}(4)^4}{4\pi \times 3} + 3 = \frac{1}{3} + 3 = 3{\cdot}333 \text{ m}$$

$$h_2 = \frac{I_{GG2}}{A_2\bar{x}_2} + \bar{x}_2 = \frac{1/36 \times (0{\cdot}8)(0{\cdot}6)^3}{0{\cdot}24 \times 3{\cdot}4} + 3{\cdot}4$$

$$= 0{\cdot}006 + 3{\cdot}4 = 3{\cdot}406 \text{ m}$$

Now, taking moments about the water surface, we have,

$$P\bar{h} = P_1h_1 - P_2h_2$$

$$\bar{h} = \frac{P_1\bar{h}_1 - P_2h_2}{P} = \frac{(37{,}704 \times 3{\cdot}333) - (816 \times 3{\cdot}406)}{36{,}888}$$

$$= \frac{125{,}800 - 2{,}780}{36{,}888} = \frac{123{,}020}{36{,}888} = \underline{3{\cdot}34 \text{ m}}$$

Problem – 29 : *A triangular plate having base width 2 m and height 3 m has semicircular shape attached with the base as shown in the figure 1–24. The diameter of the semi-circle is 4 m. Find the total pressure and centre of pressure of the composite section, when the plate is immersed vertically in water so that the vertex of the triangle is 0·5 m below the free surface of the water level and the base of the triangle is parallel to water surface.*

Area of the triangular shape, $A_1 = \frac{1}{2} \times 2 \times 3 = 3$ sq. m

Area of the semi-circular shape, $A_2 = \frac{\pi}{4} \times \frac{(4)^2}{2} = 2\pi$ sq. m

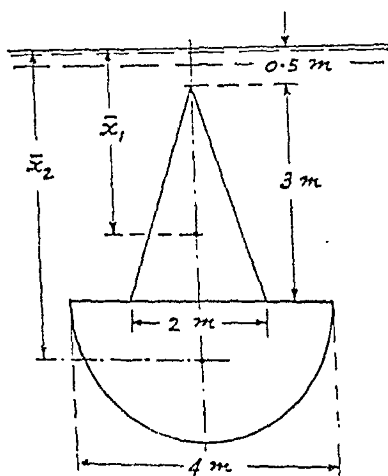


Fig. 1–24

Depth of c. g. of the triangular shape below the free surface of water level, $\bar{x}_1 = 0{\cdot}5 + 2/3 \times 3 = 2{\cdot}5$ m

Depth of c.g. of the semi-circular shape below the free surface of water level, $\bar{x}_2 = 0{\cdot}5 + 3 + 0{\cdot}424\ (\frac{4}{2}) = 4{\cdot}348$ m.

Total pressure on the triangular shape, $P_1 = wA_1\bar{x}_1$

$= 1{,}000 \times 3 \times 2{\cdot}5 = 7{,}500$ kg

Total pressure on the semi-circular shape, $P_2 = wA_2\bar{x}_2$

$= 1{,}000 \times 2\pi \times 4{\cdot}348$

$= 27{,}300$ kg

$\therefore$ total pressure on the composite section, $P = P_1 + P_2$

$= 7{,}500 + 27{,}300$

$= \underline{34{,}800 \text{ kg}}$

Depth of centre of pressure for triangular shape,

$$\bar{h}_1 = \frac{I_{GG1}}{A_1 \bar{x}_1} + \bar{x}_1 = \frac{\frac{1}{36} \times 2\,(3)^3}{3 \times 2.5} + 2.5 = 2.7 \text{ m}$$

Depth of centre of pressure for semi-circular shape,

$$\bar{h}_2 = \frac{I_{GG2}}{A_2 \bar{x}_2} + \bar{x}_2 = \frac{0.11\,(2)^4}{2\pi \times 4.348} + 4.348 = 4.4125 \text{ m}$$

Taking moments about the water surface we can find the centre of pressure of the composite section.

$$P\bar{h} = P_1 \bar{h}_1 + P_2 \bar{h}_2$$

$$\therefore \; \bar{h} = \frac{P_1\bar{h}_1 + P_2\bar{h}_2}{P} = \frac{(7{,}500 \times 2.7) + (27{,}300 \times 4.4125)}{34{,}800} = \underline{4.03 \text{ m}}$$

Problem-30. *A plate of composite section is rectangular and triangular in shape as showing fig 1-25. The rectangle is 4 m long and 2m wide while the triangle has 2 m wide base and 3 m height. If this plate is immersed inclined at an angle of 30° with the water surface, find the total pressure and centre of pressure.*

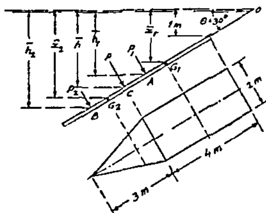


Fig 1-25

Depth of c. g of the rectangular shape below the surface of water level is

$\bar{x}_1 = 1 + 2 \sin 30° = 1 + 2 \times \frac{1}{2} = 2$ m

Depth of c. g of the triangular shape is

$\bar{x}_2 = 1 + (4 + 1) \sin 30° = 1 + 5 + \frac{1}{2} = 3.5$ m

Rectangular shape

Area, $A_1 = 4 \times 2 = 8$ sq.m, $\bar{x}_2 = 2$ m

$\therefore \quad P_1 = wA_1\bar{x}_1 = 1{,}000 \times 8 \times 2 = 16{,}000$ kg

$$\bar{h}_1 = \frac{I_{GG}\sin^2\theta}{A_1\bar{x}_1} + \bar{x}_1$$

$$= \frac{\frac{1}{12} \times 2 \times (4)^3 \times (\frac{1}{2})^2}{8 \times 2} + 2 = 2\tfrac{1}{6} \text{ m}$$

Triangular shape:

Area $A_2 = \frac{1}{2} \times 2 \times 3 = 3$ sq.m, $x_1 = 3.5$ m

$P_2 = wA_2\bar{x}_2 = 1{,}000 \times 3 \times 3.5 = 10{,}500$ kg

$$\bar{h}_2 = \frac{I_{GG}\sin^2\theta}{A_2\bar{x}_2} + \bar{x}_2 = \frac{2\,(3)^2 \times (\frac{1}{2})^2}{36 \times 3 \times 3.5} + 3.5 = 3.535 \text{ m}$$

$$OA = \frac{\bar{h}_1}{\sin\theta} = \frac{13}{6} \times \frac{1}{0.5} = \frac{13}{3} \text{ m}, \quad OB = \frac{\bar{h}_2}{\sin\theta} = \frac{3.535}{0.5} = 7.07 \text{ m}$$

$\therefore$ total pressure, $P = P_1 + P_2 = 16{,}000 + 10{,}500 = \underline{26{,}500 \text{ kg}}$

Taking moment about O, we have,

$$P \times OC = P_1 \times OA + P_3 \times OB \qquad \therefore \quad OC = \frac{P_1 \times OA + P_2 \times OB}{P}$$

$$\therefore \quad OC = \frac{16{,}000 \times \frac{13}{3} + 10{,}500 \times 7.07}{26{,}500} = 5.42 \text{ m}$$

$$\therefore \quad \bar{h} = OC\sin\theta = 5.42 \times \tfrac{1}{2} = \underline{2.71 \text{ m}}$$

The Pressure on Lock Gates

We have studied in earlier paragraphs about the centre of pressure and total pressure and one of the practical applications of it is in finding the forces on a lock gate shown in fig. 1–26. There are two gates ABCD nnd ABFE hinged at G & H and K & J respectively. These gates are held in contact (*i.e.* in closed position) at the common edge AB by the difference of the water pressures which act from the upper reach and lower reach sides, the water level being higher on the upper reach than that on the lower reach. In finding the forces we first consider what are the forces acting on the gate. Consider the gate ABCD. The water pressure acts with a resultant force P on the centre line of the gate and normal to it as shown in figs. 1–26 and 1–27. The gate ABFE exerts force on the gate ABCD with a pressure T which is normal

to the surface of contact of the two gates. The two hinges G & H on the side DC will react with a resultant force R, whose direction is still to be determined. The gate ABCD is in equilibrium under the three forces and hence these three forces must be concurrent.

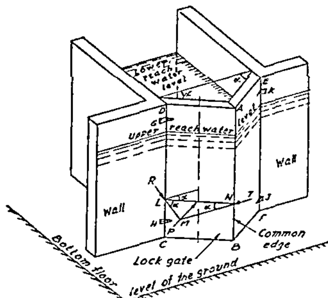


Fig. 1-26 Arrangement of lock gate

Let the forces P and T intersect at point M (fig. 1-26). The third force R must pass through M which is the point of concurrence and it also must act at the point L which is the point of its application. Join LM and the reaction R must act along the line LM.

Let α be the angle of inclination of gate of the normal of the side of lock. Δ LMN will be an isosceles triangle and its angles MLN and MNL are equal, each being equal to α.

The three forces acting at M are resolved in two directions, parallel and perpendicular to the gate. The sum of these forces so resolved should be zero in order to impart equilibrium to the gate.

Resolving the forces parallel to the gate, we have,

$$R \cos \alpha - T \cos \alpha = 0$$

$$\therefore \quad R = T \qquad \text{... (1·15)}$$

Resolving the forces in perpendicular direction to the gate, we have,

$$P - R \sin \alpha - T \sin \alpha = 0$$

$$\therefore \quad P = (R + T) \sin \alpha$$

$$\therefore \quad P = 2 R \sin \alpha$$

$$\therefore \quad R = \frac{P}{2 \sin \alpha} \qquad \text{.. (1·15 a)}$$

This reaction will act at angle α with the centre line LN of the gate.

Now consider the view of the gate shown in fig. 1-27. Here the pressure of water on the gate ABCD is presented.

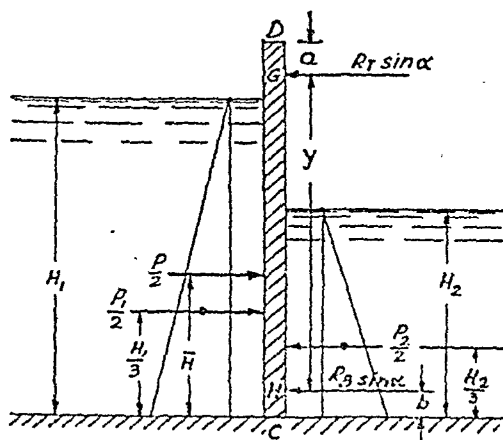


Fig. 1-27 Pressure on lock gate

Let, H_1 – height of water to left of gate CD(on the upper reach side),

H_2 – height of water to right of gate CD(on the lower reach side),

Y – height of top hinge G from the bottom hinge H,

a – distance of hinge G from the top of the gate,

b – distance of hinge H from the bottom of the gate,

B – width of the gate,

P_1 – total pressure of water from the left hand side of gate,

P_2 – total pressure of water from the right hand side of gate,

R_T – reaction of top hinge,

R_B – reaction of bottom hinge,

R – resultant reaction of the top and bottom hinges which is the sum of R_T and R_B.

Now, P_1 = average intensity of pressure × wetted area of gate

$$= \frac{wH_1}{2} \times BH_1$$

This total pressure acts at a point $\frac{H_1}{3}$ above the bottom of gate.

Similarly, $P_2 = \frac{wH_2}{2} \times BH_2$ is acting at a point $\frac{H_2}{3}$ above the bottom. The resultant of P_1 and P_2 is P which will act a point $\bar{H}$ above the bottom and $P = P_1 - P_2$

$$\therefore \quad \bar{H} = \frac{P_1 \times \frac{H_1}{3} - P_2 \times \frac{H_2}{3}}{P_1 - P_2} \quad \ldots \ldots \ldots \ldots (1{\cdot}16)$$

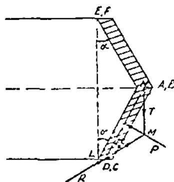


Fig. 1-28

The total pressure acts at the centre of the gate, so that half the pressure will be taken as acting on the hinge edge CD of the gate ABCD, the remaining half will be taken up by the reaction of the gate ABEF on the edge AB of the gate ABCD.

Taking moments about H, the bottom hinge

$$R_T \sin \alpha \times Y = \frac{P_1}{2}\left(\frac{H_1}{3} - b\right) - \frac{P_2}{2}\left(\frac{H_2}{3} - b\right)$$

$$\text{or } R_T \sin \alpha \times Y = \frac{P}{2}\,(\bar{H} - b) \quad \ldots \ldots \ldots \ldots (1{\cdot}17)$$

Resolving horizontally,

$$\frac{P_1}{2} - \frac{P_2}{2} = R_B \sin \alpha + R_T \sin \alpha$$

$$\text{or } \frac{P}{2} = R_B \sin \alpha + R_T \sin \alpha \quad \ldots \ldots \ldots (1{\cdot}18)$$

Then, from eqns. (1·17) and (1·18), R_T and R_B may be found out.

Problem-31 : *Each gate of a lock is 7 m high and 2 m wide, and is supported on pivots, situated 30 cm from the top and bottom. The angle between the gate when they are closed is 140°. If the depths of water on the two sides are 5 m and 2 m respectively, find (a) the magnitude and position of the resultant water pressure on each gate, (b) the magnitude of reaction between the gates, and (c) the mangnitudes and directions of the reactions of the pivots, on the assumption that the gate reaction acts in the same horizontal plane as the resultant water pressure.*

The total pressure from the upper reach side is

$$P_1 = \frac{wH_1}{2} \times H_1 \times B \times \frac{1{,}000 \times 5}{2} \times 5 \times 2$$

$= 25{,}000$ kg acting at 2 m from the bottom.

The total pressure from the lower reach side is

$$P_2 = \frac{wH_2}{2} \times H_2 \times B = \frac{1{,}000 \times 2}{2} \times 2 \times 2$$

$= 4{,}000$ kg acting at 1·66 m from the bottom.

Resultant water pressure, $P = P_1 - P_2$

$= 25{,}000 - 4{,}000 = \underline{21{,}000 \text{ kg}}$

The point of action of P is given by

$$\bar{H} = \frac{P_1 \times \frac{H_1}{3} - P_2 \times \frac{H_2}{3}}{P_1 - P_2}$$

$$= \frac{\frac{25{,}000 \times 5}{3} - 4{,}000 \times 2/8}{21{,}000} = \underline{1{\cdot}857 \text{ m}} \text{ above bottom.}$$

The reactions of the hinge of the gate are equal.

i. e. $R = T$

Resultant pressure, $P = R \sin \alpha + T \sin \alpha = 2\,T \sin \alpha$.

$$\therefore \quad T = \frac{P}{2 \sin \alpha}$$

Since, $2\alpha = 180° - 140° = 40°$,

$$T = \frac{21{,}000}{2 \sin 20°} = \frac{21{,}000}{2 \times 0{\cdot}3420} = \underline{30{,}700 \text{ kg}}$$

Further, taking moments about the lower hinge,

$$R_T \sin \alpha \times Y = \frac{P}{2}(\bar{H} - b)$$

$$\therefore \quad R_T = \frac{\frac{21,000}{2}(1{\cdot}857 - 0{\cdot}3)}{\sin 20^\circ \times 6{\cdot}4} = \underline{6,500 \text{ kg}}$$

Equating the horizontal components of forces,

$$\frac{P}{2} = R_B \sin \alpha + R_T \sin \alpha$$

$$\text{or } R_B + R_T = \frac{P}{2 \sin \alpha} = \frac{21,000}{2 \times 0{\cdot}3420} = 30,700 \text{ kg}$$

$$\therefore \quad R_B = 30,700 - R_T$$

$$= 30,700 - 6,500 = \underline{24,200 \text{ kg}}$$

Problem-32 : *The end gates of a lock are 6 m high and when closed include an angle of 120°. The width of the lock is 5·5 m. Each gate is supported on two hinges placed 60 cm from the top and bottom of the gate.*

If water levels are 5·5 m and 4·5 m on the upstream and down stream sides respectively, determine the magnitudes of the forces on the hinges due to the water pressure.

The three forces, *viz.* reaction of hinge R, reaction of gate T and total pressure, P acting on the gate meet at the point M and they are in equilibrium.

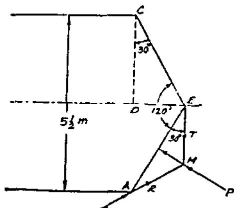


Fig. 1-29

Resolving forces along AE (fig. 1-29), we have,

$$T \cos 30^\circ - R \cos 30^\circ = 0$$

$$\therefore \quad T = R$$

Resolving forces normal to AE,

$$P = R \sin 30^\circ + T \sin 30^\circ$$

$$= \tfrac{1}{2}R + \tfrac{1}{2}R \text{ (as } T = R)$$

$$= R$$

$$\text{Now, } B = AE = CE; \text{ and } \cos 30^\circ = \frac{CD}{CE}$$

$$\therefore \quad CE = \frac{CD}{\cos 30^\circ} \text{ and } CD = 2{\cdot}75 \text{ m}$$

$$\therefore \quad B = \frac{2{\cdot}75}{0{\cdot}866} = 3{\cdot}18 \text{ m}$$

Total pressure from the upstream side is

$$P_1 = \frac{wH_1}{2} \times \text{wetted area}$$

$$= \frac{wH_1}{2} \times B \times H_1 = \frac{1{,}000 \times 5{\cdot}5}{2} \times 3{\cdot}18 \times 5{\cdot}5$$

= 48,100 kg acting at 1·83 m from the bottom.

$$\text{and, } P_2 = \frac{wH_2}{2} \times B \times H_2 = \frac{1{,}000 \times 5{\cdot}5}{2} \times 3{\cdot}18 \times 4{\cdot}5$$

= 32,200 kg acting at 1·5 m from bottom.

The resultant total pressure is

$$P = P_1 - P_2 = 48{,}100 - 32{,}200 = 15{,}900 \text{ kg}$$

$$\text{Further } \quad \overline{H} = \frac{P_1 \frac{H_1}{3} - P_2 \frac{H_2}{3}}{P} = \frac{48{,}100 \times 1{\cdot}83 - 32{,}200 \times 1{\cdot}5}{15{,}900}$$

$$= 2{\cdot}52 \text{ m}$$

$$\text{and } \quad R = T = P = \underline{15{,}900 \text{ kg}}$$

Further, taking moments about the lower hinge,

$$R_T \times \sin \alpha \times Y = \frac{P}{2} \left(\overline{H} - b \right)$$

$$\text{and } \quad Y = 6 - 0{\cdot}6 - 0{\cdot}6 = 4{\cdot}8 \text{ m}$$

$$\therefore \quad R_T = \frac{P\left(\overline{H} - b\right)}{2 \sin \alpha\, Y} = \frac{15{,}900\,(2{\cdot}52 - 0{\cdot}6)}{2 \times \sin 30^\circ \times 4{\cdot}8}$$

$$= \underline{6{,}350 \text{ kg}}$$

$$\text{Hence, } R_B = 15{,}900 - 6{,}350 = \underline{9{,}550 \text{ kg}}$$

TUTORIAL–1

Fluid pressure

1. A drum of a boiler is 3 m in diameter and 6 m long. If the pressure of steam inside the boiler is 5 kg/cm^2 (gauge), estimate the force tending to burst the drum along the longitudinal section.

[900 tonnes]

2. A hemispherical dome (ref. fig. 1–3) of radius 1 m contains a fluid under pressure of 14 kg/cm^2 gauge. Find the total force tending to lift the dome.

[440 Tonnes]

Hydraulic press

3. In a hydraulic press the area of the ram is 150 cm^2 and its weight is 10 kg., while the area of the plunger is 2 sq. cm and its weight 1 kg. Determine the weight that can be lifted by the ram if a force of 3 kg is applied to the plunger.

[290 kg]

4. The diameter of the plunger of a hydraulic press is 25 mm and that of the ram 64 mm. It is found that an effort of 23 kg just raises a load of 2·235 Tonnes. The leverage of the handle is 20 to 1. The stroke of the plunger is 100 mm. Determine the efficiency of the press and the number of strokes to be performed by the plunger to lift the weight by 60 mm.

[74 %, 40]

5. In a hydraulic press the diameters of the plunger and the ram are 3 cm and 36 cm respectively. The leverage of the handle is 18 to 1. A body weighing 10 tons is placed on the ram. The efficiency of the machine is 80%. The stroke of the plunger is 20 cm. Plunger makes 150 strokes while the load is lifted. Determine the effort applied at the end of the lever and the height through which the load is lifted.

[4·8 kg; 20·8 cm]

6. A hydraulic press has plunger diameter of 2 cm and its stroke is 30 cm. The plunger makes 400 strokes in 10 minutes. The press is raising a load of 1 ton. The force on the plunger is 10 kg. Determine the diameter of the ram and H. P. required to drive the plunger. Neglecting all losses and assuming the motion of the weight continuous, determine the lift of the weight.

[20 cm; 0·0267 H. P., 1·2 m]

7. A load of 15 tons is to be raised by means of a hydraulic press. The maximum effort applied at the end of a lever is 25 kg. If the diameter of the ram is 15 cm, determine the diameter of plunger. The leverage of the lever is 24 : 1. Determine also the number of strokes to be performed in order to lift the load through 20 cm, if the stroke of the plunger is 5 cm. Disregard the effect of friction.

[3 cm; 100]

Intensity of pressure

8. A rectangular tank, 6 m long, 2·5 m wide and 2 m deep is full of water. Find the intensity of pressure on the bottom of the tank and the total force on the side having 2·5 m width. Determine the depth of water so that the total force on this side is reduced to $\frac{3}{4}$th the total force on the same surface when the tank is full.

[0·2 kg/cm^2, 5,000 kg, 1·73 m]

Pressure gauges

9. The left limb of a mercury U-tube manometer is open to atmosphere and the right limb is connected to a pipe carrying water under pressure. The centre of the pipe is at the level of the free surface of mercury. Find the difference in levels of mercury in the limbs if the absolute pressure of water in the pipe is 14 m of water.

[19·2 cm]

10 A mercury U-tube manometer is connected to a tank which stores water under pressure. The outlet point of the tank is 2 m above the free surface of mercury in the U-tube. When a

rubber tube connected the tank to the U-tube, the difference in levels of mercury was observed to be 20 cm. Calculate the pressure of water in the tank in kg/sq. cm.

[0·052 kg/cm^2]

11. The vacuum head in the suction pipe of a centrifugal pump is measured by a U-tube vacuum gauge. The end of the tube is 2 m above the centre of the pipe at which tapping for water is provided. When water is allowed to come to the U-tube, the difference in mercury levels in the U-tube is 30 cm and there is 10 cm high water column above the mercury column. Determine the vacuum head of the pump.

[*2·18 m of water*]

12. A pressure gauge is made up of two cylindrical bulbs of diameter 2 cm and they are connected by a U-tube with vertical *limbs of diameter 5 mm. Water and oil of sp. gr. 0·75 are used* in it, which is used to measure the pressure of water flowing in a 2 cm diameter main which is connected to the limb containing oil. After the connections are made, the fall in oil level was found to be 5 cm in the U-tube. The original height of oil was 30 cm. Determine the pressure of water in the main in kg/sq. cm.

[0·00344 kg/cm^2]

Centre of pressure

13. A circular plate 3 m in diameter is placed vertically in water so that the centre of the plate is 4 m below the free surface of water. Find the depth of the centre of pressure and the total pressure on the plate.

[4·141 m; 28,278 kg]

14. A circular plate 2·5 m in diameter is immersed vertically *in a liquid having sp. gr. 0·9, so that the centre of the plate is* 2 m below the free surface of the liquid. Find the depth of the centre of the pressure and the total pressure on the plate.

[2·195 m; 8,820 kg]

15. A right angled triangular plate having base width 4 m and height 6 m is immersed vertically in water so that the base is parallel to the water surface and at a depth of 1 m below the free surface of the water level and apex is below the base. Determine the total thrust and depth of centre of pressure.

[36,000 kg, 3·67 m]

16 An isosceles triangular plate, has 3 m height and its base is 2 m. The plate is immersed vertically in water with its apex below the base which is at a distance of 4 m from the free surface. Find, (a) total pressure, and (b) centre of pressure.

[(a) 6,000 kg, (b) 2·25 m]

17. A circular plate of diameter 'D' is submerged vertically below the free surface of water so that its centre is at a depth 'd' from water surface. Prove that the depth of the centre of pressure is given by the expression $\frac{D^2}{16d} + d$.

18. A triangular plate of height 'h' is submerged vertically in water with its apex uppermost and at a depth 'd' below the free surface of water. The base is parallel to water surface. Prove that the depth of the centre of pressure is given by the expression

$$\frac{3h^2 + 8dh + 6d^2}{2(3d + 2h)}.$$

Determine its value when $h = 1$ m and $d = 60$ cm.

[1·31 m]

19. A triangular plate of height 'h' is immersed vertically in water with its apex below the base. The base is parallel to and at a depth 'd' below the free surface of water. Prove that the depth of the centre of pressure is given by the expression

$$\frac{\frac{1}{2}(h^2 + 4dh + 6d^2)}{3d + h}.$$

Determine the value if $h = 1$ m and $d = 60$ cm.

[1·54 m]

✓ 20. Find (a) the total pressure, and (b) the position of the centre of pressure on one side of the immersed rectangular plate, 3 m long and 1 m wide when the plate is perpendicular to the free surface of water, 3 m long edge is parallel to and at a depth of 1 m below the surface of water.

[4,500 kg; 1·556 m]

21. A trapezoidal plate has parallel sides of 3 m and 6 m, the height of the plate is 2 m. This plate is immersed vertically in water such that the longer of the two parallel sides is flush with the free surface of water. Find the total pressure on one side of the plate and its point of action.

[8,000 kg; 1·25 m below water surface]

22. An automatic falling shutter fitted over the crest of a waste weir holds up water to an additional depth of 2 m above the crest level. Determine the location of the hinge of the shutter below the free surface of water so that it will fall down automatically when water level is 2·5 m above the crest level.

[1·5 m]

23. A vertical shutter revolving about a horizontal axis sustains a pressure of 7 m of water on one side. At what depth should the axis be placed in order that the pressure on the portions of the shutter and below the axis may be equal ?

[4·95 m]

24. A channel is 3 m wide at the bottom, 2 m deep and its sides are sloping at an angle of 45° The channel is filled with water upto a depth of 2 m. If sluice gate is constructed across the channel, determine the total pressure on the gate and the depth of its centre of pressure.

[8,666 kg, 1·23 m]

25. A circular opening in a vertical reservoir wall is 1 m in diameter and its centre is 10 m below the water level. The opening is covered by a plate which is fixed to the masonary wall by means of two bolts placed one 75 cm vertically above and

Find the total pressure per metre length of the dam and position where it acts above the base.

[6,32,100 kg, 11·4 m above base]

28. A dock gate is to be rainforced with three equal horizontal beams (fig. 1-31). If water acts on one side only to a depth of 6 m, find the position of the beams, measured below water level, so that each will carry an equal load and find the load on each beam per metre run.

[3·46 m, 4·9 m, 6 m, 6,000 kg.]

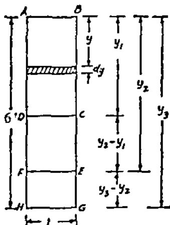


Fig. 1-31

29. An automatic falling shutter fitted over the crest of a weir holds water to an additional depth of 3 m above the crest level. Calculate the depth at which the hinge of the shutter should be located if the shutter is to fall down automatically when overtopped by one metre water ?

[1·2 m above the crest level]

30. A rectangular sluice gate 2 m broad and 2 m deep having its upper edge at a depth of 1 m is inclined at 45° to the vertical. The sluice is lifted by a force applied parallel to its plane. Determine the magnitude of the lifting force if the coefficient of friction between gate and its grooves is 0·16.

[1,546 kg]

31. A vertical gate 15 m high and 20 m wide is hinged at its bottom horizontal edge and it is maintained in vertical position by a chain attached at the top of the gate. The chain is inclined at an angle of 30° with the horizontal. Sea water stands to a depth of 12 m on the side of chain and 9 m on the other side of the gate. Assuming the gate is free to revolve about the hinge and there is no resistance at its vertical ends, find the pull in the chain which keeps the gate in vertical position. Density of sea water is 1,020 kg/cu.m [2,62,000 kg]

32. A canal having a bottom width of 7 m with sides sloping at an angle of 45° with the vertical, is 4 m deep. It is to be closed for repairs by a vertical bulk head at right angles to its centre line. Find the pressure and the depth of centre of pressure of the bulk head when there is depth of 3 m of fresh water on one side of bulk head only.

[40,500 kg; 1·89 m]

33. A tank 4 m square at bottom and 6 m square at the top is 4 m high. The four slanting sides of the tank are plane surfaces and have same trapezoidal shape. If the depth of water is 2·5 m, find the total pressure and the position of the centre of pressure on any one immersed face.

[14,050 kg; 1·61 m below water level]

34. A rectangular opening in a vertical reservoir wall is 60 cm wide and 120 cm deep and its top edge is 6 m below the free surface of water. If a rectangular gate closes this opening, find the total pressure on the gate and its point of application.

[4,750 kg; 6·82 m]

35. A vertical sided 3 m high tank is square in plan whose each side is 2 m long. The tank contains oil of sp. gr. 0·65 to a depth of 1 m floating over 2 m depth of water. Determine (a) the total pressure on one side of the tank, (b) the height of the centre of pressure above the base.

[7,250 kg; 0·935 m]

36. A hollow triangular box, the ends of which are equilateral triangles of 2 m sides, is submerged in water in such a way that one of its rectangular faces lies in the free surface of water. Find the net total pressure and the position of the centre of pressure on one of the triangular ends, (a) when the inside of the box is at atmospheric pressure and (b) when the inside of the box is at a pressure of 0.1 kg/sq. cm, above the atmospheric pressure.

[(a) 1,000 kg; 0·865 m; (b) 732 kg; 0·184 m]

Lock gates

37. Each gate of a lock is 7 m high and 2 m wide and is supported on pivots situated 0·5 m from the top and bottom. The angle between the gates when they are closed is 160°. If the depth of water on the two sides are 6 m and 3 m respectively, find, (a) the magnitude and position of the resultant water pressure on each gate, (b) the magnitude of reaction between the gates, and (c) the magnitudes and directions of the reactions at the pivots, on the assumption that the gate reaction acts in the same horizontal plane as the resultant water pressure

[(a) 27,000 kg at 2·33 m above the bottom; (b) 39,480 kg; (c) top hinge–12,040 kg; bottom hinge–27, 440 kg at 10° with the plane of gate]

38. The end gates of a lock are 6 m high and when closed include an angle of 140°. The width of the lock is 8 m. Each gate is supported on two hinges placed at the top and bottom of the gate. If water levels are 5 m and 3 m on the upstream and downstream sides respectively, determine the magnitudes of the forces on the hinges due to the water pressure.

[Reaction of the top hinge, R_T = 17,000 kg, reaction of the bottom hinge, R_B = 32,700 kg]

2
FLOATATION AND BUOYANCY

Introduction

We know that when we place any body on the surface of liquid, it will either sink or float on it. When a body is placed in the liquid, it will be acted upon by the gravitational force in the downward direction and by the upthrust of the liquid in the upward direction. If the upthrust on a body is less than the gravitational force the body will sink; if the upthrust on a body is greater than gravitational force, the body will float on the surface of the liquid.

Archimedes' Principle

The statement of Archimedes' principle is as follows :

When a body is weighed in liquid its weight is found to be less than its weight in air. This apparent loss in weight is equal to the weight of the volume of liquid displaced by the body.

The above statement can be rewritten in different form as follows :

If a body is partially or totally immersed in a fluid it experiences an upthrust equal to the weight of the fluid displaced.

A body immersed in a fluid is therefore acted upon by its own weight $W = V_1 w_1$ tending to sink the body, and upthrust $U = V_2 w_2$ tending to float it. V_1 is the volume of body and w_1 its density. V_2 is the volume of fluid displaced and w_2 its density.

The body will float in equilibrium, when $W < U$; if $W > U$ it will sink.

If a body is allowed to float in two different liquids, the depth of immersion in the liquids will be inversely proportional to their density.

Thus, if a ship moves from sea-water to fresh water its depth of immeresion will increase and vise versa.

Buoyancy and centre of Buoyancy

When a body is kept in liquid, it is subjected to the pressure of liquid. The resultant upward pressure of the liquid in which the body is floating is known as the *buoyancy*.

The force of buoyancy acts through the centre of gravity of the displaced liquid and this point is known as the *centre of buoyancy*.

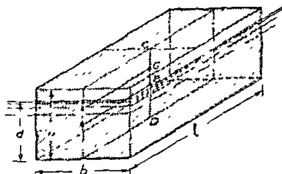


Fig. 2-1

A floating body is shown in fig. 2-1. A portion of the body is above the free surface of water. The height or depth of immersion of a floating body is called its *draught* and this is denoted by the letter *d* as indicated in fig. 2-1.

The value of displacement which is denoted by the letter *V* is the volume of liquid displaced by the floating body. The mass of the liquid displaced by the floating body is known as the

displacement which is denoted by the letter U and this is equal to the mass W of the floating body.

$$\therefore \quad V \times w = U = W$$

$$\therefore \quad V = \frac{W}{w} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots (2\cdot1)$$

Also, $V = a \times d$

where a is uniform area of cross-section.

$$\therefore \quad d = \frac{V}{a} = \frac{W}{wa}$$

In fig. 2-1, B is the centre of buoyancy which lies on the axis CD such that it is at half the depth of immersion, d from the bottom. G is the c. g. of the body which lies on the axis CD and is situated at mid-point of it *i. e.* at a height of $h/2$ from the bottom.

Problem - 1 : *A baloon weighing 50 gm and of capacity 200 litres, is filled with Hydrogen. What is the minimum load which will prevent the baloon from rising ? Density of Hydrogen = 0·00009 gm/cm³. Density of air = 0·00129 gm/cm³.*

Let the load required to prevent the baloon from rising be W gm.

The total downward force = $50 + W + 200 \times 1{,}000 \times 0{\cdot}00009$
$= (68 + W)$ gm

Upthrust = $200 \times 1{,}000 \times 0{\cdot}00129 = 258$ gm

If the baloon is stationary,

upthrust = total downward force
or $258 = 68 + W$

$\therefore$ load required, W = 190 gm

Problem - 2 : *A barge is 12 m long, 4 m wide and 2 m deep. The thickness of the sides and bottom of the barge which is constructed of iron sheet is 5 mm. If the density of the material is 7·5 gm/cm³, what extra load is required to wet the barge upto 1·6 m in*

water? Determine also the depth of immersion without the extra load.

Area of two longitudinal surfaces = $12 \times 2 \times 2 = 48$ m^2

Area of two end surfaces = $4 \times 2 \times 2 = 16$ m^2

Area of the bottom = $12 \times 4 = 48$ m^2

$\therefore$ total area = $48 + 16 + 48 = 112$ m^2

As the thickness of the metal is same throughout, volume of the metal $= 112 \times \frac{5}{1,000} = 0.56$ m^3

$\therefore$ weight of the barge, W = vloume × density

$$= 0.56 \times \frac{7.5 \times 10^6}{1,000} = 4,200 \text{ kg}$$

Let x m be the depth of immersion without the load. Then,

$W = 12 \times 4 \times x \times 1,000$ (according to Archimedes' law)

$$\therefore \quad x = \frac{W}{12 \times 4 \times 1,000} = \frac{4,200}{48,000} = \underline{0.0875 \text{ m}}$$

When the barge sinks upto 1.5 m in water, weight of water displaced, $U = 12 \times 4 \times 1.5 \times 1,000 = 72,000$ kg

The displacement U which is upthrust on the barge must be equal to the sum of weight of the barge and the load placed on it.

$\therefore$ load on the barge $= U - W$

$= 72,000 - 4,200 = \underline{67,800}$ kg

Problem-3 : *A log of wood 3 m in diameter and 5 m long is placed horizontal in sea water. If the specific gravity of wood is 0.5, determine the depth of immersion. Density of sea water is 1,040 kg/m³.*

The weight of the log is

$$W = \frac{\pi}{4} d^2 \times l \times \rho \qquad \text{where } \rho = sw$$

$$= \frac{\pi}{4} 3^2 \times 5 \times 0.5 \times 1,040 = 18,400 \text{ kg}$$

The upthrust U should also be 18,400 kg.

Volume of sea water displaced,

$$V = \frac{U}{w} = \frac{18,400}{1,040} = 17.7 \text{ m}^3$$

Volume of immersed portion of log = area ACBA × length.

Now, area ACB = area of sector ACBO – area of triangle OAB

$$= \frac{1}{2} r^2 \, 2\theta \frac{\pi}{180} - \frac{r^2 \sin\theta \cos\theta \times 2}{2}$$

(where r = radius of the log)

$$= r^2 \frac{\pi\theta}{180} - \frac{r^2 \sin 2\theta}{2}$$

$$= r^2 \left(\frac{\pi\theta}{180} - \frac{\sin 2\theta}{2} \right)$$

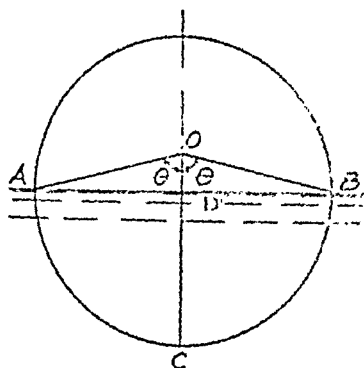


Fig. 2-2 End view of the log

∴ volume of immersed portion of log

$$= r^2 \left(\frac{\pi\theta}{180} - \frac{\sin 2\theta}{2} \right) \times l$$

This volume should be equal to the volume of water displaced.

$$\therefore \quad 17{\cdot}7 = 1{\cdot}5^2 \left(\frac{\pi\theta}{180} - \frac{\sin 2\theta}{2} \right) \times 5$$

$$\therefore \quad \sin 2\theta = \frac{\pi\theta}{90} - 1{\cdot}575$$

By trial and error method, $\theta = 70°$

and depth of immersion,

$$DC = r - r\cos\theta = r(1 - \cos\theta)$$

$$= 1{\cdot}5\,(1 - \cos 70°)$$

$$= \underline{0{\cdot}99 \text{ m}}$$

Problem-4 : *A piece of stone weighs 50 kg in air and 37·5 kg in water when kept immersed in it. Calculate the volume and specific gravity of stone. Density of water is 1,000 kg/m³.*

When the stone piece is kept immersed in water it displaces volume of water equal to its own volume and the upthrust (upward force) equal to the weight of the volume of water displaced acts on the stone. If V is the volume of stone, the upthrust is $U = wV$. This upthrust reduces the weight of stone in water.

$$\therefore \; 50 = 37.5 + U$$

$$\text{or} \quad U = wV = 50 - 37.5 = 12.5 \text{ kg}$$

$$\therefore \quad V = \frac{12.5}{1{,}000} = 0.0125 \text{ m}^3 \text{ or } 12{,}500 \text{ cm}^3$$

The specific gravity, s is given by

$$s \times 1{,}000 \times V = 50$$

$$\therefore \quad s = \frac{50}{1{,}000 \times 0.0125} = 4$$

Problem-5 : *A log of wood having square cross - section and weight of 20 kg float vertically with 20 cm out of water and 40 cm out of liquid of density 1,200 kg/m³ Determine the size and density of the log. This log is floating vertically in a liquid of density 1,500 kg/m³ when 5 kg weight is placed on its top. Determine its depth of immersion*

Let L may be the length of the log in m and A its cross-sectional area in sq. m

weight of log = weight of water displaced
= weight of liquid displaced

Volume of water displaced × density of water
= volume of liquid displaced × density of liquid

$$\text{i. e. } A\,(L - 0.2) \times 1{,}000 = A\,(L - 0.4) \times 1{,}200$$

$$\therefore \quad L - 0.2 = (L - 0.4)\, 1.2$$

$$\therefore \quad L = \frac{0.28}{0.2} = 1.4 \text{ m}$$

$$\text{Volume of water displaced} = \frac{\text{weight of water displaced}}{w}$$

$$= \frac{20}{1{,}000} = 0.02 \text{ m}^3$$

Now, depth of immersion in water = 1·4 – 0·2 = 1·2 m

$$\therefore A = \frac{0{\cdot}02}{1{\cdot}2} \times 100 \times 100 = 167 \text{ cm}^2$$

$$\therefore \text{each side, } S = \sqrt{167} = \underline{12{\cdot}9 \text{ cm}}$$

$$\text{Volume of log} = A \times L = S^2 \times L = \left(\frac{12{\cdot}9}{100}\right)^2 \times 1{\cdot}4$$

$$= 0{\cdot}0234 \text{ m}^3$$

$$\text{Density of log} = \frac{\text{weight}}{\text{volume}} = \frac{20}{0{\cdot}0234} = 855 \text{ kg/m}^3$$

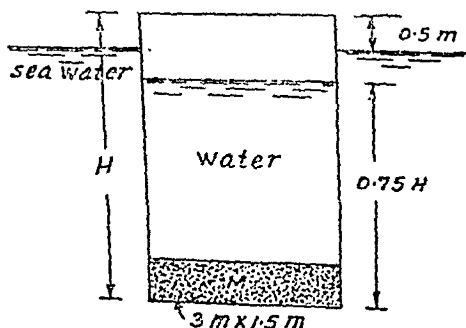
Weight of liquid displaced = 20 + 5 = 25 kg

$$\text{Volume of liquid displaced, } V = \frac{25}{1{,}500}$$

$$\text{Depth of immersion} = \frac{\text{volume of liquid displaced}}{\text{area of cross-section}}$$

$$\text{Depth of immersion} = \frac{25}{1{,}500} \Big/ \frac{167}{10{,}000} = 1 \text{ metre}$$

Problem-6 : *A rectangular vessel made of thin metal sheet has its base 3 m long and 1·5 m wide. It is filled with fresh water and mercury upto its ¾th height ; the volume of water being 49 times that of mercury. It is now placed in sea-water so that it floats with 0·5 m height outside the free water surface. If the sp. gr. of mercury is 13·6; determine the maximum height of the vessel. Density of sea-water is 1,023 kg /m³, density of fresh water is 1,000 kg/m³.*



Fig, 2·3

Area of cross-section of the vessel,

$$A = L \times b = 3 \times 1.5 = 4.5 \text{ m}^2$$

Let the height of the vessel be H m.

Then, volume of water and mercury filled upto three-fourth height is equal to $(4.5 \times \frac{3}{4}H) = 3.375H \text{ m}^3$.

Let V_1 and V_2 be volumes of fresh water and mercury respectively.

$$\text{Then, } V_1 + V_2 = 3.375H$$

$$\text{But, } V_1 = 49V_2$$

$$\therefore\ 49V_2 + V_2 = 3.375H$$

$$\therefore\ V_2 = \frac{3.375}{50} = 0.0675 \text{ m}^3$$

$$\therefore\ V_1 = 0.0675\ H \times 49 = 3.3075H \text{ m}^3$$

Let w_1 be density of fresh water.

$$\therefore\ w_1 = 1{,}000 \text{ kg/m}^3$$

and w_2 = density of mercury,

$$\therefore\ w_2 = 1{,}000 \times 13.6 = 13{,}600 \text{ kg/m}^3$$

$$\begin{aligned}\text{Then, } W &= W_1 + W_2 \\ &= V_1 w_1 + V_2 w_2 \\ &= (3.3075H \times 1{,}000) + (0.0675 \times 13{,}600) \\ &= 4225.5H \text{ kg}\end{aligned}$$

Weight of sea water displaced = $4225.5H$ kg

$$\therefore \text{ volume of water displaced} = \frac{4225.5H}{1{,}020} \text{ m}^3$$

$$\therefore \text{ depth of immersion} = \frac{\text{volume of water displaced}}{\text{area of cross-section}}$$

$$= \frac{4225.5H}{1{,}020 \times 4.5} = 0.92\ H \text{ m}$$

Now as the height of the vessel outside the free water surface is 0.5 m, the depth of immersion corresponds to $(H - 0.5)$ m.

$$\therefore\ H - 0.5 = 0.92H$$

$$\text{or } H = \frac{0.5}{0.08} = \underline{6.25 \text{ m}}$$

Problem-7 : *A buoy carrying a bacon light has the upper portion cylindrical 3 m diameter and 2 m deep. The lower portion which is curved, displaces a volume of 0·5 m³. Determine the weight of the buoy and height outside the free surface of water when it floats in the sea water having density of 1,020 kg/m³. The density of the cylindrical portion is 350 kg/m³, while that of the lower curved portion is 400 kg/m³.*

Let, W – weight of the whole buoy,

W_1 – weight of the cylindrical portion, and

W_2 – weight of the lower curved portion.

Volume of cylindrical portion, $V_1 = \frac{\pi}{4} D^2 \times h$

$$= \frac{\pi}{4} (3)^2 \times 2 = 4{\cdot}5\pi \text{ m}^3$$

$\therefore\ W_1 = w_1 V_1 = 350 \times 4{\cdot}5\pi = 4{,}950$ kg

$W_2 = w_2 V_2 = 400 \times 0{\cdot}5 = 200$ kg

$\therefore\ W = W_1 + W_2 = 4{,}950 + 200 = \underline{5{,}150 \text{ kg}}$

Now, weight of water displaced = 5,150 kg

and volume of water displaced $= \frac{5{,}150}{1{,}020} = 5{\cdot}05 \text{ m}^3$

$\therefore$ volume of immersed cylindrical portion

= volume of water displaced – volume of water displaced by the lower curved portion

$= 5{\cdot}05 - 0{\cdot}5 = 4{\cdot}55 \text{ m}^3$

$\therefore$ depth of immersion of cylindrical portion

$$= \frac{4{\cdot}55}{\frac{\pi}{4}(3)^2} = \frac{4{\cdot}55 \times 4}{9\pi} = 0{\cdot}645 \text{ m}$$

$\therefore$ height of the buoy outside the free water surface

$= 2 - 0{\cdot}645 = \underline{1{\cdot}355 \text{ m}}$

Problem-8 : *A rectangular pontoon, 14 m long, 10 m wide and 3 m deep weighs 100 tonnes. It carries on its upper deck a boiler of 20 m diameter and weighing 60 tonnes. If the density of sea water is 1,020 kg/m³, find the depth of immersion of the pontoon. Determine the weight of water to be pumped in the pontoon if it is to be on the point of sinking.*

Let, W_1 – weight of the pontoon,

W_2 – weight of the boiler,

W_3 – weight of water to be pumped into the pontoon.

The total weight of the pontoon before pumping of water is

$$W = W_1 + W_2 = 100 + 60 = 160 \text{ tonnes}$$

Displacement, $U = 160$ tonnes

$\therefore$ volume of water displaced, $V = \frac{W}{w} = \frac{160 \times 1{,}000}{1{,}020} = 157 \text{ m}^3$

$\therefore$ depth of immersion, $d = \frac{V}{\text{area}} = \frac{157}{14 \times 10} = \underline{1{\cdot}12 \text{ m}}$

When the pontoon is just on the point of sinking, the volume of water displaced is

$$V_1 = \text{area} \times \text{height} = 14 \times 10 \times 3 = 420 \text{ m}^3$$

$\therefore$ displacement, $U_1 = \frac{420 \times 1{,}020}{1{,}000} = 428$ tonnes

$\therefore$ weight of water to be pumped is given by

$$U_1 - U = 428 - 160 = \underline{268 \text{ tonnes}}$$

Problem-9 *A solid cone of diameter 'D' and height 'h' is floating vertically in water with its apex below the base. The semi cone angle is α, and the specific gravity of the material of cone is s. Prove that the depth of centre of gravity of the cone below the free surface of water is given by the expression* $\frac{D}{2\tan\alpha}\left(s^{1/3} - \frac{3}{4}\right)$

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Volume of cylindrical portion, $V_1 = \frac{\pi}{4} D^2 \times h$

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$$\therefore W_1 = w_1 V_1 = 350 \times 4{\cdot}5\pi = 4{,}950 \text{ kg}$$

$$W_2 = w_2 V_2 = 400 \times 0{\cdot}5 = 200 \text{ kg}$$

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Now, weight of water displaced = 5,150 kg

and volume of water displaced $= \frac{5{,}150}{1{,}020} = 5{\cdot}05 \text{ m}^3$

$\therefore$ volume of immersed cylindrical portion

= volume of water displaced – volume of water displaced by the lower curved portion

$= 5{\cdot}05 - 0{\cdot}5 = 4{\cdot}55 \text{ m}^3$

$\therefore$ depth of immersion of cylindrical portion

$$= \frac{4{\cdot}55}{\frac{\pi}{4}(3)^2} = \frac{4{\cdot}55 \times 4}{9\pi} = 0{\cdot}645 \text{ m}$$

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Problem-9 : *A solid cone of diameter 'D' and height 'h' is floating vertically in water with its apex below the base. The semi cone angle is α, and the specific gravity of the material of cone is s. Prove that the depth of centre of gravity of the cone below the free surface of water is given by the expression* $\dfrac{D}{2 \tan\alpha}\left(s^{1/3} - \frac{3}{4}\right)$

Let d be the depth of immersion.

Now, $\tan \alpha = \dfrac{D}{2h}$.

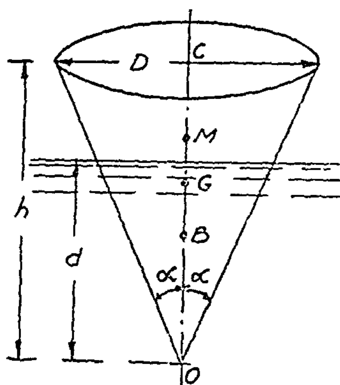


Fig. 2-4

Weight of the cone, W = volume of cone × density

$$= \frac{1}{3}\pi\left(\frac{D}{2}\right)^2 h \times sw$$

$$= \frac{1}{3}\pi h^2 \tan^2 \alpha \times hsw \text{ as } D = 2h \tan \alpha$$

∴ volume of water displaced,

$$V = \frac{W}{w} = \frac{1}{3}\pi h^3 s \tan^2 \alpha$$

Also $V = \frac{1}{3}\pi d^3 \tan^2 \alpha$

$$\therefore \quad \frac{1}{3}\pi h^3 s \tan^2 \alpha = \frac{1}{3}\pi d^3 \tan^2 \alpha$$

$$\therefore \quad h^3 s = d^3$$

$$\therefore \quad d = hs^{\frac{1}{3}}$$

Now, distance of c.g. from apex, $OG = \frac{3}{4}h$

Distance of c.g. below surface of water level is say y.

$$\therefore \quad y = d - OG$$

$$= hs^{\frac{1}{3}} - \frac{3}{4}h = h\left(s^{\frac{1}{3}} - \frac{3}{4}\right)$$

But, $h = \dfrac{D}{2 \tan \alpha}$

$$\therefore \quad y = \frac{D}{2 \tan \alpha}\left(s^{\frac{1}{3}} - \frac{3}{4}\right)$$

Problem – 10 : *A wooden cone weighing 15 kg floats in liquid of specific gravity 0·9. The apex of the cone is downward and is below the level of the liquid. If specific gravity of wood is 0·6, find what weight of steel of specific gravity 8, should be suspended with the help of thread tied to the apex of the cone to just submerge it. Determine also the tension in the thread.*

Volume of the cone, $V = \dfrac{W}{\rho} = \dfrac{15}{0{\cdot}6 \times w} = \dfrac{25}{w}$ m^3

If the cone is completely submerged, the weight of the volume of liquid that will get displaced is

$$\text{volume} \times \text{density} = \frac{25}{w} \times 0{\cdot}9 \times w = 22{\cdot}5 \text{ kg}$$

If T is tension in the string,

$$T + 15 = \text{upthrust of liquid} = 22{\cdot}5$$

$$\therefore \quad T = 22{\cdot}5 - 15 = \underline{7{\cdot}5 \text{ kg}}$$

Let, W_1 be the weight of the steel piece suspended. Then, volume of liquid displaced by it is $\dfrac{W_1}{8w}$ m^3 and upthrust of liquid on the suspended body is $\left(\dfrac{W_1}{8w}\right) \times 0{\cdot}9\ w = W_1 \times 0{\cdot}1125$ kg. For equilibrium, weight of the steel piece minus upthrust on the piece is equal to the tension in the string, *i e*

$$W_1 - W_1 \times 0{\cdot}1125 = 7{\cdot}5$$

$$\therefore \quad 0{\cdot}8875\ W_1 = 7{\cdot}5$$

$$\therefore \quad W_1 = \underline{8{\cdot}45 \text{ kg}}$$

Metacentre and Metacentric Height

When a body is floating in liquid its centre of gravity G, and centre of buoyancy B, are situated as shown in fig. 2-5.

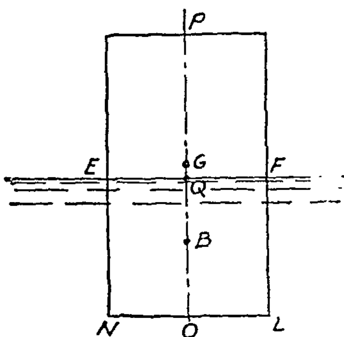


Fig. 2-5 Body floating in upright position

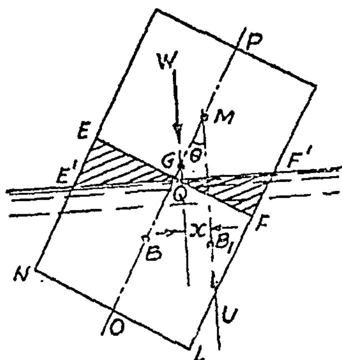


Fig. 2-6 Body is given a heel

Now let the body be tilted as shown in fig. 2-6, then the c. g. will remain on the axis PO at the same place, but the centre of buoyancy will now not be on this axis. It will be the new centre of buoyancy, B_1. The line of action of upthrust *U*, if produced, will intersect the axis PO at the point M. This point M is called the *metacentre*. Thus, metacentre can be defined as the point of intersection of the axis of the body with the line of action of the upthrust in the displaced position, when the floating body is displaced through a small angle of heel. The distance between the metacentre, M and the centre of gravity, G is known as the *metacentric height.*

Equilibrium of a Floating Body

There are three kinds of equilibrium of a floating body :

(i) Stable equilibrium,

(ii) Unstable equilibrium, and

(iii) Neutral equilibrium.

Stable equilibrium : If a body is displaced from its equilibrium position and if it tries to return to the original position, it is said to be in stable equilibrium. When M is above G, *i.e.* when meta-

centric height GM is positive the body is in stable equilibrium. Stiffer is the floating body, as greater is the value of meta-centric height ; tender is the floating body as less is the value of meta-centric height.

Unstable equilibrium : When a body is displaced from its equilibrium position, it is said to be in unstable equilibrium. This happens if metacentre M is below the centre of gravity G, *i. e.* metacentric height GM is negative.

Neutral equilibrium : When a body is displaced from its equilibrium position, and if it remains in that displaced position, it is said to be in neutral equilibrium. In this case the metacentre M coincides with G, *i. e.* metacentric height is zero.

Determination of Metacentric Height

Experimental determination : Let W be the total weight of the floating body, including two movable weights each equal to m and placed at equal distances x from the centre line as shown in fig. 2-7.

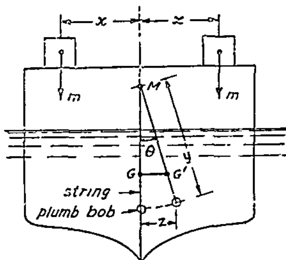


Fig. 2-7

Now, let us move the two weights to the right by an amount dx, so that the body will tilt and its C. G. will move to the right to position G'.

Taking moments about the metacentre, M, we have,

$$W \times GG' = m(x + dx) - m(x - dx)$$

$$\therefore \quad GG' = \frac{2m dx}{W} \qquad [\text{neglecting } (dx)^2 \text{ term}]$$

Also, $GG' = GM \tan\theta$

$$\therefore \quad GM = \frac{GG'}{\tan\theta} = \frac{2m dx}{W \tan\theta} \qquad ..(2{\cdot}2)$$

A plumb bob is shown hanging at the end of a plumb line of length y and its other end is fixed to some point on the floating body. The angle of heel of the floating body can be found from the apparent displacement of the body relative to the vertical. Let this apparent displacement be z. The plumb line in the tilted position of the floating body will make angle θ with the direction of the vertical. Hence, we have,

$$\tan\theta = \frac{z}{y}$$

$$\therefore \quad GM = \frac{2m dx \times y}{W \cdot z} \qquad ..(2{\cdot}3)$$

Analytical determination : When a body is floating in upright position, *pr* (fig. 2-8) is the trace of the water level on the body. Now let this body be tilted so that the trace of the water level is *qs*. When a floating body is given a small angle of heel θ, the volume of water displaced, V remains the same in the tilted position as that in the upright position. There is more displaced liquid on the right of the vertical axis EF than that on the left of it. It appears that the volume of water occupied by the triangular wedge *opq* in the upright position is shifted to the position of the triangular wedge *osr* in the tilted position of the floating body. Let g_1 and g_2 be respectively the centres of gravity of liquid wedges *opq* and *osr*.

Due to the disturbing of twisting couple, the weight of water wedge *opq* has been shifted from g_1 to g_2 over a distance of $\frac{2}{3}b$.

$\therefore$ disturbing moment = weight of water wedge $opq \times \frac{2}{3}b$

$$= w \times (\tfrac{1}{2} \times op \times pq \times l) \times \tfrac{2}{3}b$$

Since the angle of tilt is small, pq may be taken as an arc of a circle of radius, op.

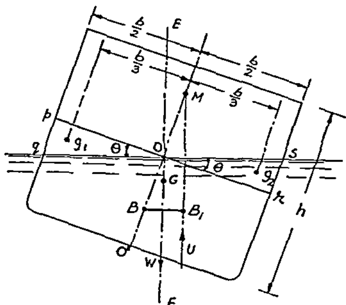


Fig. 2-8

$\therefore$ $pq = op \times \theta$; where θ is the angle in radians.

$\therefore$ disturbing moment = $w\,(\frac{1}{2} \times op \times op \times \theta \times l)\,\frac{2}{3}b$

$$= w \left\{ \frac{1}{2} \cdot \frac{b}{2} \times \frac{b}{2} \times \theta \times l \right\} \frac{2b}{3} \text{ as } op = \frac{b}{2}$$

$$= w\theta \frac{lb^3}{12} \qquad (2\text{-}4)$$

Let I be the moment of inertia of horizontal section of the body at water level about the longitudinal axis and

$$I = \frac{lb^3}{12}$$

$\therefore$ disturbing couple = $w\,\theta\,I$ (2-5)

Due to the shifting of liquid wedge *opq* from left of the vertical axis to the right of this axis in the position *osr* the point of application of upthrust U has been shifted from B to B_1.

$$\therefore \quad \text{restoring couple} = U \times BB_1$$
$$= U \times MB \sin\theta$$
$$= U \times MB \times \theta \qquad \ldots (2\cdot6)$$

(when θ is very small, $\sin\theta \simeq \theta$)

Equating the two couples, we have,

$$w \times \theta \times I = U \times MB \times \theta$$

$$\therefore \quad MB = \frac{w \times \theta \times I}{U/\theta} = \frac{I}{U/w} = \frac{I}{V}\left(\text{as } V = \frac{U}{w}\right) \ldots (2\cdot7)$$

Further, $O'B = \dfrac{d}{2}$

where, $d = \dfrac{\text{displacement volume, } V}{\text{area of cross section}}$

$$\text{and} \quad BG = O'G - O'B \qquad \ldots (2\cdot8)$$

Hence, metacentric height is given by

$$GM = MB - BG \qquad \ldots (2\cdot9)$$

Problem – 11 : *A ship has displacement of 2,000 tonnes in sea water. A weight of 15 tonnes is moved 8 m across the deck causing 2 m long pendulum to move 10 cm horizontally. Find the metacentric height of the ship.*

Referring to fig. 2–7,

$$\tan\theta = \frac{10/100}{2} = 0\cdot05$$

Again, referring to eqn. (2·2),

$$GM = \frac{2mx}{W\tan\theta} = \frac{2 \times 15 \times 1{,}000 \times 8/2}{2{,}000 \times 1{,}000 \times 0\cdot05} = \underline{1\cdot2 \text{ m}}$$

Problem-12: *A cylindrical buoy 2 m in diameter, and 4 m long is floating vertically in sea. The density of sea water is 1,020 kg/m³. If the metacentric height of the buoy is 0·5 m, determine the density of its material and the weight of the buoy.*

Let W be the weight of the buoy and ρ the density of its material in kg/m³.

Then, $W = \rho \times \frac{\pi}{4} d^2 h = \rho \times \frac{\pi}{4} (2)^2 \times 4 = 4\pi\rho$ kg .. (i)

Displacement, $U = W$

Volume of displacement, $V = \frac{U}{w} = \frac{4\pi\rho}{1{,}020}$ m³

Depth of immersion $= \frac{4\pi\rho}{1{,}020 \times \pi (2)^2} = \frac{4\rho}{1{,}020}$ m .. (ii)

Hence O′B = ½ × depth of immersion

$$= \frac{1}{2} \times \frac{4\rho}{1{,}020} = \frac{\rho}{510} \text{ m}$$

O′G = ½ × height of the buoy = ½ × 4 = 2 m

$\therefore$ BG = O′G − O′B $= \left(2 - \frac{\rho}{510}\right)$ m

Distance $BM = \frac{I}{V} = \frac{\frac{\pi}{64}(2)^4}{\frac{4\pi\rho}{1{,}020}} = \frac{1{,}020}{16\rho}$

But, $GM = BM - BG$

$$\therefore \quad 0{\cdot}5 = \frac{1{,}020}{16\rho} - \left(2 - \frac{\rho}{510}\right)$$

$$\therefore \quad 2{\cdot}5 = \frac{1{,}020}{16\rho} + \frac{\rho}{150}$$

$$\therefore \quad 2{\cdot}5 = \frac{63{\cdot}6}{\rho} + \frac{\rho}{510}$$

$$\therefore \quad 510 \times 2{\cdot}5\rho = 63{\cdot}6 \times 510 + \rho^2$$

$$\therefore \quad \rho^2 - 1{,}275\rho + 32{,}400 = 0$$

$$\therefore \quad \rho = 32{\cdot}5 \text{ or } 1242{\cdot}5 \text{ kg/m}^3$$

From eqn. (ii) of depth of immersion it is seen that only the first value of density is acceptable. Hence density is 32·5 kg/m³.

The weight of the buoy as given by eqn. (i) above is

$$W_1 = 4\pi\rho = 4\pi \times 32{\cdot}5 = \underline{406 \text{ kg}}$$

Problem–13 : *A rectangular pontoon, 13 m long, 10 m broad and 3 m deep weighs 100 tonnes. It carries on its upper deck a boiler of 7 m diameter and weighing 60 tonnes. The centres of gravity of the boiler and pontoon may be assumed to be at their geometrical centres and in the same vertical line. Find the metacentric height. Density of sea water is 1,020 kg/m³.*

Let W_1 – weight of the pontoon, and

W_2 – weight of the boiler.

The total weight of the two is

$W = W_1 + W_2 = 100 + 60 = 160$ tonnes

Displacement, $U = 160$ tonnes

Volume of water displaced,

$$V = \frac{W}{w} = \frac{160 \times 1{,}000}{1{,}020} = 157 \text{ m}^3.$$

Depth of immersion,

$$d = \frac{V}{\text{area}} = \frac{157}{13 \times 10} = 1{\cdot}208 \text{ m}$$

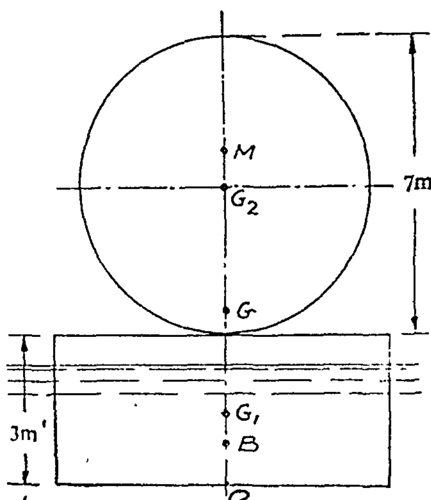


Fig. 2–9

Referring to fig. 2–9,

$$\text{distance OB} = \frac{d}{2} = \frac{1{\cdot}208}{2} = 0{\cdot}604 \text{ m}$$

where B is centre of buoyancy.

In fig. 2–9, G_1, G_2 are the respective centres of gravity of the pontoon and boiler and G is the common c.g.

$$\text{Then, } OG = \frac{W_1 \times OG_1 + W_2 \times OG_2}{W_1 + W_2}$$

$$= \frac{100 \times 1{\cdot}5 + 60 \times (3 + 3{\cdot}5)}{160} = 3{\cdot}375 \text{ m}$$

The metacentre M is given by eqn. (2·7) as

$$BM = \frac{I}{V} = \frac{\frac{1}{12} \times 13 \times 10^3}{157} = 6{\cdot}9 \text{ m}$$

Now, $BG = OG - OB = 3{\cdot}375 - 0{\cdot}604 = 2{\cdot}771$ m

and the metacentric height,

$$GM = BM - BG = 6{\cdot}9 - 2{\cdot}771 = \underline{4{\cdot}129 \text{ m}}$$

Problem-14 : *A vessel has length of 55 m, a beam of 9 m, and a displacement of 1,200 tonnes. A weight of 18 tonnes, when moved 7 m across the deck, inclines the vessel by 5°. The second moment of the water level plane about its fore and aft axis is 60% of the second moment of the circumscribing rectangle, and position of the centre of buoyancy is 1·2 m below the water line. Find the position of the metacentre and the centre of gravity of the vessel. The weight of one cu. m of sea water be taken as 1,020 kg.*

Since the upthrust is 1,200 tonnes, the volume of water displaced is

$$V = \frac{U}{w} = \frac{1{,}200 \times 1{,}000}{1{,}020} = 1{,}178 \text{ m}^3$$

The moment of inertia of the water level cross-section of vessel is

$$I = 0{\cdot}6 \times \tfrac{1}{12} l b^3 = 0{\cdot}6 \times \tfrac{1}{12} \times 55 \times 9^3 = 2{,}040 \text{ m}^4$$

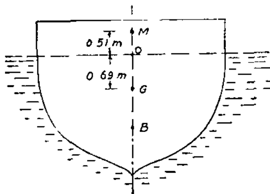


Fig. 2-10

The distance between the centre of buoyancy and the metacentre is given by

$$BM = \frac{I}{V} = \frac{2{,}040}{1{,}178} = 1{\cdot}71 \text{ m}$$

The distance between the centre of gravity and metacentre is given by eqn. (2·2) as

$$GM = \frac{2mx}{W \tan\theta} = \frac{2 \times 18 \times \frac{7}{2}}{1{,}200 \times \tan 5^\circ} = 1{\cdot}2 \text{ m}$$

Hence, $OM = BM - BO$

$= 1{\cdot}71 - 1{\cdot}2 = 0{\cdot}51$ m above the water level

Further, $OG = GM - OM = 1{\cdot}2 - 0{\cdot}51 = 0{\cdot}69$ m

Hence, it is seen that the metacentre M is 0·51 m above the water level and the centre of gravity, G is 0·69 m below the level of water.

Problem - 15 : *A solid cone is floating in water with its axis vertical such that its apex is below the base. If the cone angle is 40°, determine the sp.gr. or the cone material such that it remains in stable equilibrium.*

Let, h - height of the cone,

D - diameter of the base of the cone,

α - semi-cone angle,

d - depth of immersion of the cone,

s - sp.gr. of the material of the cone.

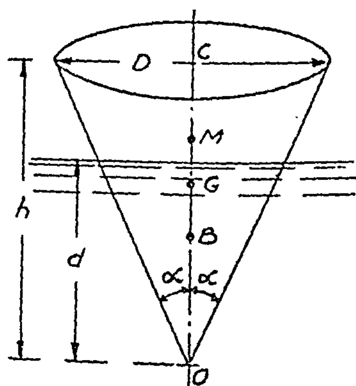


Fig. 2-11

Now, $\tan\alpha = \dfrac{D}{2h}$

Weight of cone, $W = (\frac{1}{3}\pi h^3 \tan^2\alpha)\, sw$

Displacement, $U = (\frac{1}{3}\pi d^3 \tan^2\alpha)\, w$

Now, for a floating body, $W = U$

i.e. $\left(\frac{1}{3}\pi h^3 \tan^2\alpha\right) sw = \left(\frac{1}{3}\pi d^3 \tan^2\alpha\right) w$

$$\therefore \quad d = hs^{\frac{1}{3}}$$

For the cone, $OB = \frac{3}{4}d = \frac{3}{4}hs^{\frac{1}{3}}$

and $OG = \frac{3}{4}h$

Hence $BG = OG - OB = \frac{3}{4}h\left(1 - s^{\frac{1}{3}}\right)$

$$\text{and } BM = \frac{I}{V} = \frac{\frac{\pi}{4} d^4 \tan^4\alpha}{\frac{\pi}{3} d^3 \tan^2\alpha}$$

$$= \frac{3}{4} d \tan^2\alpha$$

$$= \frac{3}{4} hs^{\frac{1}{3}} \frac{D^2}{4h^2}$$

$$= \frac{3\,D^2 s^{\frac{1}{3}}}{16 \times h}$$

For stable equilibrium $BM > BG$

$$\text{or } \frac{3\,D^2 s^{\frac{1}{3}}}{16 \times h} \geqslant \frac{3}{4}h\left(1 - s^{\frac{1}{3}}\right)$$

$$\therefore \quad h^2 \leqslant \frac{1}{4} \frac{D^2 s^{\frac{1}{3}}}{1 - s^{\frac{1}{3}}}$$

$$\therefore \quad s^{\frac{1}{3}} \tan^2\alpha \geqslant 1 - s^{\frac{1}{3}}$$

$$\therefore \quad s^{\frac{1}{3}}\left(1 + \tan^2\alpha\right) \geqslant 1$$

$$\therefore \quad s^{\frac{1}{3}} \geqslant \frac{1}{1 + \tan^2\alpha}$$

$$\text{or } \quad s^{\frac{1}{3}} = \frac{1}{1 + \tan^2\alpha} = \frac{1}{1 + 0.3640^2}$$

$$\text{or } \quad s^{\frac{1}{3}} = 0.8824$$

$\therefore$ sp. gr. of the cone material, $s = 0.6873$

Problem – 16 : *A cylindrical buoy 3 m diameter and 4 m long weighs 4 tonnes. Show that it will not float in sea water with its longitudinal axis vertical.*

Find the vertical pull applied to the chain attached to the centre of the base that will keep the buoy vertical. Density of sea water be taken as 1,020 kg/m³.

Since the weight of the buoy is 4 tonnes,

volume of sea water displaced, $V = \frac{U}{w} \times \frac{4 \times 1{,}000}{1{,}020} = 3{\cdot}92 \text{ m}^3$

$$\text{Depth of immersion, } d = \frac{\text{volume}}{\text{area of cross-section}}$$

$$= \frac{3{\cdot}92}{\frac{\pi}{4}(3)^2} = 0{\cdot}556 \text{ m}$$

Referring to fig. 2–5,

$$\text{OB} = \frac{d}{2} = 0{\cdot}278 \text{ m}$$

$$\text{and OG} = \frac{h}{2} = 4/2 = 2 \text{ m}$$

$$\text{Now, BG} = \text{OG} - \text{OB} = 2 - 0{\cdot}278 = 1{\cdot}722 \text{ m}$$

$$\text{and BM} = \frac{I}{V} = \frac{\frac{\pi}{64}D^4}{\frac{\pi}{4}D^2 \times d} = \frac{\frac{\pi}{64}(3)^4}{\frac{\pi}{4}(3)^2 \times 0{\cdot}556}$$

$$= 1{\cdot}01 \text{ m}$$

$$\therefore \quad \text{GM} = \text{BM} - \text{BG} = 1{\cdot}01 - 1{\cdot}722 = -0{\cdot}712$$

The negative sign indicates that M lies below G and hence the cylinder is in unstable equilibrium. It, therefore, cannot float in sea water with its longitudinal axis vertical.

Let P kg be the pull applied to the chain attached to the centre of the base.

New displacement, $U_1 = 4 \times 1{,}000 + P = (4{,}000 + P)$ kg

New volume of sea-water displaced, $V_1 = \frac{U_1}{w} = \frac{4{,}000 + P}{1{,}020}$ m³

$$\text{and new depth of immersion, } d_1 = \frac{\text{volume, } V_1}{\text{cross-sectional area}}$$

$$= \frac{4{,}000 + P}{1{,}020 \times 2{\cdot}25\pi} = \frac{4{,}000 + P}{7{,}200} \text{ m}$$

$$\text{Now, } OB_1 = \frac{d_1}{2} = \frac{4{,}000 + P}{2 \times 7{,}200} = \frac{4{,}000 + P}{14{,}400} \text{ m}$$

$$B_1M_1 = \frac{I}{V} = \frac{\frac{\pi}{64}(3)^4}{\frac{4{,}000 + P}{1{,}020}} = \frac{4{,}070}{4{,}000 + P} \text{ m}$$

Taking moments about the base of the cylinder,

$$(4{,}000 + P)\ OG_1 = 4{,}000 \times OG + P \times 0 = 8{,}000$$

$$\therefore \quad OG_1 = \frac{8{,}000}{4{,}000 + P} \text{ m}$$

$$\text{Now, } B_1G_1 = OG_1 - OB_1 = \frac{8{,}000}{4{,}000 + P} - \frac{4{,}000 + P}{14{,}400}$$

For stable equilibrium, $B_1M_1 > B_1G_1$

$$i.e. \quad \frac{4{,}070}{4{,}000 + P} > \frac{8{,}000}{4{,}000 + P} - \frac{4{,}000 + P}{14{,}400}$$

$$\therefore \quad \frac{4{,}000 + P}{14{,}400} > \frac{8{,}000 - 4{,}070}{4{,}000 + P}$$

$$\therefore \quad \frac{4{,}000 + P}{14{,}400} > \frac{3{,}930}{4{,}000 + P}$$

$$\therefore \quad (4{,}000 + P)^2 > 14{,}400 \times 3{,}930$$

$$\therefore \quad 4{,}000 + P > \sqrt{56{,}500{,}000} = 7{,}520$$

$$\therefore \quad P > 7{,}520 - 4{,}000$$

$$\therefore \quad P > 3{,}580 \text{ kg}$$

$\therefore$ minimum pull required, P = 3,580 kg.

Problem – 17 : *A solid cylinder 30 cm in diameter is composed of two materials. Its lower part of 5 cm length is of a material of specific gravity 4, while the remainder of it is of material of specific gravity 0·4. Find the maximum total length of the cylinder such that it may float with stable equilibrium with its axis vertical.*

Let, y = length of cylinder of material sp. gr. 0·4, cm

W_1 = weight of lower portion of cylinder, gm

W_2 = weight of upper portion of cylinder, gm

Then, $W_1 = \frac{\pi}{4} d^2 l s w = \frac{\pi}{4} 30^2 \times 5 \times 4w = 14{,}150w$ gm

and $W_2 = \frac{\pi}{4} 30^2 \times y \times 0{\cdot}4w = 283yw$ gm

Total weight of the cylinder,

$$W = W_1 + W_2 = 14{,}150w + 283yw = 283w\,(50 + y)$$

Displacement, $U = 283w\,(50 + y)$ gm

Volume of water displaced, $V = \frac{U}{w} = 283\,(50 + y)$ cm^3

Depth of immersion, $d = \frac{V}{\text{area}} = \frac{283\,(50 + y)}{\frac{\pi}{4} 30^2} = 0{\cdot}4\,(50 + y)$ cm

Now, $OB = \frac{d}{2} = 0{\cdot}2\,(50 + y)$ cm

and $BM = \frac{I}{V} = \frac{\frac{\pi}{64} 30^4}{283\,(50 + y)} = \frac{140}{(50 + y)}$ cm

The height of the C.G. from the bottom is given by

$$x = \frac{W_1 x_1 + W_2 x_2}{W_1 + W_2} = \frac{14{,}150w \times \frac{5}{2} + 283yw \times (5 + y/2)}{283w(50 + y)}$$

$$= \frac{125 + 5y + 0{\cdot}5y^2}{(50 + y)}$$

Hence, $OG = \bar{x} = \frac{125 + 5y + 0{\cdot}5y^2}{(50 + y)}$

and $BG = OG - OB$

For stable equilibrium, $BM > BG$ *i.e.* $BM > OG - OB$

$$\text{i.e. } \frac{140}{50 + y} > \frac{125 + 5y + 0{\cdot}5y^2}{50 + y} - 0{\cdot}2\,(50 + y)$$

$$\text{or } 0{\cdot}2\,(50 + y) > \frac{125 + 5y + 0{\cdot}5y^2}{50 + y} - \frac{140}{50 + y}$$

$$\text{or } 50 + y > \frac{2{\cdot}5y^2 + 25y - 75}{50 + y}$$

$$\text{or } (50 + y)^2 > 2{\cdot}5y^2 + 25y - 75$$

Simplifying this further, we have,

$$1{\cdot}5y^2 - 75y > 2{,}575$$

Trying values of y as 70, 71 and 72, it is seen that $y = 72$ cm, the value of L. H. S. is near the value of R. H. S. though less. Hence y may be kept less than 72. Hence overall height of the cylinder may be kept as $5 + 72 = \underline{77 \text{ cm}}$

TUTORIAL - 2

Buoyancy

1 A barge is 14 m long, 4 m wide and 2·5 m deep. The thickness of the sides and bottom of the barge which is constructed of sheet iron is 10 mm. If the density of the material is $7{\cdot}75 \times 10^{-3}$ kg/cm^3, what extra load is required to wet the barge upto 1·5 m in water? Determine also the depth of immersion without the extra load.

[72,700 kg, 20·2 cm]

2 A log of wood 3 m in diameter and 7 m long is placed horizontally in water. If the specific gravity of wood is 0·5, determine the depth of immersion. Density of sea water is 1,040 kg/cu. m.

[69 cm]

3 A solid cube of wood of sp gr. of 0·9 floats in water with a face parallel to water plane. If the length of each edge is 25 cm, determine the depth of immersion. What will be the extra load to be placed on the cube to just sink it in water ?

[22·5 cm; 1·56 kg]

4 A ship has displacement of 2,000 tonnes in sea water with 1·5 m free board. Determine the volume of the ship below the water line. The vessel has a length of 70 m and beam of 10 m at the surface of water level and this is constant above water level. If the free board is to be 0·5 metre, determine the weight of the extra cargo carried by the ship.

[1,960 cu.m; 714 tonnes]

5 A cylindrical buoy 3 m long and weighing 400 kg floats vertically in water so that it is immersed upto 2 m height. When the same buoy was placed in a liquid having specific gravity s its depth of immersion was 1·5 m. Determine sp. gr. s and density of the material of the buoy.
[1·335; 66 kg/cu.m]

6 A rectangular pontoon has length of 20 m and breath 5 m when the pontoon floats vertically in sea water. The free-board must not be less than 1 m. The pontoon just sinks in fresh water when a weight of 92 tonnes is placed on its top. Determine the weight of the pontoon and the height of the pontoon. Density of sea water is 1,040 kg/cu.m. Density of fresh water is 1,000 kg/cu.m. [208 tonnes, 3 m]

7 A wooden cone weighing 20 kg floats in liquid of specific gravity 0·8. The apex of the cone is downward and is below the level of liquid. If specific gravity of wood is 0·6, find what weight of steel of specific gravity 8 should be suspended with the help of thread tied to the apex of the cone to just submerge it. Determine also the tension in the thread.
[7·33 kg; 6·6 kg]

8 A solid cylinder 30 cm in diameter is composed of two materials. Its lower part of 5 cm length is of a material of sp. gr. 4, while the remainder of it is of material of sp. gr. 0·4. If the total length of the cylinder is 80 cm, determine the position of the C. G. of the whole cylinder with respect to water surface.
[23·5 cm]

9 A cylindrical buoy 2·6 m diameter and 4 m long weighs 4 tonnes. Find the vertical pull applied to the chain attached to the centre of the base that will keep the buoy floating vertically in sea water with 0·4 height outside the water surface. Density of sea water may be taken as 1,020 kg/cu.m.

10 A solid square pyramid is floating in water with its axis vertical such that its apex is below the base. The height of the pyramid is 2 m and the length of each side of the base is 1·5 m. If the density of the material is 690 kg/cu.m, determine the depth of immersion. [1·77 m]

11 A rectangular pontoon 13 m long, 10 m broad and 4 m deep weighs 200 tonnes. It carries on its upper deck a boiler of 7 m diameter and weighing 60 tonnes. The centres of gravity of the boiler and pontoon may be assumed to be at their geometrical centres and in the same vertical plane. Find the distance of the C. G. of the combined body from the water line. Density of water 1,000 kg/cu.m.

[2·06 m above the water line]

12 A buoy carrying a beacon light has the upper portion cylindrical of 2 m diameter and 4 m deep. The lower portion which is hemispherical has a sp. gr. of 1·5. The upper portion has sp. gr. of 0·3. The centre of gravity of whole buoy and beacon is situated 2·4 m below the top of the cylinder and the water line is 2 m below the top of the cylinder. Determine the weight of the beacon and its C. G. from the top of the cylinder. Density of sea water is 1,020 kg/cu.m.

[1,648 kg, 51·6 cm]

Metacentric height

13 A cylinder has diameter of 45 cm and relative density 0·9 If the cylinder is floating in water with its axis vertical, determine the maximum permissible length of the cylinder. [53 cm]

14 A solid square pyramid is floating in water with its axis vertical such that its apex is below the base. The height of the pyramid is 2 m and the length of each side of the base is 1·5 m If the density of the material is 690 kg/m^3 determine the metacentric height. [7·3 cm]

15 A ship has displacement of 2,000 tonnes in sea water. A weight of 15 tonnes is moved 8 m across the deck causing 3 m long pendulum to move 18 cm horizontally. Find the metacentric height of the ship [1 m]

16 A cylindrical buoy 2 m diameter and 3 m long is floating vertically in sea The density of sea water is 1,040 kg/m^3. If the metacentric height of the buoy is 0·5 m, determine the density of its material and the weight of the buoy.

[45 kg/m^3, 424 kg]

17 A rectangular pontoon 13 m long, 10 m broad and 4 m deep weighs 100 tonnes. It carries on its upper deck a boiler of 7 m diameter and weighing 60 tonnes. The centres of gravity of the boiler and pontoon may be assumed to be at their geometrical centres and in the same vertical line. Find the metacentric height. Density of sea water is 1,020 kg/m³.

[3·455 m]

18 A wooden cone weighing 30 kg floats in liquid of sp. gr 0·9 with its apex below the free surface of the liquid. If the sp. gr. of wood is 0·6, find what weight of steel of sp. gr. 8 must be suspended from the apex of the cone by a thread just to submerge the cone. Determine also the tension in the thread.

[16·9 kg, 15 kg]

19 A cylindrical buoy 3 m diameter and 4 m long, weighs 4 tonnes. Show that it will not float in sea water with its axis vertical.

Find the vertical pull applied to the chain attached to the centre of the base, that will keep the buoy vertical. Density of sea water may be taken as 1,020 kg/m³.

[3,500 kg]

20 A vessel has a length of 60 m, a beam of 10 m and a displacement of 1,200 tonnes. A weight of 18 tonnes, moved 7 m across the deck, inclines the vessel 5°. The second moment of the level water plane about its fore and aft axis is 60 per cent of the second moment of the circumscribing rectangle, and the position of the centre of buoyancy is 1·54 m below the water line. Find the position of the metacentre and the centre of gravity of the vessel. The weight of one m³ of sea water can be taken as 1,020 kg. (ref.fig. 2–12)

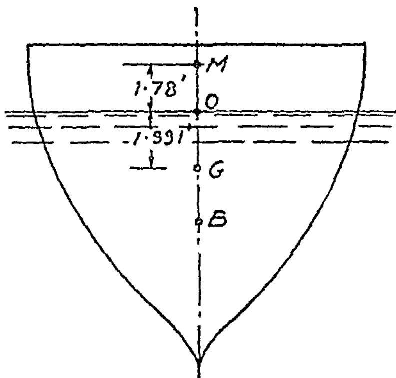


Fig. 2–12

Stable equilibrium

21 A wooden cylinder of circular section and uniform density and of sp. gr. 0·5 is required to float in oil of sp. gr. 0·9. If *the radius of the cylinder is r and its height is h*, show that h cannot be greater than $1{\cdot}42\,r$ for cylinder to float with its axis vertical.

22 A rectangular wooden block of of unit length has specific gravity s. Prove that the ratio of breadth to depth for the block to be in stable equilibrium is given by the expression, $\frac{B}{D} > \sqrt{6s\,(1-s)}$.

23 Show that a solid cylinder of length l, radius r and sp. gr. s, *floating with its axis vertical is in stable equilibrium if*

$$r > l\sqrt{2s\,(1-s)}.$$

24 A cone floating in water with its apex downwards has radius R and vertical height H. Show that for stable equilibrium

$$H^2 < \frac{R^2 s^{\frac{1}{3}}}{1 - s^{\frac{1}{3}}}$$

where s is the sp. gr. of the cone material.

25 A solid cylinder 30 cm diameter is composed of two materials. Its base for 5 cm length is of material of sp. gr. 4, while the remainder of the cylinder is of material of sp. gr. 0·4. Find the maximum overall length of the cylinder so that it may float in stable equilibrium with its axis vertical.

[86 cm]

26 State the conditions which give the stability or instability of a floating vessel.

A buoy carrying a beacon light has the upper portion cylindrical of 4 m diameter and 2 m deep The lower portion which is curved, displaces a volume of 2 m³ *and its centre* of buoyancy is situated at 2·2 m below the top of the cylinder. The centre of gravity of the whole buoy and beacon is situated at 1·6 m *below the top of the cylinder* and the total displacement is 4 tonnes. Find the metacentric height. Density of sea water is 1,020 kg/m³.

[2·73 m]

3
HYDRODYNAMICS

Introduction

It is a fundamental fact that whenever there is motion of gas or liquid, the motion is governed by basic principles of mechanics *viz* the principles of conservation of mass, conservation of energy and conservation of momentum. The principle of conservation of mass leads to the development of the equation of mass continuity. The principle of conservation of energy helps to establish the energy equation. The principle of conservation of momentum helps in establishing equation evaluating the dynamic forces exerted by flowing fluids.

Fluid Flow

When fluid flow takes place in pipe lines or in open channels, it meets with resistance of the sides of the pipe lines and internal resistance of one layer with another layer. Thus, the type of flow may be according to the conditions under which it takes place. Hence the fluid flow may be steady or unsteady ; uniform or non-uniform ; laminar or turbulent, one dimensional, two-dimensional or three dimensional ; rotational or irrotational.

Steady flow : Let a fluid flow be taking place, and let a point be selected at a station in the flow. If at this station point, the velocity of the fluid remains constant at successive periods of time, the flow is said to be a steady flow. This statement leads to the concept that the velocity is constant with respect to time *i.e.* $\frac{dV}{dt} = 0$. This does not mean that the velocity at any other station point is

the same as at this station ; it may vary at different points (*i. e.* it may vary with respect to distance). This statement also infers that other parameters of the fluid will not vary with time, *i.e.* $\frac{dp}{dt} = 0$; $\frac{d\rho}{dt} = 0$ and $\frac{dQ}{dt} = 0$ where p is pressure of the fluid; ρ is density of the fluid and Q is rate of discharge of the fluid. The steady fluid flow may be uniform or non-uniform.

With unsteady flow the fluid variables change with the time *i e. velocity, pressure, density, rate of discharge, etc. differ at the same* station point at successive periods of time. This type of flow is beyond the scope of this volume.

Uniform flow : Uniform flow occurs when magnitude and direction of the velocity do not change from point to point in the fluid *i.e.* $\frac{dV}{dS} = 0$, *where S is the distance. This statement also* suggests that other fluid variables do not change with distance *i.e.* $\frac{dp}{dS} = 0$, $\frac{d\rho}{dS} = 0$. The flow of liquids under pressure through long pipe lines of constant diameter is uniform. This flow may be steady or unsteady.

If the velocity, depth, pressure, etc. change from point to point in the fluid flow, it is known as non-uniform flow. This type of flow takes place in the open channels.

Stream line flow · Stream lines are imaginary curves drawn through a fluid to indicate the direction in *various sections* of the flow. The particle at any point will move instantaneously in the direction of the tangent drawn to the curve at that point.

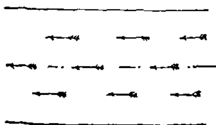


Fig. 3-1 Stream line flow

If we draw the number of tangents to any one streamline at different sections we shall get the average direction of velocity of the fluid. If the velocity is small, the average direction of velocity of the fluid will be parallel to the sides of the passage and the liquid will flow in parallel lines, which states that there is no component of the velocity perpendicular to the direction of flow. Thus stream line flow can be defined as the flow of fluid in which the directions of velocities are parallel to the sides of passage and in the form of lines, *i e.* particles move in straight lines.

Turbulent or eddy flow : If the velocity of the particles of the fluid is large, the particles will not move in straight lines as discussed above but will move in cross currents or eddies; resulting in turbulent or eddy flow. This type of phenomena causes greater resistance to flow and the velocity of the liquid is not uniform over the cross-section, it is small near the sides of the passage. The cross-currents will cause an additional loss of energy due to friction, generated by the cross-flow of the fluid particles.

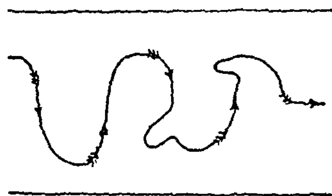


Fig. 3-2 Turbulent flow

Equation of Mass Continuity

The principle of conservation of mass gives rise to the equation of continuity. For a steady flow, the mass of fluid passing through each transverse section (fig. 3-3) in a given interval of time must be equal.

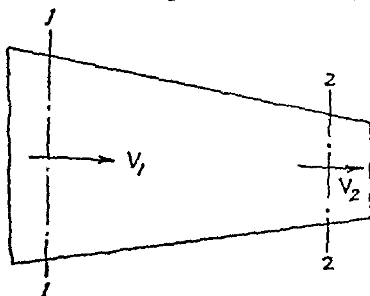


Fig. 3-3

Thus, mass $= \rho_1 A_1 V_1$

$= \rho_2 A_2 V_2 = \text{const.}$

or $w_1 A_1 V_1 = w_2 A_2 V_2$

For an incompressible fluid like water in which the density remains constant at all sections at all times ; $w_1 = w_2 = w$.

Then, we have

$$\text{discharge, } Q = A_1V_1 = A_2V_2 = \text{const.} \quad \quad .. (3\cdot1)$$

where A_1 and A_2 are the cross-sectional *areas of section 1-1 and* section 2-2 respectively, and V_1 and V_2 are the average velocities of stream at those sections.

Energies of Fluid

Kinetic energy of liquid in motion: Let V be velocity of the liquid and W the weight of liquid.

The kinetic energy of the liquid is given by

$$\left.\begin{aligned} \text{K. E.} &= \frac{WV^2}{2g} \\ &= \frac{V^2}{2g} \text{ per unit weight of the liquid.} \end{aligned}\right\} \quad .. (3\cdot2)$$

The kinetic (velocity) head of the liquid will be $\frac{V^2}{2g}$.

Potential energy of liquid: The potential energy (P.E.) is by virtue of the position of the liquid with respect to some datum level. If Z is the height (in metres) for a point in a flowing or steady liquid above a horizontal datum, and W is weight (kg) of liquid at the point considered,

$$\left.\begin{aligned} &\text{the P E.} = WZ \text{ (kg-m)} \\ &\text{or P.E.} = Z \text{ (kg-m per kg of liquid)} \end{aligned}\right\} \quad .. (3\cdot3)$$

The potential head of liquid at that point will be Z (m) of liquid.

Pressure energy of liquid: If in addition to the static head the liquid has pressure also, there is additional static energy known as pressure energy. Let p be intensity of pressure (in kg/m²) at *a certain point in flowing or steady liquid* and w be its density (in kg/m³).

$$\left.\begin{aligned} &\text{Then, pressure energy} = \frac{p}{w} \text{ (kg-m per kg of liquid)} \\ &\text{and pressure head} = \frac{p}{w} \text{ (m of liquid)} \end{aligned}\right\} \quad .. (3\cdot4)$$

Pressure head is $\frac{10^4 \times p}{w}$ m of liquid where p is pressure in kg/cm²

and w is density of the liquid in kg/m³.

Total energy of flowing liquid : The total energy of flowing liquid is the sum of its pressure energy, kinetic energy, and potential energy. If E is total energy of a flowing liquid,

$$E = \left\{ \frac{p}{w} + \frac{V^2}{2g} + Z \right\} \text{(kg-m/kg of liquid)} \quad .. \ (3\cdot5)$$

Total head of flowing liquid : It is the sum of its pressure head, the velocity head, and potential or static head. If H is total head of a flowing or steady liquid,

$$H = \left(\frac{p}{w} + \frac{V^2}{2g} + Z \right) \text{ m of liquid} \quad .. \quad .. \quad .. \ (3\cdot6)$$

Energy Equation

The energy equation results from the application of the principle of conservation of energy of flowing fluid. When there is continuous flow of the fluid, the total energy at every section must remain the same.

Let, H_a – energy added to a flowing fluid per kg,
H_l – energy lost in a flowing fluid per kg, *and*
H_e – energy extracted from a flowing fluid per kg.

Then, $\left\{ \begin{matrix} \text{energy at} \\ \text{section 1–1} \end{matrix} \right\} + \left\{ \begin{matrix} \text{energy} \\ \text{added} \end{matrix} \right\} - \left\{ \begin{matrix} \text{energy} \\ \text{extracted} \end{matrix} \right\} - \left\{ \begin{matrix} \text{energy} \\ \text{lost} \end{matrix} \right\}$
$= \left\{ \begin{matrix} \text{energy at} \\ \text{section 2–2} \end{matrix} \right\}$

The equation for steady flow of incompressible fluid can be written as

$$\left(\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 \right) + H_a - Hl - H_e = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2 \quad .. \ (3\cdot7)$$

If there is no addition, loss or extraction of energy while the flow is taking place, the above equation reduces to

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2 \quad .. \quad .. \quad .. \quad .. \ (3\cdot8)$$

This equation is known as *Bernoulli's theorem.*

Bernoulli's theorem : In 1738 Daniell Bernoulli (a Swiss engineer) gave the theorem for frictionless and incompressible liquid. This theorem states that whenever there is a continuous flow of liquid the total energy at every section remains the same provided that there is no loss or addition of the energy. It can be verified with the help of the diagram of fig. 3-4.

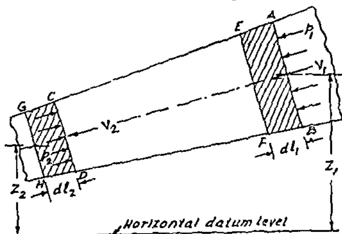


Fig. 3-4

Consider water flowing through a tapering pipe in such a way that pipe is running full and is under pressure. Let us consider small volume of water in section ABCD of the pipe, and let it move to a position EFGH through a small movement, such that distance between section AB and EF is dl_1 and the distance between the sections CD and GH is dl_2. The above movement of water may be looked upon as the water contained between the sections EF and CD as steady while water between the sections AB and EF is bodily transferred to the position between sections CD and GH. This shows that weight of water, W_1 contained between the sections AB and EF is the same as the weight of water between sections CD and GH. Let Z_1 and Z_2 be the heights above horizontal datum of section AB and CD respectivety, p_1 and p_2 be pressures at sections AB and CD respectively, V_1 and V_2 be velocities of water at sections AB and CD respectively and A_1 and A_2 be areas of cross-sections of the pipe at sections AB and CD respectively.

Then, $W_1 = w \times A_1 \times dl_1$

and $W_2 = w \times A_2 \times dl_2$

But $W_1 = W_2 = W$ (say)

Then, $\frac{W}{w} = A_1 \, dl_1 = A_2 dl_2$ .. ..(i)

Total pressure of water in the forward direction at section AB is

$$P_1 = p_1 A_1$$

Work done in the forward direction at section AB is

$$P_1 \times dl_1 = p_1 A_1 dl_1$$

Total pressure of water in the backward direction at section 2 is

$$P_2 = p_2 A_2$$

The work done in the backward direction at section CD is

$$P_2 \times dl_2 = p_2 A_2 dl_2$$

∴ net work done by pressure, W.D. $= p_1 A_1 dl_1 - p_2 A_2 dl_2$

Since, $A_1 dl_1 = A_2 dl_2$ from eqn. (i) above,

$$\text{W. D.} = (p_1 - p_2) A_1 dl_1$$

$$= \frac{W}{w} (p_1 - p_2) \text{ as } A_1 dl_1 = \frac{W}{w}.$$

P.E. at section AB is $W_1 Z_1$ and P.E. at section CD is $W_2 Z_2$.

∴ loss of P.E. $= W_1 Z_1 - W_2 Z_2$

$= W(Z_1 - Z_2)$

K.E. at section AB is $\frac{W_1 V_1^2}{2g}$ and K.E. at the section CD is $\frac{W_2 V_2^2}{2g}$.

∴ gain in K.E. $= \frac{W_2 V_2^2}{2g} - \frac{W_1 V_1^2}{2g}$

$= \frac{W}{2g} (V_2^2 - V_1^2)$ as $W_1 = W_2 = W$.

The loss of potential energy and work done by pressure have been converted into additional kinetic energy.

∴ loss of P.E. + work done by pressure = gain of K.E.

$$\text{or} \quad W(Z_1 - Z_2) + \frac{W}{w}(p_1 - p_2) = \frac{W}{2g}(V_2^2 - V_1^2)$$

$$\therefore \quad Z_1 + \frac{p_1}{w} + \frac{V_1^2}{2g} = Z_2 + \frac{p_2}{w} + \frac{V_2^2}{2g} \quad \text{.. (3 9)}$$

Problem – 1 : *Water is flowing into a horizontal tapered pipe having 15 cm diameter at the larger end and 5 cm diameter at the smaller end. The velocity of water at the larger end is 2·5 m/sec. Determine the rate of discharge in litres/min. and the velocity head at the small end.*

Quantity of water flowing through the pipe is

$$Q = A \times V = \frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times 2{\cdot}5$$

$$= 0{\cdot}0441 \text{ cu. m./sec.}$$

$$\text{or} = 0{\cdot}0441 \times 1{,}000 \times 60 = \underline{2{,}642 \text{ litres/min.}}$$

Since $A_1V_1 = A_2V_2$

$$V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{D_1}{D_2}\right)^2 \times V_1 = \left(\frac{15}{5}\right)^2 \times 2{\cdot}5$$

$$= 22{\cdot}5 \text{ m/sec.}$$

$$\therefore \quad \textit{velocity head, } \frac{V_2^2}{2g} = \frac{(22{\cdot}5)^2}{2 \times 9{\cdot}81} = \underline{26{\cdot}05 \text{ m of water}}$$

Problem – 2 : *The diameter of a pipe changes uniformly from 45 cm at a point A, 6 m above the datum level to 15 cm at B, 2·5 m*

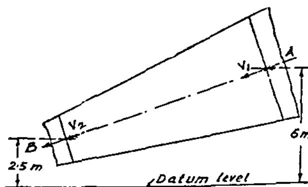


Fig. 3-5

above the datum as shown in fig. 3-5. The pressure at A is 55 kg/cm². and velocity of flow is 1·4 m/sec. Assuming that no loss takes place between A and B, find the pressure at B. Take gravitational acceleration, g = 9·81 m/sec².

Here density of water, w = 1,000 kg/m³ and 1 kg/cm² = 10,000 kg/m².

Since the discharge is constant at all sections,

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{D_1}{D_2}\right)^2 \times V_1$$

$$= \left(\frac{45}{15}\right)^2 \times 1{\cdot}4 = 12{\cdot}6 \text{ m/sec.}$$

From Bernoulli's theorem, (eqn. 3·9)

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2$$

$$i.e. \; \frac{55 \times 10{,}000}{1{,}000} + \frac{1{\cdot}4^2}{2 \times 9{\cdot}81} + 6 = \frac{10{,}000 p_2}{1{,}000} + \frac{12{\cdot}6^2}{2 \times 9{\cdot}81} + 2{\cdot}5$$

$$\text{or } 550 + 0{\cdot}1 + 6 = 10p_2 + 8{\cdot}1 + 2{\cdot}5$$

$\therefore$ pressure at B, p_2 = 54·55 kg/cm²

Problem – 3 : *A pipe line is used to carry oil of specific gravity 0·877, and it changes in size from 22 cm at section P to 60 cm at section Q. Section P is 3 m lower than section Q and the pressures are 1 kg/cm² and 0·7 kg/cm² respectively. If the discharge is 0·17 m³/sec, determine the loss of head and the direction of flow.*

The average velocity at each cross section = $\dfrac{Q}{A}$

Let, V_1 – velocity of oil at the section P, and

V_2 – velocity of oil at section Q.

$$\text{Then, } V_1 = \frac{Q}{A_1} = \frac{0{\cdot}17}{\frac{\pi}{4}\left(\frac{22}{100}\right)^2} = 4{\cdot}48 \text{ m/sec}$$

$$\text{and } V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{D_1}{D_2}\right)^2 \times V_1 = \left(\frac{22}{60}\right)^2 \times 4{\cdot}48$$

$$= 0{\cdot}587 \text{ m/sec}$$

Using the section P as datum,

$$\text{Total energy at P} = \frac{10{,}000 p_1}{sw} + \frac{V_1^2}{2g} + Z_1$$

$$= \frac{10{,}000 \times 1}{0.877 \times 1{,}000} + \frac{4.48^2}{2 \times 9.81} + 0$$

$$= 11.4 + 1.02 + 0$$

$$= 12.42 \text{ kg-m/kg of oil}$$

$$\text{Total energy at Q} = \frac{10{,}000 p_2}{sw} + \frac{V_2^2}{2g} + Z_2$$

$$= \frac{10{,}000 \times 0.7}{0.877 \times 1{,}000} + \frac{0.587^2}{2 \times 9.81} + 3$$

$$= 7.98 + 0.0176 + 3$$

$$= 10.997 \text{ kg-m/kg of oil}$$

Since the total energy at P is greater than that at Q, <u>the flow takes place from P towards Q.</u>

Again, loss of head, $h_l = 12.42 - 10.977 = $ <u>1.423 m of oil</u>

Problem-4 · *A conical pipe has diameter 60 cm at the larger end and 15 cm at the smaller end and forms part of a vertical main. The pressure head at the larger end is found to be 20 m and at the small end 15 m of water. Find the discharge through the pipe if the length of the conical portion is 1.5 m.*

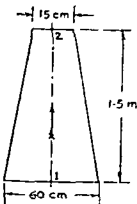


Fig. 3-6

Since discharge, $Q = A_1V_1 = A_2A_2$

$$V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{D_1}{D_2}\right)^2 \times V_1$$

$$= \left(\frac{60}{15}\right)^2 \times V_1$$

$$= 16\ V_1$$

Writing Bernoulli's theorem as

$$\frac{10,000p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{10,000p_2}{w} + \frac{V_2^2}{2g} + Z_2$$

and considering the larger end as datum, $Z_1 = 0$ and $Z_2 = 1{\cdot}5$ m

$$20 + \frac{V_1^2}{2g} + 0 = 15 + \frac{(16\ V_1)^2}{2g} + 1{\cdot}5$$

$$\therefore \quad 3{\cdot}5 = \frac{256V_1^2}{2g} - \frac{V_1^2}{2g} = \frac{255V_1^2}{2g}$$

$$\therefore \quad V_1 = \sqrt{\frac{2 \times 9{\cdot}81 \times 3{\cdot}5}{255}} = 0{\cdot}2695 \text{ m}^3/\text{sec}$$

$$\text{Hence, } Q = A_1V_1 = \frac{\pi}{4}\left(\frac{60}{100}\right)^2 \times 0{\cdot}2695 = \underline{0{\cdot}0761 \text{ m}^3/\text{sec}}$$

Problem-5 : *A 30 m long pipe CD is inserted in a pipe line inclined at 60° to the horizontal. At C, which is at higher level, the diameter is 15 cm. At D the diameter is 30 cm, the pressure is 45 kg/cm², and velocity of flow is 3 m/sec. Neglecting all losses, determine the pressure head at C, when the flow is downward.*

If water flows from lower to higher level and the head loss due to friction is 1·5 m of water, find the difference of pressure heads at C and D.

Determine also the energy lost per second and the horse power required to lift water from D to C.

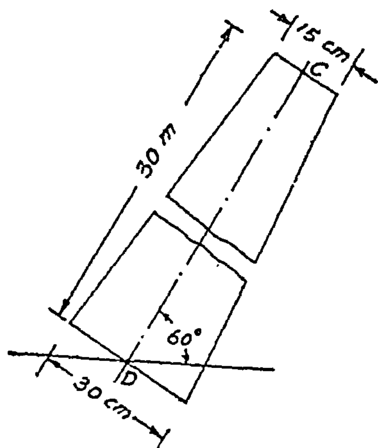


Fig. 3-7

Consider the lower end as datum. Height of the upper end C above the lower end D is

$$Z = Z_C - Z_D = l \sin\theta$$
$$= 30 \times \sin 60° = 26 \text{ m}$$

(Below suffix 1 & 2 refer to D & C respectively)

Hence, $Z_1 = 0, Z_2 = 26$ m

Since, $Q = A_1V_1 = A_2V_2$

$$\therefore \quad V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1 = \left(\frac{30}{15}\right)^2 \times 3 = 12 \text{ m/sec}$$

Applying Bernoulli's theorem to sections at D & C,

$$\frac{45 \times 10{,}000}{1{,}000} + \frac{3^2}{2 \times 9{\cdot}81} + 0 = \frac{10{,}000 p_2}{1{,}000} + \frac{12^2}{2 \times 9{\cdot}81} + 26$$

$$\therefore \quad 10p_2 = 417{\cdot}1$$

$$\therefore \quad p_2 = \underline{41{\cdot}71 \text{ kg/cm}^2}$$

For flow from lower to higher level, we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2 + \text{losses}$$

$$\text{or} \quad \frac{10{,}000\,(p_1 - p_2)}{w} = (Z_2 - Z_1) + \frac{V_2^2 - V_1^2}{2g} + \text{losses}$$

$$= 26 + \frac{12^2 - 3^2}{2 \times 9{\cdot}81} + 1{\cdot}5$$

$$= \underline{34{\cdot}38 \text{ m of water}}$$

Further, weight of water flowing per sec. is

$$W = wA_1V_1 = 1{,}000 \times \frac{\pi}{4}\left(\frac{30}{100}\right)^2 \times 3$$

$$= 212 \text{ kg/sec}$$

$$\text{Energy lost} = W \times \text{head lost}$$

$$= 212 \times 1{\cdot}5 = \underline{317 \text{ kg-m/sec}}$$

Horse power required to lift water from D to C is

$$\frac{W(Z_2 - Z_1)}{75} = \frac{212 \times 26}{75} = \underline{73{\cdot}2}$$

Problem – 6 : *In a vertical pipe carrying water, pressure gauges are inserted at A and B, where the diameters are 10 cm and 5 cm respectively. The point B is 3 m below A and when the rate of*

flow down the pipe is 0·01 m³/sec, the pressure at B is 0·15 kg/cm² greater than that at A. Assuming that losses in the pipe between A and B can be expressed as $\frac{KV_A^2}{2g}$, *where* V_A *is the velocity at A, find the value of K. If the gauges at A and B are replaced by tubes filled with water and connected to a U-tube containing mercury of specific gravity 13·6, give a sketch showing how the levels of the two limbs of the U-tube stand and calculate the value of this difference, measured in cm.*

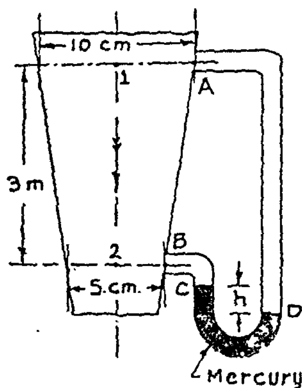


Fig. 3-8

Applying Bernoulli's eqn. to A & B,

$$\frac{10{,}000 p_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{10{,}000 p_B}{w} + \frac{V_B^2}{2g} + Z_B + K\frac{V_A^2}{2g}$$

and $A_1 V_A = A_2 V_B$

$$\therefore \quad V_B = \frac{A_1}{A_2} V_A = \left(\frac{10}{5}\right)^2 \times V_A = 4V_A$$

$$\therefore \quad \frac{10{,}000\,(p_A - p_B)}{w} + (Z_A - Z_B) = (K-1)\frac{V_A^2}{2g} + \frac{16V_A^2}{2g}$$

$$\text{or} - \frac{10{,}000 \times 0{\cdot}15}{1{,}000} + 3 = (K+15)\frac{V_A^2}{2g}$$

$$\text{Now,} \quad V_A = \frac{Q}{A} = \frac{0{\cdot}01}{\frac{\pi}{4}\left(\frac{10}{100}\right)^2} = 1{\cdot}275 \text{ m/sec}$$

$$\therefore \quad -1{\cdot}5 + 3 = (K+15)\frac{(1{\cdot}275)^2}{2 \times 9{\cdot}81}$$

$$\therefore \quad \frac{1{\cdot}5}{0{\cdot}0828} = (K+15)$$

$$\therefore \quad K = 3{\cdot}1$$

Let h cm be difference in mercury level. Total pressures in both the limbs must be equal.

$$\text{Thus,}\quad \frac{10{,}000 p_A}{w} + 3 + \frac{h}{100} = \frac{10{,}000 p_B}{w} + \frac{13{\cdot}6h}{100}$$

$$\therefore\quad \frac{10{,}000\,(p_A - p_B)}{w} + 3 = \frac{12{\cdot}6}{100}\,h$$

$$\text{or} - 1{\cdot}5 + 3 = 0{\cdot}126h$$

$$\therefore\quad h = \frac{1{\cdot}5}{0{\cdot}126} = \underline{11{\cdot}9 \text{ cm of mercury}}$$

Venturimeter

This is an instrument in which the practical application of Bernoulli's theorem is found. It is used for measuring the rate of flow of the liquid. It consists of a short piece of pipe which is converging upto certain length and then diverging for remaining length. The section where the converging and diverging parts meet

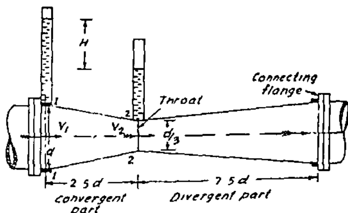


Fig. 3-9 Venturimeter

is known as the *throat* of the venturimeter. It has also two piezometer tubes, one at the entrance and other at the throat as shown in fig. 3-9. It may have pressure gauges fitted in place of these tubes. The venturimeter is bolted in a pipe line by two flanges at the two ends. In some venturimeters, tubes may be connected to a U-tube containing mercury to read difference of pressure. As liquid flows through the meter, velocity will increase

at the throat owing to the reduction of area; consequently the pressure will be reduced which may be read by the device fitted to it.

Let, H – difference of pressure head of water in the piezometer tubes,

A_1 – area of enlarged end (*i.e.* area of pipe for which the discharge is to be measured)

A_2 – area of throat,

Q – rate of discharge,

V_1 – velocity of water at entrance to venturimeter, and

V_2 – velocity of water at throat.

The theoretical discharge of water through the venturimeter is

$$Q_{th} = A_1V_1 = A_2V_2 \qquad \text{(law of mass continuity)}$$

$$\therefore \quad V_2 = \frac{A_1}{A_2}V_1$$

Applying Bernoulli's theorem to sections 1–1 and 2–2,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2$$

$$\therefore \quad \frac{p_1}{w} - \frac{p_2}{w} + Z_1 - Z_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

But, $\frac{p_1}{w} - \frac{p_2}{w} = H$, the difference of heights in the piezometer tubes

$$\therefore \quad H + Z_1 - Z_2 = \frac{V_2^2 - V_1^2}{2g} = \frac{V_1^2}{2g}\left\{\left(\frac{A_1}{A_2}\right)^2 - 1\right\}$$

$$\therefore \quad V_1 = \frac{\sqrt{2g\,(H + Z_1 - Z_2)}}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

Hence, $Q_{th} = A_1V_1 = \dfrac{A_1\sqrt{2g\,(H + Z_1 - Z_2)}}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}}$

Now, actual discharge, $Q = C_d \times Q_{th}$

where C_d is coefficient of discharge of the meter.

$$\therefore \quad Q = \frac{C_d A_1 \sqrt{2g\,(H + Z_1 - Z_2)}}{\sqrt{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

$$= \frac{C_d A_1 \sqrt{2g\,(H + Z_1 - Z_2)}}{\sqrt{\left(\frac{d_1}{d_2}\right)^4 - 1}} \quad \ldots (3\cdot10)$$

If the meter is placed in the horizontal position as shown in fig. 3-9, $Z_1 - Z_2 = 0$.

$$\therefore \quad Q = \frac{C_d A_1 \sqrt{2gH}}{\sqrt{\left(\frac{d_1}{d_2}\right)^4 - 1}}$$

(a) If the piezometer tubes of a horizontal meter are connected to a U-tube (fig. 3-10) containing mercury and the difference of pressure as indicated by height, is h cm of mercury, then, considering the equilibrium at level $x - x$.

$$\frac{p_1}{w} + \frac{h}{100} = \frac{p_2}{w} + \frac{13\cdot6h}{100} \text{ where}$$

there is water above the columns of mercury in the two limbs.

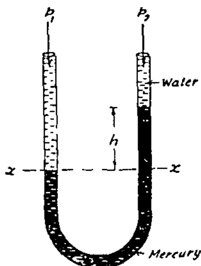


Fig. 3-10

$$\therefore \quad \frac{p_1}{w} - \frac{p_2}{w} = \frac{h}{100}\,(13\cdot6 - 1)$$

$$= \frac{12\cdot6h}{100}$$

If $\frac{p_1}{w} - \frac{p_2}{w} = H$ m of water,

then, $H = \frac{h}{100}\,(13\cdot6 - 1)$ m of water .. (3·11)

If the discharge is large with the result that pressure difference is considerable, use of mercury manometer reduces the height of tubes which would otherwise be very long if water alone is used.

(b) If the U-tube is used for measuring the flow of liquid having sp. gr. s_l, then sp. gr. of mercury with respect to that of liquid is $\frac{13 \cdot 6}{s_l}$

$$\text{and } H = \frac{h}{100}\left(\frac{13 \cdot 6}{s_l} - 1\right) \text{ m of oil } (h \text{ in cm of Hg}) \quad .. \quad (3 \cdot 12)$$

(c) If the meter is connected to an inverted U-tube manometer containing a liquid lighter than the liquid flowing in the horizontal meter, and if

s_l - sp. gr. of the liquid flowing through the meter,

s_u - sp. gr. of the liquid used in the U-tube, and

h - deflection (difference in levels) of the liquid in U-tube in centimetres.

$$\text{then, } H = \frac{h}{100}\left(1 - \frac{s_u}{s_l}\right) \text{ m of liquid} \quad .. \quad .. \quad (3 \cdot 13)$$

After finding the value of H, shown by any one of the above methods, discharge can be found with the help of eqn. (3·10a).

This will be true for any meter weather horizontal, vertical or inclined as the differential manometer takes into account the value of $(Z_1 - Z_2)$ also. This is illustrated by problem-9. In case the value of the venturi-head, H is determined by piezometer tube or pressure gauge the value of $(Z_1 - Z_2)$ must be taken into account and in that case discharge is determined by using eqn. (3·10).

Problem-7 : *A horizontal venturimeter 16 cm × 8 cm is used for measuring the flow of oil of sp. gr. 0·8. The oil is discharged at the rate of 0.05 m³/sec. If the coefficient of the meter is unity, determine the deflection of the oil-mercury gauge in centimetres.*

$$\text{Since discharge, } Q = \frac{C_d a_1 \sqrt{2gH}}{\sqrt{\left(\frac{d_1}{d_2}\right)^4 - 1}} \quad \{ \text{ref. eqn. } (3 \cdot 10a) \}$$

$$i.e.\ 0.05 = \frac{1 \times \frac{\pi}{4}\left(\frac{16}{100}\right)^2 \sqrt{19.62 \times H}}{\sqrt{\left(\frac{16}{8}\right)^4 - 1}}$$

$\therefore \quad H = 4.75$ m of oil.

Further, referring to eqn. (3·12),

$$H = \frac{h}{100}\left(\frac{13.6}{s_l} - 1\right)$$

$$\text{or} \quad 4.75 = \frac{h}{100}\left(\frac{13.6}{0.8} - 1\right)$$

$$\therefore \quad h = \frac{4.75 \times 100}{16} = 29.7 \text{ cm of mercury}$$

Problem – 8 : *A venturimeter has diameters of 1·2 m in the main and 0·6 m at the throat. With water flowing through it the difference of pressures between the main and the throat is measured by a differential mercury gauge which shows a deflection of 5·1 cm. Find the discharge through the meter and also calculate the velocity at throat. The coefficient of the meter may be taken as 0·98.*

Converting the cm of mercury in metres of water,

$$H = \frac{h}{100}\ (13.6 - 1) = \frac{5.1}{100} \times 12.6 = 0.642 \text{ m}$$

The discharge, Q is given by eqn. (3·10a) as

$$Q = \frac{C_d A_1 \sqrt{2gH}}{\sqrt{\left(\frac{d_1}{d_2}\right)^4 - 1}} = \frac{0.98 \times \frac{\pi}{4} \times 1.2^2 \times \sqrt{19.62 \times 0.642}}{\sqrt{\left(\frac{1.20}{0.6}\right)^4 - 1}}$$

$= \underline{1.038 \text{ m}^3\text{/sec.}}$

Further, since $Q = A_2 V_2$

$$V_2 = \frac{Q}{A_2} = \frac{1.038}{\frac{\pi}{4}(0.6)^2} = 3.66 \text{ m/sec.}$$

Problem – 9 : *For the venturimeter shown in fig. 3–11, the deflection of mercury in the differential gauge is 36 cm. Determine the flow of water through the meter if no energy is lost between A and B. Assume coefficient of discharge for the meter as unity.*

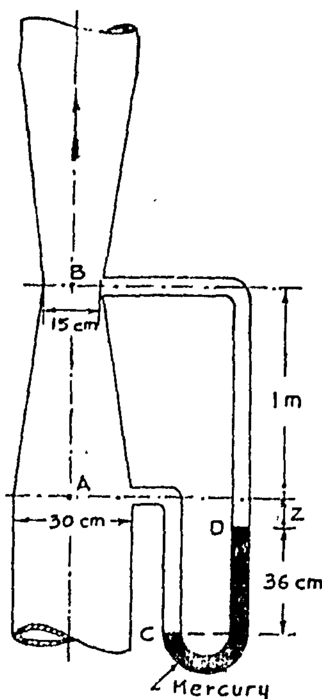


Fig. 3–11

Comparing the total pressure head in the two limbs at level C–C { ref. eqn. (3·9) }

$$\frac{p_A}{w} + \frac{36}{100} + Z = \frac{p_B}{w} + 1 + Z + \frac{36 \times 13.6}{100}$$

$$\therefore \quad \frac{p_A}{w} - \frac{p_B}{w} = \frac{36}{100}(13.6 - 1) + 1$$

$$= 5.54 \text{ m of water} \quad ..(i)$$

Applying Bernoulli's theorem and taking A as datum, we have,

$Z_A = 0, Z_B = 1$ m

$$\text{and} \quad \frac{p_A}{w} + \frac{V_A^2}{2g} + 0 = \frac{p_B}{w} + \frac{V_B^2}{2g} + 1$$

$$\therefore \quad \frac{p_A}{w} - \frac{p_B}{w} = \frac{V_B^2 - V_A^2}{2g} + 1 \quad ..(ii)$$

From eqns. (i) and (ii) we have

$$\frac{V_B^2 - V_A^2}{2g} + 1 = 5.54$$

Also, $A_A V_A = A_B V_B$

$$\therefore \quad V_B = \frac{A_A}{A_B} \times V_A = \left(\frac{30}{15}\right)^2 \times V_A = 4V_A$$

Hence, $\frac{16V_A^2 - V_A^2}{2g} = 5{\cdot}54 - 1 = 4{\cdot}54$

$\therefore \quad V_A = \sqrt{\frac{2 \times 9{\cdot}81 \times 4{\cdot}54}{15}} = 2{\cdot}44$ m/sec.

Discharge, $Q = 1 \times \frac{\pi}{4}\left(\frac{30}{100}\right)^2 \times 2{\cdot}44 = \underline{0{\cdot}172 \text{ m}^3/\text{sec.}}$

Alternately, as the meter is connected to a differential manometer, the reading of this manometer takes into account the difference of the levels between the throat and entrance hence the discharge can be assessed in a simple manner as under.

Head, $H = \frac{h}{100}(13{\cdot}6 - 1) = \frac{36}{100} \times 12{\cdot}6 = 4{\cdot}54$ m of water

and $Q = \frac{C_d A\sqrt{2gH}}{\sqrt{\left(\frac{d_A}{d_B}\right)^4 - 1}}$

$$= \frac{1 \times \frac{\pi}{4}\left(\frac{30}{100}\right)^2 \times \sqrt{2 \times 9{\cdot}81 \times 4{\cdot}54}}{\sqrt{\left(\frac{30}{15}\right)^4 - 1}} = \underline{0{\cdot}172 \text{ m}^3/\text{sec.}}$$

Problem – 10 : *A venturi contraction is introduced in a 75 cm diameter horizontal pipe. The area of the pipe is six times the area of the throat. The upper end of a vertical cylinder 30 cm in diameter is connected by a pipe to the throat and the lower end to the beginning of the convergence. Neglecting friction losses and ~~and~~ the thickness of the piston in the cylinder, determine the flow through the pipe in m³/sec. at which the piston begins to rise when the gross effective load due to piston, piston rod and external weight on the piston rod is 200 kg. The piston rod is 4 cm diameter, and passes through both ends of the cylinder.*

Since, ratio of areas $\frac{A_1}{A_2}$ is 6,

$$\left(\frac{d_1}{d_2}\right)^4 = 6^2 = 36$$

Considering equilibrium of the piston,

$$(p_1 - p_2)(A - a) = W$$

$$\therefore \quad p_1 - p_2 = \frac{W}{A - a} = \frac{200}{\frac{\pi}{4}(30^2 - 4^2)} = 0.288 \text{ kg/cm}^2$$

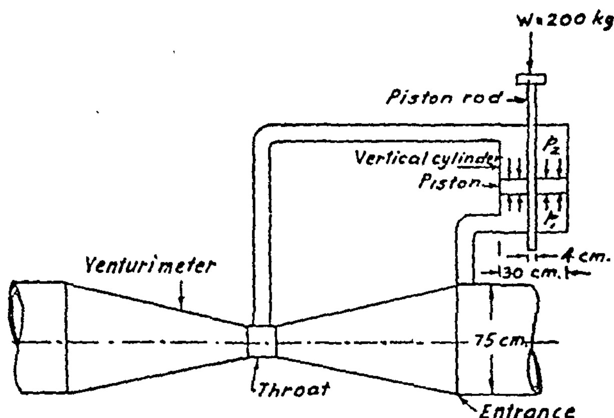


Fig. 3-12

The difference of head causing this net pressure is

$$H = \frac{(p_1 - p_2)\,10{,}000}{w} = \frac{10{,}000 \times 0.288}{1{,}000} = 2.88 \text{ m of water}$$

The discharge is then given by,

$$Q = \frac{C_d A_1 \sqrt{2gH}}{\sqrt{\left(\frac{d_1}{d_2}\right)^4 - 1}} = \frac{1 \times \frac{\pi}{4}\left(\frac{75}{100}\right)^2 \times \sqrt{2 \times 9.81 \times 2.88}}{\sqrt{36 - 1}}$$

$$= 0.562 \text{ m}^3\text{/sec.}$$

Radial Flow

In this type of flow the path of the liquid changes. For example, water in a centrifugal pump enters the pump in the axial direction and leaves it in the radial direction. During the whole

passage, the parameters viz. pressure, velocity, head, etc. remain changing. The flow, hence, is in two directions. Liquid flows over a large cross-section. Hence for stationary parts, mean values are used for the given section. Consider that water is passing up central pipe and then comes out radially between two parallel a stationary plates shown in fig. 3-13.

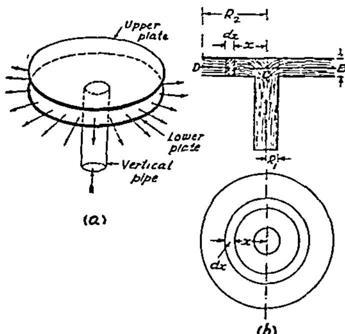


Fig. 3-13 Radial flow

We know that in the same horizontal line, if the velocity decreases the pressure will increase and vice versa. As water flows radially, the area of flow will increase and to keep the flow constant, the velocity may be kept less. Hence the pressure will increase. This is true for all positions according to Bernoulli's equation. If we apply this equation to two points C and D, we have, neglecting the losses,

$$\frac{p_c}{w}+\frac{V_c^2}{2g}=\frac{p_d}{w}+\frac{V_d^2}{2g}$$

Now as the discharge is in the atmosphere, the pressure at D will be atmospheric, hence p_d is known. If we know the values of p_c and V_c, we can find V_d from the above equation.

If pressure or velocity is required at any radius x from the axis of the central pipe, we can write, from equation of conservation of mass, $A_x V_x = 2\pi R_2 V_d \times B$.

where, V_x – average velocity of flow of water at radius x,

B – width of flow (distance between plates)

Area of flow A_x at radius x is given by $2\pi x \times B$

$$\therefore \quad \text{discharge, } Q = 2\pi x \times B \times V_x = 2\pi R_2 \times B \times V_d$$

$$\therefore \quad V_x = V_d \times \frac{R_2}{x}$$

Further, total energy at $x = \dfrac{p_x}{w} + \dfrac{V_x^2}{2g}$ = total energy at C or D

$$\therefore \quad \frac{p_x}{w} = \text{total energy at C or D} - \frac{V_x^2}{2g}$$

Let K be the total energy at C or D.

$$\text{Then, } \frac{p_x}{w} = K - \frac{V_x^2}{2g}$$

$$= K - \frac{V_d^2 \times R_2^2}{2gx^2}$$

$$\therefore \quad p_x = w\left(K - \frac{V_d^2 R_2^2}{2g \cdot x^2}\right) = w\left(K - \frac{m}{x^2}\right) \qquad \text{.. (3·14)}$$

where, $m = \dfrac{V_d^2 R_2^2}{2g}$

Consider now a small ring of thickness dx at a radius x.

Area of this thin ring $dA = 2\pi x dx$

Total pressure on this thin ring of the upper plate is

$$d_p = p_x \times dA$$

$$= 2\pi w\left(K - \frac{m}{x^2}\right) x \times dx$$

The total pressure on the plate is then given by

$$\int_{o}^{P_1} d_p = 2\pi w \int_{R_1}^{R_2} \left[Kx dx - \frac{m dx}{x} \right]$$

$$\therefore \; P_1 = 2\pi w \left[\frac{K}{2} (R_2^2 - R_1^2) - m \log_e \left(\frac{R_2}{R_1} \right) \right]$$

Further, let P_2 be total pressure due to incoming water in the central portion.

Then $P_2 = p_c \times \pi R_1^2$

$\therefore$ total upward pressure on the upper plate is

$$= 2\pi w \left[\frac{K}{2} (R_2^2 - R_1^2) - m \log_e \left(\frac{R_2}{R_1} \right) \right] + p_c \pi R_1^2 \quad ..(3\cdot15)$$

$$= P_1 + P_2$$

There is the atmospheric pressure acting on the upper plate and this pressure is p_d as a result of which the total downward pressure on the upper plate is equal to $p_d \pi R_2^2$,

$\therefore$ net pressure on the upper plate is

$$P = 2\pi w \left[\frac{K}{2} (R_2^2 - R_1^2) - m \log_e \left(\frac{R_2}{R_1} \right) \right] + p_c \pi R_1^2 - p_d \pi R_2^2 \quad ..(3\cdot16)$$

Problem – 11 : *Two horizontal parallel circular discs are placed 5 cm apart and each plate has diameter of 15 cm. The lower plate has a central hole of 2·5 cm diameter and 2·5 cm diameter vertical pipe is connected centrally to it. Water flows through the pipe at the rate of 300 litres per minute. If the pressure at the outer edge of the discs is 1·03 kg/cm², determine . (i) pressure inside the vertical pipe, (ii) pressure and velocity at a radius of 5 cm from the centre and (iii) the resultant pressure on the upper disc.*

Now, discharge, $Q = \frac{300}{1{,}000 \times 60} = 0\cdot005 \text{ m}^3/\text{sec.}$

Referring to fig. 3–13,

velocity in the pipe line, $V_c = \frac{Q}{A} = \frac{0\cdot005}{\frac{\pi}{4}\left(\frac{25}{100}\right)^2} = 10\cdot1$ m/sec

As $Q = 2\pi R_2 B V_d$

$$i.e.\ 0{\cdot}005 = 2\pi \times \frac{7{\cdot}5}{100} \times \frac{5}{100} \times V_d$$

$$\therefore \quad V_d = 0{\cdot}21 \text{ m/sec.}$$

Further, $\dfrac{p_c}{w} + \dfrac{V_c^2}{2g} = \dfrac{p_d}{w} + \dfrac{V_d^2}{2g}$

$$\therefore \quad \frac{p_c}{w} = \frac{1{\cdot}033 \times 10{,}000}{1{,}000} + \frac{(0{\cdot}21)^2}{19{\cdot}62} - \frac{(10{\cdot}1)^2}{19{\cdot}62} = 5{\cdot}112$$

$$\therefore \quad p_c = \frac{5{\cdot}112 \times 1{,}000}{10{,}000} = 0{\cdot}5112 \text{ kg/cm}^2$$

Also, $\quad V_x = V_d \dfrac{R_2}{x} = 0{\cdot}21 \times \dfrac{7{\cdot}5}{5} = 0{\cdot}315$ m/sec.

Total energy at D is say, $K = \dfrac{10{,}000 \times 1{\cdot}033}{1{,}000} + \dfrac{(0{\cdot}21)^2}{19{\cdot}62}$

$$= 10{\cdot}332 \text{ kg-m}$$

and $p_x = w \left\{ K - \dfrac{V_d^2 R_2^2}{2\,gx^2} \right\} = 1{,}000 \left\{ 10{\cdot}332 - \dfrac{(0{\cdot}21)^2}{19{\cdot}62} \times \left(\dfrac{7{\cdot}5}{5} \right)^2 \right\}$

$$= 1{,}000 \times 10{\cdot}337 \text{ kg/m}^2$$

$$\text{or } = \frac{1{,}000 \times 10{\cdot}327}{10{,}000} = 1{\cdot}0327 \text{ kg/cm}^2$$

The value of m is

$$m = \frac{V_d^2}{2g} \times R^2 = \frac{(0{\cdot}21)^2}{2 \times 9{\cdot}81} \left(\frac{7{\cdot}5}{100} \right)^2 = 12{\cdot}6 \times 10^{-6}$$

Total upward pressure

$$= 2\pi w \left\{ \frac{K}{2} (R_2^2 - R_1^2) - m \log_e \frac{R_2}{R_1} \right\} + p_c \times \pi R_1^2$$

$$= 2\pi \times 1{,}000 \left\{ \frac{10{\cdot}327}{2} (0{\cdot}00562 - 0{\cdot}000156) - 12{\cdot}6 \times 10^{-6} \times 2{\cdot}3 \log_{10} 6 \right\}$$

$$+ 0{\cdot}5112 \times \pi \times \left(\frac{7{\cdot}5}{100} \right)^2$$

$$= \underline{177{\cdot}12 \text{ kg}}$$

Downward atmospheric pressure $= \pi R_2^2 \times 1{\cdot}033$

$$= \pi \times 7{\cdot}5^2 \times 1{\cdot}033 = 182 \text{ kg}$$

Resultant pressure $= 177{\cdot}12 - 182 = -4{\cdot}88$ kg

Negative sign shows that the pressure is downward on the top plate.

Vortex Flow and Whirling of Liquids

Due to difference of pressures between layers of liquid the flow is constrained to take curved path leaving the straight stream-

lined path. The rotation of the liquid about some axis is known as vortex. This is caused by imparting the liquid an external torque or due to some previous rotation. The vortex is classified as *free* and *forced*. If, for example, water is allowed to drain out from a basin, filled with water, through the discharge pipe at the bottom, a free vortex is created. To create the forced vortex, the liquid is given external rotation. Forced vortex is also noticed in the water of the river where cross currents are present.

Vortex flow : There is no external torque applied on this type of flow. The force acting on such a flow is proportional to the mass × velocity, or

$$F = \frac{d}{dt}\left(\frac{W}{g} \times V\right)$$

where F is resultant force due to change in momentum of a body of mass $M = \frac{W}{g}$ where W is its weight.

Further, torque, $T = F \times R = \frac{d}{dt}(M \times V \times R)$

The product $M{\cdot}V{\cdot}R$ is called angular momentum or moment of momentum, If no torque is applied,

$$\frac{d}{dt}(M{\cdot}V{\cdot}R) = 0$$

Integrating it,

$$V{\cdot}R = C \text{ (constant)}, \qquad \ldots.(3{\cdot}17)$$

The constant C can be evaluated from known velocity at given radius R. From eqn. (3·17) it may be seen that velocity changes inversely as R.

$$\text{Hence, velocity, } V = \frac{C}{R} \qquad \ldots.(3{\cdot}18)$$

Now, in a rotating flow, acted upon by centrifugal force, the change in pressure due to increase in centrifugal force is given by

$$\frac{dp}{dR} = \frac{\rho V^2}{R} \qquad \ldots.(3{\cdot}19)$$

(This is subsequently proved to be so in eqn. (i) of the following paragraph on whirling of liquids. Here ρ is the mass density of the substance.)

Substituting value of V from eqn. (3·18) in eqn. (3·19), and integrating, we have,

$$\int_{p_1}^{p_2} dp = \rho C^2 \int_{R_1}^{R_2} \frac{1}{R^3}\, dR$$

$$\text{or } p_2 - p_1 = \left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)\frac{\rho C^2}{2} \qquad \ldots(3{\cdot}20)$$

Since, $C = V_1 R_1 = V_2 R_2$ and $\rho = \dfrac{w}{g}$

$$\frac{p_2}{w} - \frac{p_1}{w} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} \qquad \ldots(3{\cdot}21)$$

Eqn. (3·21) has the same form as Bernoulli's equation. Hence, for such steady flow, Bernoulli's equation gives relationship between heads at any two points in the liquid.

Problem–12 : *In a free vortex, at a radius of 0·61 m, the velocity is 45·7 m/sec. and pressure is 1 kg/cm² absolute. What is the pressure at a radius of 1·22 m ? Assume the fluid as water with density of 1,000 kg/m³.*

For a free vortex,

$$V_1 R_1 = C = V_2 R_2$$

$$\text{or } C = V_1 R_1 = 45{\cdot}7 \times 0{\cdot}61 = 27{\cdot}8$$

Therefore, $V_2 = \dfrac{27{\cdot}8}{1{\cdot}22} = 22{\cdot}85$ m/sec.

Writing the pressure – velocity relations, (eqn. 3·21)

$$\frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

$$\text{or } \frac{p_2}{w} - \frac{p_1}{w} = \frac{1}{2g}\left(V_1^2 - V_2^2\right)$$

$$= \frac{1}{2 \times 9{\cdot}81}\left(45{\cdot}7^2 - 22{\cdot}85^2\right) = 80{\cdot}4 \text{ m of water}$$

$$\therefore \quad p_2 - p_1 = w \times 80{\cdot}4$$

$$\text{or } (p_2 - p_1) = 1{,}000 \times 80{\cdot}4 = 8{\cdot}04 \times 10^4 \text{ kg/m}^2$$

$$\text{or } p_2 - p_1 = 8{\cdot}04 \text{ kg/cm}^2$$

$$\therefore\ p_2 = 8{\cdot}04 + p_1 = 8{\cdot}04 + 1 = \underline{9{\cdot}04 \text{ kg/cm}^2}$$

Whirling of liquids : Consider a cylinder of radius R which contains liquid and revolves about its vertical axis as shown in fig. 3-14. Let its angular velocity be ω. Consider a thin ring of liquid of radius x and thickness dx. Let a portion on this thin ring subtend a small angle $d\theta$ at the centre (fig. 3-15).

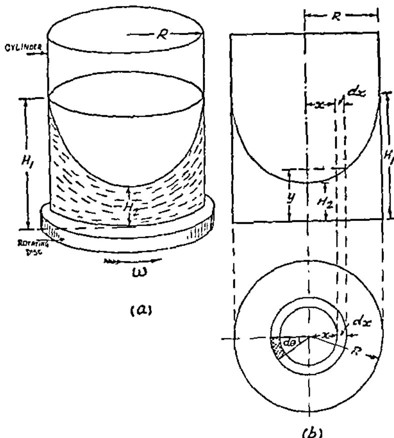


Fig. 3-14

This ring will be acted on by pressure p on its inner surface and by pressure $p + dp$ on its outer surface created by the action of centrifugal forces. The pressure on the outer side of the ring is greater since its radius is greater by dx. Let us consider the equilibrium of this elementary ring as shown in fig. 3-15.

Area of the inside curved surface BCGF of the element

$$= \text{arc BC} \times y = x d\theta \times y$$

Area of the outside curved surface ADHE of the element

$$= \text{arc AD} \times y = (x + dx)\, d\theta \times y$$

Intensity of pressure on the outside surface of the element

$$= p + dp$$

Intensity of pressure on the inside surface of the element $= p$

Average intensity of pressure on sides $= p + \dfrac{dp}{2}$

Weight of element, dW = area of top surface ABCD $\times y \times w$

$$= (x d\theta)\, dx\, y \cdot w$$

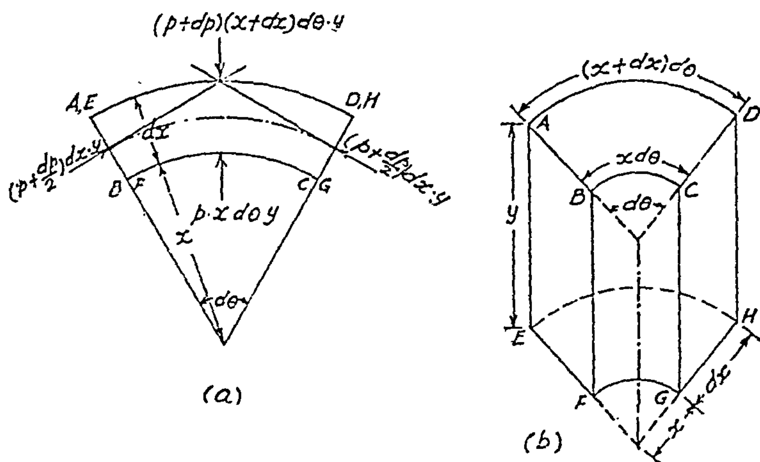


Fig. 3-15

$$\text{Centrifugal force on the element} = \frac{dW}{g}\,\omega^2 x$$

$$= \frac{(x \cdot d\theta)\, dx \times yw}{g}\,\omega^2 x$$

Resolving the force on the element in the outward radial direction (fig. 3-15 a) we have,

$$\text{difference in thrust} = (p + dp)(x + dx)\, d\theta \times y - p \cdot x d\theta \cdot y$$

$$- 2\left(p + \frac{dp}{2}\right) dx \cdot y\, \frac{d\theta}{2}$$

Problem–13 : *Point out the difference between free vortex and forced vortex with examples.*

A cylindrical vessel 25 cm in diameter and 75 cm high contains water to a depth of 50 cm. At what speed in r.p.m. should it be rotated so that water just reaches the top ? What should be the speed when 25% of the original volume has spilled out over the vessel?

Since water just reaches the top edge of the cylinder without spilling off, the volumes of water before and after rotation are equal.

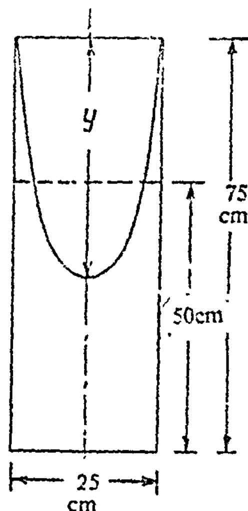


Fig. 3-16

Volume of water before rotation =

$$\frac{\pi}{4} \times \left\{\frac{25}{100}\right\}^2 \times 50/100 = \frac{\pi}{128}\ m^3$$

Volume of paraboloidal space after rotation=

$$\frac{\pi}{4}\left(\frac{25}{100}\right)^2 \times \frac{y}{2}$$

as the volume of paraboloid of height h is equal to area of the base $\times \dfrac{\text{height}}{2}$

$\therefore$ $\pi/128$ = gross volume of cylinder–volume of the paraboloid

$$= \frac{\pi}{4}\left(\frac{25}{100}\right)^2 \times \frac{75}{100} - \frac{\pi}{4}\left(\frac{25}{100}\right)^2 \times \frac{y}{2}$$

$\therefore$ $y = 0{\cdot}5$ m

Now, $y = \dfrac{\omega^2 R^2}{2g}$ (ref. eqn. 3·22)

$$\therefore \quad 0{\cdot}5 = \frac{\omega^2\,(12{\cdot}5/100)^2}{2 \times 9{\cdot}81}$$

$\therefore$ $\omega^2 = 625$

$\therefore$ $\omega = 25$ rad./sec.

$\therefore$ $N = \dfrac{30\omega}{\pi} = \underline{239}$ r.p.m.

When 25% of the original volume has spilled over due to increase of speed,75% of the original volume remains in the cylinder.

Hence, volume of paraboloid formed in the cylinder = gross volume of cylinder minus quantity remaining in the cylinder.

$$i.e.\ \frac{\pi}{4}\left(\frac{25}{100}\right)^2 \times \frac{y_1}{2}$$

$$= \frac{\pi}{4}\left(\frac{25}{100}\right)^2 \times \frac{75}{100} - \frac{3}{4} \times \frac{\pi}{4}\left(\frac{25}{100}\right)^2 \times \frac{50}{100}$$

$$\therefore\ y_1 = 0{\cdot}75$$

Hence, corresponding speed of rotation is given by

$$y_1 = \frac{\omega_1^2 R^2}{2g}.$$

$$\text{or } 0{\cdot}75 = \frac{\omega_1^2}{2 \times 9{\cdot}81} \times \left(\frac{12{\cdot}5}{100}\right)^2$$

$$\therefore\ \omega_1 = 30{\cdot}6 \text{ rad/sec.}$$

$$\therefore\ N_1 = \frac{30\omega_1}{\pi} = \frac{30 \times 30{\cdot}6}{\pi} = \underline{292 \text{ r.p.m.}}$$

TUTORIAL – 3

Bernoulli's equation

1. Water is flowing into a horizontal tapered pipe having 15 cm diameter at the large end and 5 cm diameter at the smaller end. The velocity of water at the larger end is 3 m/sec. Determine the rate of discharge in tons/hour and the velocity head at the small end.

[19·1; 37·2 m of water]

2. The diameter of a pipe changes gradually from 45 cm at a point A, 6 m above datum level to 15 cm at B, 2·5 m above datum. The pressure at A is 1·4 kg/cm^2 and velocity of flow is 1 m/sec. Assuming no loss between A and B compute the pressure at B. Take g = 9·81 m/sec^2.

[1·342 kg/cm^2]

3. A pipe line is used to carry oil of sp. gr. 0·896 and it changes in size from 25 cm at section P to 60 cm at section Q. Section P is 4 m lower than section Q and the pressures are 1 kg/cm^2 and 0·7 kg/cm^2 respectively. If the discharge is 0·17 m^3/sec.; determine the loss of head and the direction of flow.

[0·056 m of oil from Q towards P]

4. A conical pipe has diameter 80 cm at the large end and 20 cm at the smaller end and forms part of a vertical main. The pressure head at the larger end is found to be 20 m and at the smaller end 16 m. Find the discharge through the pipe if the length of the conical portion is 2 m.

[0·19 m^3/sec]

5. A 30 m long pipe CD is inserted in a pipe line inclined at 60° to the horizontal. At C which is at a higher level, the diameter is 15 cm. At D the diameter is 50 cm, the pressure is 4·5 kg/cm^2 and velocity of flow is 2·5 m/sec. Neglecting all losses, find the pressure head at C.

If water flows from lower to higher level and the head lost due to friction is 4 m of water, find the difference of pressure heads at C and D. Also determine the energy lost per second and the horse power required to lift the water from D to C.

[1·422 kg/cm^2; 31·76 m of water;
176·7 kg. m, 61·6 H.P.]

6. In a vertical pipe conveying water, pressure gauges are inserted at A and B, where the diameters are 10 cm and 5 cm respectively. The point B is 4 m below A and when the rate of flow down the pipe is 0·013 m^3/sec., the pressure at B is 0·14 kg/cm^2 greater than that at A. Assuming that the losses in the pipe between A and B can be expressed as $\frac{KV_A^2}{2g}$ where V_A is the velocity at A find the value of K. If the gauges at A and B are replaced by tubes filled with water and connected to a U-tube containing mercury of sp. gr. 13·6 give a sketch showing how the levels in the two limbs of the U-tube stand and calculate the value of this difference measured in cm.

[K = 3·5; 20·6]

7. A tapered pipe is fitted vertically and forms part of a pipe line. The pressure at the upper end is 6 m of water and velocity 4 m/sec. The velocity at the bottom end which is larger in

area is 1 m/sec. There is a loss of energy due to friction and this loss is expressed in metre head by relation $h = \frac{1.92\ (V_1 - V_2)^2}{2g}$ where $V_1 = 4$ m/sec. and $V_2 = 1$ m/sec. If the length of the tapered section is 5 m, determine the pressure at the bottom end of the pipe. The flow is in the downward direction. Take, $g = 9.81$ m/sec^2.

[1.147 kg/cm^2]

8 A portion of a pipe for conveying water is vertical and the diameter of the upper part of the pipe is 10 cm and the section is gradually reduced to 5 cm diameter at the lower part. A pressure gauge is inserted where the diameter is 10 cm and a second gauge is placed 2 m below the first and where the pipe is 5 cm diameter. When the quantity of water flowing up through the pipe is 1 m^3 per minute, the gauges show a pressure difference of 0.3 kg/cm^2. Assuming that the frictional losses vary as the square of the velocity, determine the quantity of water passing through the pipe when two gauges show no pressure difference and water is flowing downwards.

[0.504 m^3/minute]

9. A portion of pipe for conveying water is vertical and the diameter of the upper part of the pipe is 10 cm and the section is gradually reduced to 5 cm diameter at the lower part. A pressure gauge is inserted where the diameter is 10 cm and a second gauge is placed 'X' m below the first and where the pipe is 5 cm diameter. When the quantity of water flowing up through the pipe is 1 m^3/min., the gauges show a pressure difference of 0.3 kg/cm^2; and when the quantity of water flowing through the pipe is 0.504 m^3/minute, the gauges show no pressure difference. Assuming that frictional losses vary as the square of the velocity, determine the value of 'X'.

[2 m]

Venturimeter

10. Through a horizontal venturimeter 16 cm × 8 cm oil of sp. gr. 0.5 is flowing at the rate of 4 m^3/minute. If the coefficient of the meter is 0.98, determine the deflection of the oil mercury gauge in cm.

[33.2 cm]

11. A venturimeter has a diameter of 1 m in the main and 60 cm at the throat. With water flowing through it, the difference of pressures between the entrance and the throat is measured by a differential mercury gauge which shows a deflection of 5 cm. Find the discharge through the meter and also calculate the velocity at the throat. Take coefficient of the meter as 0·98.

[1·045 m^3/sec.; 3·695 m/sec.]

12. A venturimeter has diameters of 120 cm in the main and 60 cm at throat. With water flowing through it, the difference of pressures at the entrance and throat is measured by a differential mercury gauge which shows a deflection of 4 cm. Find the discharge through the meter and also calculate the velocity at the throat. The coefficient of the meter is unity.

[0·915 m^3/sec; 3·14 m/sec.]

13. A venturi contraction is introduced in a 75 cm diameter horizontal pipe. The area of the pipe is six times the area of the throat. The upper end of the vertical cylinder 30 cm in diameter (ref. fig. 3–12) is connected by a pipe to the throat and the lower end to the beginning of the convergence. Neglecting friction losses and thickness of the piston in the cylinder, when the flow through the pipe is 0·536 m^3 /sec., the piston begins to rise, find the gross effective load due to piston, piston rod and external weight on the piston rod. The piston rod is 4 cm in diameter and passes through both ends of the cylinder. Take C_d = 0·96.

[200 kg]

14 A venturimeter is installed in a pipe line so that the difference of pressures at entrance and throat read by mercury manometer is 5 cm when water flows at the rate of 0·05 m^3/sec. If the diescharge coefficient for the meter is 0·98, and the diameter at the throat is 13·5 cm, find the diameter of the pipe line.

[30 cm]

15. A venturimeter is installed in a pipe line 30 cm diameter. The difference of pressures at entrance and throat read by mercury manrmeter is 5 cm when water flows at the rate of 0·05 m^8/sec. If the discharge coefficient for the meter is 0·96, determine the diameter at the throat.

[13·6 cm]

16. A venturimeter is installed in a horizontal pipe line 30 cm in diameter in which the maximum flow is 0·19 m^3/sec. when the pressure difference between the entrance and the throat is 6 m of water. If the diameter of the throat is 15 cm, determine the coefficient of the meter. What will be the discharge when a differential mercury gauge shows a deflection of 20 cm ?

[0 956; 0·123 m^3/sec.]

17. A venturimeter is used for measuring the flow of petrol in a pipe line inclined at 45° to the horizontal. Specific gravity of petrol is 0·8. The difference in mercury levels in the gauge is 6 cm. The pipe line is 40 cm in diameter. If the discharge is 100 kg/sec; determine the throat diameter of the meter. Take C_d = 0 96.

[17·35 cm]

18. A venturimeter is used for measuring the flow of petrol in a pipe line inclined at 35° to the horizontal. Specific gravity of petrol is 0 8. The difference in mercury levels in the gauge is 5 cm. The pipe line is 30 cm in diameter. If the discharge is 75 litres/sec., determine the throat diameter of the meter. Take C_d = 1. [15·25 cm]

19. A venturimeter is introduced in a pipe line 70 cm in diameter having a slope of 1 in 15. The pipe line diameter is twice the diameter at the throat. An oil having sq gr. of 0·84 flows through the meter. Connections are made from the entrance (to the meter) and throat to an inverted U-tube containing liquid of sp. gr. 0·5. If the gauge reading is 40 cm, and the coefficient of the meter is unity, determine the discharge per second

[0·177 m^3/sec.]

20. A venturimeter is introduced in a pipe line having a slope of 1 in 10 to discharge 2,05,800 litres per hour. The pipe line diameter is three times the diameter at the throat. An oil having sq. gr. of 0·81 flows through the meter. Connections are made from the entrance (to the meter) and throat to an inverted U-tube containing liquid of sq gr 0 5. If the gauge reading is 45 cm and the coefficient of the meter is 0 98, determine the diameter of the pipe line. [60 cm]

Radial flow

21. Two horizontal circular discs are placed 5 cm apart and each plate has diameter of 20 cm. The lower plate has central hole of 5 cm diameter and 5 cm diameter vertical pipe is connceted centrally to it. Water flows through the pipe at the rate of 500 kg/minute. If the pressure at the outer edge of disc is 1 kg/cm^2, determine, (i) pressure inside the vertical pipe, (ii) velocity and pressure at a radius of 6 cm from the centre, and (iii) the resultant pressure on the upper disc.

[(i) 0·9007 kg/cm^2; (ii) 0·44 m/sec., 0·999 kg/cm^2; (iii) 2·4 kg in the downward direction]

Vortex flow

22. In a free vortex at a radius of 10 cm. the velocity is 10 m/sec. and the pressure is 1·76 kg/cm.^{2} absolute. What is the pressure at the radius of 20 cm. ? Assume the fluid as water with density of 10 kg/m^3.

[2·08 kg/cm^2]

23. Point out the difference between the free vortex and forced vortex with examples.

A cylinder vessel 40 cm in diameter and 1 m high contains water to a depth of 60 cm. At what speed in rpm. should be the rotated so that water just reaches the top ? What should it be speed when 30% of the original volume has spilled out over the vessel ?

[188; 222]

4

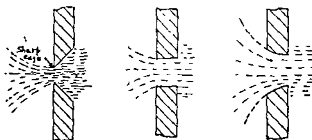
FLOW THROUGH ORIFICES

Introduction

An orifice is a sharp-edged circular opening in a thin plate which is attached to the side or bottom of a tank and through which liquid will flow out. The hole behaves as an orifice so long as liquid level is above its upper edge. It is one of the devices which are used to measure the rate of flow of liquids. The study of orifices is helpful in understanding the behaviour of the flow of the fluid which occurs under different conditions, *i. e.* free flow, flow through partially submerged orifices, flow through submerged orifices etc.

Types of Orifices

Under the conception of types of orifices we mean the various shapes of the openings The usual and the most common shape is the circular one, but other shapes such as square, rectangular, and triangular are also in use. The orifice is further sub-divided into



(a) Sharp edged (b) Square edged (c) Bell mouthed

Fig 4-1 Types of orifices

classes such as *sharp edged orifice*, *square edged orifice* and *bell mouthed orifice*. The sharp edged orifice has its inner edge made sharp or in other words sharp angles are made round the periphery at entrance. This orifice is made tapered with increased diameter at

the outer edge in order that the jet of water issuing from the tank does not touch the orifice. In the square edged orifice the passage through the parent plate is straight, parallel one. The bell-mouthed orifice has the rounded passage due to which the friction in it is reduced.

The orifice is further sub-divided as *small* orifice and *large* orifice. This type of classification depends on the orifice size relative to depth of liquid in the tank. In the case of small orifice the velocities of the liquid particles near the bottom edge and top edge do not differ appreciably, and hence the velocity of the liquid particles at the centre of the orifice is taken as the velocity of flow. In the case of large orifice, the velocity is not constant between the top edge of the orifice and its bottom edge. The velocity of liquid particles near the bottom edge is greater than that near the top edge.

If the jet of water through the orifice issues in the atmosphere or into a tank, the orifice is said to be *free* orifice and its discharge is *free* discharge. If water is discharged into a tank in which the water level is above the top edge of the orifice, the orifice is termed as *submerged* orifice.

Coefficients of Orifice

Water particles in a tank fitted with an orifice move first vertically downward and then take a turn to enter the orifice and leave it. In taking this turn some energy is lost and hence the original properties are not retained by the jet of water. Hence, we observe in practice that the cross-sectional area of the jet does not confirm to the orifice area and the actual velocity of the jet is less than that theoretically possible. To account for these facts the various coefficients of the orifice are defined as under:

Coefficient of contraction : Water in the tank around the sides of the orifice has motion parallel and perpendicular to the direction of the jet. The velocity in the perpendicular direction is destroyed on reaching the orifice. This causes lateral force on the jet and consequently the area of cross-section of the jet is reduced.

Water particles take a turn round the orifice edge on issuing and then they flow parallel to the centre line of the orifice at some distance away from the sharp edge. The section where these particles become parallel to the centre line of the orifice is termed as *vena contracta.*

The ratio of the area of cross-section of the jet at vena contracta to the area of the orifice is called the *coefficient of contraction.* If C_c is coefficient of contraction, a_j area of cross-section of the jet at vena contracta and a_0 area of cross-section of the orifice, then,

$$C_c = \frac{\text{area of jet at the vena contracta}}{\text{area of the orifice}} = \frac{a_j}{a_0} \qquad ..(4\cdot1)$$

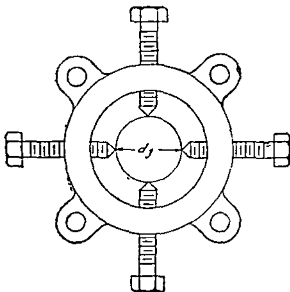


Fig. 4-2 Measuring ring

The area of the orifice is determined by measuring its diameter with the help of an inside calliper. The diameter of the jet cannot be measured with calliper and hence a special device is required for it. This can be done with an instrument shown in fig. 4-2. This consists of a small collar or ring having four radial screws which

are equally spaced. The ring is held at the vena contracta so that the jet passes through its centre. The screws are adjusted by turning them so long as their points or tips are in contact with the surfaces of the jet. Once this is done, the instrument is removed and the distances between the screw points are measured. The distance between the opposite screw tips gives the diameter of the jet. As there are four screws, we will get two readings. The average of the two readings gives the average diameter of the jet at vena contracta from which area of cross-section of the jet can be found.

This method is not very accurate as the section of the jet is not absolutely regular; also it is difficult to adjust the instrument so that all four screws just touch the jet surface simultaneously.

The coefficient of contraction varies slightly with the head and with the size and shape of the orifice. An average value for small sharp edged orifices is 0·64.

Coefficient of velocity : As there is friction taking place between water and sides of the orifice, there is some reduction in the velocity of water. The ratio of actual velocity of the jet to its theoretical velocity at the vena contracta is termed as the coefficient of velocity. If C_v is coefficient of velocity, V the actual velocity of jet and V_t the theoretical jet velocity, then,

$$C_v = \frac{\text{actual velocity}}{\text{theoretical velocity}} = \frac{V}{V_t} \qquad \text{.. (4·2)}$$

The coefficient of velocity will vary slightly for different orifices, depending on the shape and size of the orifice and on the available head. An average value of C_v is about 0·97.

There are three methods of determining the coefficient of velocity. They are: Trajectory method, Use of momentum equation, Pitot tube method.

Trajectory method : So long as water particle is inside the tank it is well supported, but when it comes out of the tank thro-

ugh the orifice, it is free to move in any direction. The particles just coming out of the orifice will be thrown out due to static pressure behind it. Hence, the free particle will have constant velocity which is the velocity at vena contracta and at the same time it will be under the effect of gravity. Consequently the path followed by the particle will be parabolic as shown in fig. 4-3.

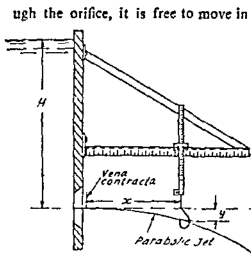


Fig. 4-3 Trajectory method

Let, V – velocity of the particle at vena contracta, m/sec,
t – time taken to travel a particular distance, sec.
x – horizontal distance travelled, m
y – vertical distance travelled, m
H – available head, m of water.

Hence, $x = V \times t$

$$\therefore \quad \frac{x^2}{V^2} = t^2$$

Also, $y = \frac{1}{2}gt^2$

$$= \frac{1}{2}g \times \frac{x^2}{V^2}$$

$$\therefore \quad V = \sqrt{\frac{gx^2}{2y}}$$

$\therefore$ actual velocity, $V = \sqrt{\frac{gx^2}{2y}}$

Theoretical velocity, $V_t = \sqrt{2gH}$

$$\therefore \quad C_v = \frac{V}{V_t} = \frac{\sqrt{\frac{gx^2}{2y}}}{\sqrt{2gH}} = \sqrt{\frac{x^2}{4yH}} \qquad .. (4\text{·}3)$$

Thus, if x and y are measured, C_v can easily be determined. The limitations of this method are : (a) There is difficulty in locating vena contracta, and (b) it is difficult to measure y from the centre of the jet.

Use of momentum equation : According to Newton's third law of motion, action and reaction are equal and opposite. If a vessel is free to move and liquid discharged from it, then the vessel will move in the direction opposite to the direction in which the liquid

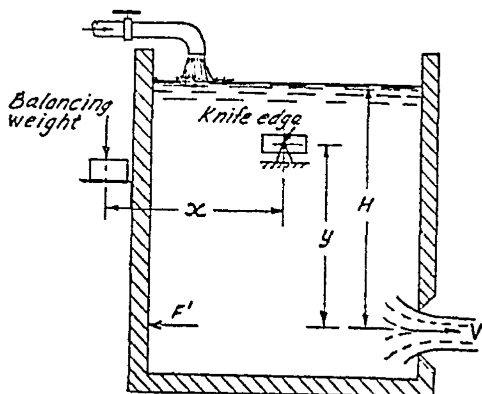


Fig. 4-4 Momentum method

is discharged. If the body is not allowed to move, we have to apply force on it to keep it stationary. This force will be proportional to the velocity of water. Hence, by measuring the force, we can easily determine the velocity at vena contracta.

A tank is suspended on knife edge as shown in fig. 4-4. First by closing the orifice the tank is levelled on the knife edge. By opening the plug, water is allowed to leave the tank with a velocity V and a force, $F=\dfrac{wQV}{g}$ creates the momentum in jet. An equal and opposite force F, acts on the opposite wall of the tank. This force F, disturbs the leveling of the tank, which is again adjusted by adding known weight W at a lever arm of x.

$$\text{Then, } F \times y = W \times x$$

$$\text{But, } F=\frac{wQV}{g}$$

$$\therefore \quad \frac{wQV}{g} \times y = W \times x$$

$$\therefore \quad V = \frac{W \times g}{wQy} \quad \text{.. (4·4)}$$

$$\text{Hence, } C_v = \frac{V}{\sqrt{2gH}} = \frac{W\sqrt{g}}{wQy\sqrt{2H}} \quad \text{.. (4·5)}$$

This method will be useful only for small tanks.

Pitot tube method : If the pitot tube is held at the vena contracta, the height of the liquid in it will be proportional to the velocity of the jet at the vena contracta. If h_p is the height of the liquid in pitot tube, then the velocity of the jet is given by

$$V = \sqrt{2gh_p}$$

$$\therefore \quad C_v = \frac{V}{\sqrt{2gH}} = \frac{\sqrt{2gh_p}}{\sqrt{2gH}} = \sqrt{\frac{h_p}{H}} \quad \text{.. (4·6)}$$

This method is easier and more accurate.

Coefficient of discharge : The actual discharge through an orifice is always found to be less than the theoretical discharge. This happens on account of the reduction in the area of the jet and the reduction in jet velocity. The ratio of the actual discharge to the theoretical discharge is defined as the coefficient of discharge, C_d.

$$C_d = \frac{\text{actual discharge}}{\text{theoretical discharge}} = \frac{Q}{Q_t} \quad \text{... (4·7)}$$

where, Q_t = theoretical discharge,

= area of the orifice × theoretical velocity

$= a_0 \times \sqrt{2gH}$

and Q = actual discharge

$$= \left\{\begin{array}{c}\text{area of jet at vena} \\ \text{contracta}\end{array}\right\} \times \left\{\begin{array}{c}\text{actual velocity of jet} \\ \text{at vena contracta}\end{array}\right\}$$

$= a_j \times V$

$$\therefore \quad C_d = \frac{a_j \times V}{a_0 \times \sqrt{2gH}}$$

$$\text{Now, } C_c = \frac{a_j}{a_0}$$

Thus, if x and y are measured, C_v can easily be determined. The limitations of this method are : (a) There is difficulty in locating vena contracta, and (b) it is difficult to measure y from the centre of the jet.

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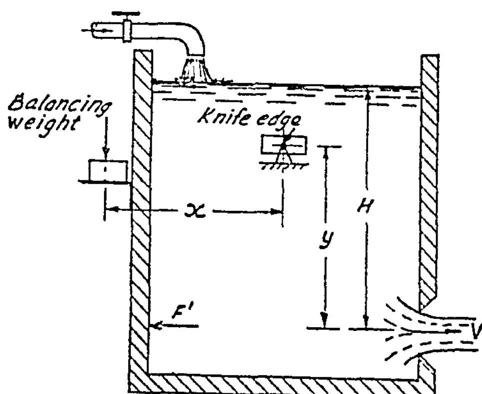


Fig. 4-4 Momentum method

is discharged. If the body is not allowed to move, we have to apply force on it to keep it stationary. This force will be proportional to the velocity of water. Hence, by measuring the force, we can easily determine the velocity at vena contracta.

A tank is suspended on knife edge as shown in fig. 4-4. First by closing the orifice the tank is levelled on the knife edge. By opening the plug, water is allowed to leave the tank with a velocity V and a force, $F = \frac{wQV}{g}$ creates the momentum in jet. An equal and opposite force F, acts on the opposite wall of the tank. This force F, disturbs the leveling of the tank, which is again adjusted by adding known weight W at a lever arm of x.

$$\text{Then, } F \times y = W \times x$$

$$\text{But, } F = \frac{wQV}{g}$$

$$\therefore \quad \frac{wQV}{g} \times y = W \times x$$

$$\therefore \quad V = \frac{W \times g}{wQy} \qquad \text{.. (4·4)}$$

$$\text{Hence, } C_v = \frac{V}{\sqrt{2gH}} = \frac{W\sqrt{g}}{wQy\sqrt{2H}} \qquad \text{.. (4·5)}$$

This method will be useful only for small tanks.

Pitot tube method : If the pitot tube is held at the vena contracta, the height of the liquid in it will be proportional to the velocity of the jet at the vena contracta. If h_p is the height of the liquid in pitot tube, then the velocity of the jet is given by

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$$\therefore \quad C_v = \frac{V}{\sqrt{2gH}} = \frac{\sqrt{2gh_p}}{\sqrt{2gH}} = \sqrt{\frac{h_p}{H}} \qquad \text{.. (4·6)}$$

This method is easier and more accurate.

Coefficient of discharge : The actual discharge through an orifice is always found to be less than the theoretical discharge. This happens on account of the reduction in the area of the jet and the reduction in jet velocity. The ratio of the actual discharge to the theoretical discharge is defined as the coefficient of discharge, C_d.

$$C_d = \frac{\text{actual discharge}}{\text{theoretical discharge}} = \frac{Q}{Q_t} \qquad \text{... (4·7)}$$

where, Q_t = theoretical discharge,
= area of the orifice × theoretical velocity
$= a_0 \times \sqrt{2gH}$

and Q = actual discharge

$$= \left\{\begin{matrix}\text{area of jet at vena} \\ \text{contracta}\end{matrix}\right\} \times \left\{\begin{matrix}\text{actual velocity of jet} \\ \text{at vena contracta}\end{matrix}\right\}$$

$= a_j \times V$

$$\therefore \quad C_d = \frac{a_j \times V}{a_0 \times \sqrt{2gH}}$$

$$\text{Now, } C_c = \frac{a_j}{a_0}$$

$$\therefore \quad a_j = C_c a_o$$

$$\text{and } C_v = \frac{V}{\sqrt{2gH}}$$

$$\therefore \quad V = C_v \sqrt{2gH}$$

$$\therefore \quad C_d = \frac{C_c a_o \times C_v \sqrt{2gH}}{a_o \sqrt{2gH}}$$

$$= C_c \times C_v \qquad .. \text{ (4·8)}$$

This expression shows the relation among the three coefficients. If any two are known, the third can be determined.

Coefficient of resistance : This coefficient indicates the measure of the energy lost while passing through the orifice. When water is discharged through a horizontal orifice in the vertical direction under a head H, the jet would not rise to the level of water in the tank. The arrangement is pictured in fig. 4–5.

The difference between the two levels is due to the head lost on overcoming resistance to flow at the orifice. This loss of head

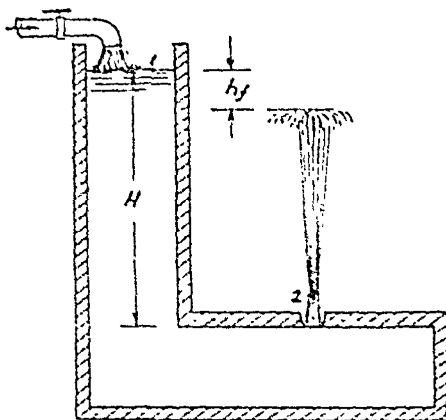


Fig. 4–5

can be calculated by applying Bernoulli's equation between the point 1 on the liquid surface and the point 2 at vena contracta of the jet. The loss of head can be expressed in terms of either the head, H in the tank or the actual velocity head, $\frac{V^2}{2g}$. Applying Ber-

noulli's equation to those points and neglecting the velocity of approach at the orifice, we have,

$$H = \frac{V^2}{2g} + h_l$$

$$\therefore \quad h_l = H - \frac{V^2}{2g}$$

$$\text{But, } V = C_v \sqrt{2gH}$$

$$\text{Hence, } h_l = H - \frac{\{C_v \sqrt{2gH}\}^2}{2g}$$

$$= H(1 - C_v^2) \qquad \text{.. (4·9a)}$$

$\therefore$ loss of energy at orifice will be $H(1-C_v^2)$ m-kg per kg of liquid

$$\text{Further, since } V = C_v \sqrt{2gH}$$

$$H = \frac{V^2}{2gC_v^2}$$

$$\therefore \quad h_l = \left(\frac{1}{C_v^2} - 1\right)\frac{V^2}{2g} \qquad \text{.. (4·9b)}$$

It should be noted that the loss of head is dependent on the coefficient of velocity only. The coefficient of contraction is not at all involved. The coefficient of contraction is mainly dependent on the geometry of orifices as against the coefficient of velocity which is dependent on the loss of head in orifice. On the other hand, coefficient of discharge depends on both geometry of orifice and loss of head.

Sometimes the word coefficient of resistance is used in connection with the orifices. It may be defined as ratio of the loss of head in the orifice to the head above the orifice. That is, coefficient of resistance is

$$C_r = \frac{h_l}{H} \qquad \text{.. (4·9c)}$$

Substituting for h_l from equation (4·9 a) we have

$$C_r = \frac{H(1 - C_v^2)}{H}$$

$$= 1 - C_v^2$$

$$\text{or } C_r + C_v^2 = 1$$

Problem-1 : *In making an experiment to find different coefficients of an orifice a jet of water is made to flow through a circular orifice of 2·5 cm diameter. The orifice is fitted in the vertical side of a tank. The head of water above the centre of the orifice is 1 m. The jet is passed through a ring whose centre is found to be 35 cm horizontally away and 3·5 cm vertically down from the centre of vena contracta. The actual discharge found by measuring with collecting tank is 1·35 litres/sec. Determine the coefficients of the orifice.*

Referring to fig. 4-3 and eqn. (4·3), we have,

$$x = \frac{35}{100} = 0{\cdot}35 \text{ m}, \; y = \frac{3{\cdot}5}{100} = 0{\cdot}035 \text{ m, and } H = 1 \text{ m.}$$

$$\text{Hence, } C_v = \sqrt{\frac{x^2}{4yH}} = \frac{x}{2\sqrt{yH}} = \frac{0{\cdot}35}{2\sqrt{0{\cdot}035 \times 1}}$$

$$= 0{\cdot}935$$

Theoretical discharge = area of orifice × theoretical velocity

$$= \frac{\pi}{4} \times \left(\frac{2{\cdot}5}{100}\right)^2 \times \sqrt{2 \times 9{\cdot}81 \times 1}$$

$$= 0{\cdot}00217 \text{ cu.m/sec.} = 2{\cdot}17 \text{ litres/sec.}$$

$$\text{Hence, } C_d = \frac{1{\cdot}35}{2{\cdot}17} = 0{\cdot}62$$

$$\text{Since, } C_d = C_c \times C_v$$

$$C_c = \frac{C_d}{C_v} = \frac{0{\cdot}620}{0{\cdot}935} = \underline{0{\cdot}664}$$

The coefficient of resistance, $C_r = 1 - C_v^2 = 1 - 0{\cdot}935^2 = 0{\cdot}126$

Problem-2 : *The centre of an orifice is situated 15 cm above the bottom of a vertical vessel containing water to a depth of 75 cm. Assuming a coefficient of velocity of 0·975, estimate the change in the height of the jet for a horizontal travel of 65 cm from vena contracta when the orifice is fitted in the side of the vessel.*

Here, we have, $x = 65$ cm, and $H = 75 - 15 = 60$ cm,

$$\text{Since, } C_v = \frac{x}{2\sqrt{yH}} \qquad \text{..(ref. eqn. 4·3)}$$

$$\text{i.e. } 0{\cdot}975 = \frac{65}{2\sqrt{y \times 60}}$$

$$\therefore \quad \sqrt{y} = \frac{65}{2 \times \sqrt{60} \times 0.975} = 4.3$$

$$\therefore \quad y = \underline{18.5 \text{ cm}}$$

Problem-3 : *Water issues vertically up from an orifice in a horizontal plane of a tank (ref. fig. 4-5). The jet is under a pressure of 0·8 kg/ cm². If the coefficient of velocity of the orifice is 0·98, find, the height through which the jet rises, the actual velocity of the jet at the vena contracta, and the head lost in friction.*

To convert the pressure in kg/sq. cm² into equivalent head of water, we have,

$$H = \frac{100^2 p}{w} = \frac{100^2 \times 0.8}{1,000} = 8 \ m$$

The actual velocity of water is $V = C_v \sqrt{2gH}$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 8}$$
$$= 12.3 \text{ m/sec.}$$

The height to which jet rises is given by

$$H' = \frac{V^2}{2g} = \frac{12.3 \times 12.3}{19.62} = 7.72 \text{ m}$$

Head lost in friction = $H - H' = 8 - 7.72 = 0.28$ m

Problem-4 : *A 5 cm diameter orifice is provided in a tank containing water to a height of 1·5 m above the centre of the orifice. If the coefficients of contraction and velocity for the orifice are 0·62 and 0·98 respectively, determine, (i) the coefficient of discharge, (ii) theoretical discharge, (iii) actual discharge, (iv) loss of head, and (v) coefficient of resistance.*

(i) Coefficient of discharge, $C_d = C_c \times C_v$

$$= 0.62 \times 0.98 = \underline{0.608}$$

(ii) Theoretical discharge is

Q_t = area of orifice × theoretical velocity

$$= \frac{\pi}{4}\left(\frac{5}{100}\right)^2 \sqrt{2 \times 9.81 \times 1.5} = \underline{0.0107 \text{ m}^3/\text{sec.}}$$

(iii) Actual discharge $= C_d \times$ theoretical discharge

$$= 0.608 \times 0.0107 = \underline{0.0065 \text{ m}^3/\text{sec.}}$$

(iv) Loss of head, $h_f = (1 - C_v^2)H$ (ref. eqn. 4·9a)

$$= (1 - 0.98^2) \times 1.5$$

$$= 0.06 \text{ m of water}$$

(v) Coefficient of resistance $C_r = \dfrac{h_f}{H} = \dfrac{0.06}{1.5} = \underline{0.04}$

Problem-5 : *A large vertical plate is held perpendicular to a jet issuing from a sharp-edged orifice, 2·6 cm diameter, in the side of a tank. The force exerted on the plate is 2 kg. The measured discharge is 2·6 litres/sec. The constant head of water on the orifice is 3 m. Determine the coefficients of the orifice.*

We have, the actual discharge

$$Q = \frac{2.6}{1,000} = 0.0026 \text{ m}^3/\text{sec.}$$

The force exerted by the jet is

$$F = \frac{wQV}{g}$$

$$\therefore \quad V = \frac{Fg}{wQ}$$

$$= \frac{2 \times 9.81}{1,000 \times 0.0026} = 7.5 \text{ m/sec}$$

Theoretical velocity of the jet is

$$V_t = \sqrt{2gH} = \sqrt{19.62 \times 3} = 7.65 \text{ m/sec.}$$

Hence, coefficient of velocity is

$$C_v = \frac{V}{V_t} = \frac{7.5}{7.65} = \underline{0.98}$$

Theoretical discharge is

$$Q_t = a_0 \sqrt{2gH} = \frac{\pi}{4} \times \left(\frac{2.6}{100}\right)^2 \times \sqrt{2 \times 9.81 \times 3}$$

$$= 0.0041 \text{ m}^3/\text{sec.}$$

Hence coefficient of discharge is

$$C_d = \frac{Q}{Q_t} = \frac{0.0026}{0.0041} = \underline{0.634}$$

Hence, coefficient of contraction is

$$C_c = \frac{C_d}{C_v} = \frac{0.634}{0.98} = \underline{0.647}$$

Problem-6 : *A vessel contains water and is fitted with an orifice of 2·5 cm diameter on one side. The orifice is at first plugged and the vessel is suspended so that the displacing force could be measured. On removal of plug, the horizontal force required to keep the vessel in place, was found to be 1·15 kg. This force was applied on the side of vessel opposite the orifice. The constant level of water above the centre of the orifice is 2 m. The discharge collected in the measuring tank was 222 litres in 2 minutes. Determine the coefficients of the orifice.*

The discharge from the orifice is

$$Q = \frac{222}{2 \times 60 \times 1{,}000} = 0.00185 \text{ m}^3/\text{sec.}$$

$$\therefore \quad V = \frac{gF}{wQ} = \frac{9.81 \times 1.15}{1{,}000 \times 0.00185} = 6.1 \text{ m/sec.}$$

Theoretical velocity of the jet is

$$V_t = \sqrt{2gH} = \sqrt{19.62 \times 2} = 6.25 \text{ m/sec.}$$

$$\text{Hence, } C_v = \frac{V}{V_t} = \frac{6.1}{6.25} = \underline{0.975}$$

Theoretical discharge is

$$Q_t = \text{area of the orifice} \times \text{theoretical velocity}$$

$$= \frac{\pi}{4} \times \left(\frac{2.5}{100}\right)^2 \times 6.25 = 0.00307 \text{ m}^3/\text{sec.}$$

$$\text{Hence, } C_d = \frac{Q}{Q_t} = \frac{0.00185}{0.00307} = 0.60$$

$$\text{Further, } C_c = \frac{C_d}{C_v} = \frac{0.60}{0.975} = 0.617$$

The coefficient of resistance of the orifice is

$$C_r = 1 - C_v^2 = 1 - 0.975^2 = 0.05$$

Problem-7 : *A 7 cm diameter orifice is provided in a closed tank containing water to a constant height of 3 m above the orifice. The coefficients of contraction and velocity for the orifice are 0·62 and 0·97 respectively. The pressure of air above water surface is 0·3 kg/cm² which is maintained constant. Calculate, (i) the actual discharge, (ii) the theoretical discharge, (iii) coefficient of discharge, (iv) loss of head, and (v) coefficient of resistance.*

The flow of water will take place through the orifice under a total head which is the sum of the head due to water and the head due to air pressure.

$$i.e. \quad H = 3 + \frac{0{\cdot}3 \times 100^2}{1{,}000} \qquad \left(\text{since } H = \frac{p}{w}\right)$$

$$= 6 \text{ m of water}$$

(i) The actual discharge is

$$Q = \text{area of jet} \times \text{actual velocity}$$

$$= C_c \times a_o \times C_v \times \sqrt{2gH}$$

$$= 0{\cdot}62 \times \frac{\pi}{4}\left(\frac{7}{100}\right)^2 \times 0{\cdot}97 \times \sqrt{2 \times 9{\cdot}81 \times 6}$$

$$= \underline{0{\cdot}026 \text{ m}^3/\text{sec.}}$$

(ii) The theoretical discharge is

$$Q_t = \frac{\pi}{4}\left(\frac{7}{100}\right)^2 \times \sqrt{2 \times 9{\cdot}81 \times 6} = \underline{0{\cdot}042 \text{ m}^3/\text{sec.}}$$

(iii) Coefficient of discharge is

$$C_d = \frac{Q}{Q_t} = \frac{0{\cdot}026}{0{\cdot}042} = \underline{0{\cdot}62}$$

(iv) Loss of head through the orifice is (ref. eqn. 4.9a)

$$h_f = (1 - C_v^2)\, H$$

$$= (1 - 0{\cdot}97^2) \times 6 = \underline{0{\cdot}36 \text{ m of water}}$$

(v) Coefficient of resistance is

$$C_r = \frac{h_f}{H} = \frac{0{\cdot}36}{6} = \underline{0{\cdot}06}$$

Problem – 8 : *In order to determine the different coefficients of an orifice, a tank is suspended on knife-edges of 2·5 m above the centre line of the orifice which is 6 5 cm² in area. When the head of water in the tank is 1·25 m, water is discharged at the rate of 120 kg/min. and a weight of 2·5 kg at a leverage of 1 m is required to keep the tank vertical Find the coefficients of velocity, contraction and discharge.*

When water is discharged from a tank through the orifice, a force F is produced on the opposite side of the orifice. In order to balance this force, external force W is applied to the tank.

Taking moments about the turning point (fig. 4–4) *i.e.* knife edge, we have,

$$W \cdot x = F \cdot y$$

$$\therefore \quad F = \frac{W \cdot x}{y} = \frac{2 \cdot 5 \times 1}{2 \cdot 5} = 1 \text{ kg}$$

Also, we have,

$$F = \frac{wQV}{g} \text{ and } Q = \frac{120}{60 \times 1{,}000} = 0 \cdot 002 \text{ m}^3/\text{sec.}$$

$$\therefore \quad V = \frac{Fg}{wQ}$$

$$= \frac{1 \times 9 \cdot 81}{1{,}000 \times 0 \cdot 002} = 4 \cdot 905 \text{ m}^3/\text{sec.}$$

Theoretical velocity is

$$V_t = \sqrt{2gH} = \sqrt{19 \cdot 62 \times 1 \cdot 25} = 4 \cdot 96 \text{ m/sec.}$$

$$\text{Hence, } C_v = \frac{V}{V_t} = \frac{4 \cdot 905}{4 \cdot 96} = 0 \cdot 988$$

Theoretical discharge, $Q_t = a_0 \times V_t$

$$= \frac{6 \cdot 5}{(100)^2} \times 4 \cdot 96$$

$$= 0 \cdot 00323 \text{ m}^3/\text{sec.}$$

$$\text{or} \quad = 0 \cdot 00323 \times 1{,}000 \times 60 = 193 \cdot 8 \text{ kg/min.}$$

$$\text{Hence, } C_d = \frac{120}{193 \cdot 8} = \underline{0 \cdot 62} \text{ and } C_c = \frac{C_d}{C_v} = \frac{0 \cdot 62}{0 \cdot 988} = \underline{0 \cdot 63}$$

Problem-9: *Two orifices are placed in a tank as shown in the fig. 4-6. If it is given that $x_2 = 3x_1$ determine the head of water above the centre of the top orifice.*

Let 'H' be the head of water above the centre of the top orifice. Let v_1 and v_2 be the velocity of water through the bottom and top orifice respectively.

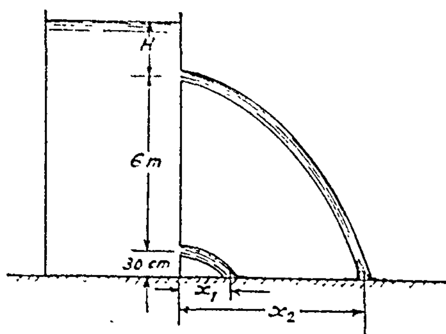


Fig. 4-6

Assume that 'C_v' for both orifices is the same,

$$C_{v1} = \sqrt{\frac{gx_1^2}{4y_1H_1}}$$

$$C_{v2} = \sqrt{\frac{gx_2^2}{4y_2H_2}}$$

$$\therefore \quad \frac{gx_1^2}{4y_1H_1} = \frac{gx_2^2}{4y_2H_2} \qquad \text{i.e.} \quad \frac{x_1^2}{y_1H_1} = \frac{x_2^2}{y_2H_2}$$

Now $x_2 = 3x_1$, $y_1 = 0{\cdot}3m$; $y_2 = 6{\cdot}3$ m

$H_1 = (6 + H)$ m and $H_2 = H$ m

$$\therefore \quad \frac{x_1^2}{0{\cdot}3\,(6 + H)} = \frac{(3x_1)^2}{6{\cdot}3\,(H)}$$

$$\therefore \quad x_1^2 \times 6{\cdot}3H = 0{\cdot}3\,(6 + H) \times 9x_1^2$$

$$6{\cdot}3H = 16{\cdot}2 + 2{\cdot}7H$$

$$\therefore \quad 3{\cdot}6H = 16{\cdot}2$$

$$\therefore \quad H = \frac{16{\cdot}2}{3{\cdot}6} = \underline{4{\cdot}5 \text{ m}}$$

Problem-10 : *Oil of specific gravity 0·75 flows through a 10 cm diameter orifice having $C_v = 0{\cdot}95$, $C_c = 0{\cdot}65$. If the power delivered by the jet is 10 H.P., determine the reading of the pressure gauge A fitted on the top of the tank as shown in fig. 4–7.*

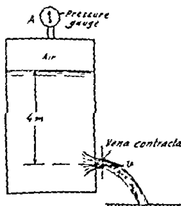


Fig. 4–7

Let V be the velocity of the jet at the vena contracta.

Area of jet at vena contracta,

$$A_1 = C_c \times \frac{\pi}{4} \times d_1^2 = 0{\cdot}65 \times \frac{\pi}{4}\left(\frac{10}{100}\right)^2$$

$$= 0{\cdot}0051 \text{ m}^2$$

Weight of oil discharged by the jet per second,

$$W = 0{\cdot}0051 \times V \times 0{\cdot}75 \times 1{,}000 = 3{\cdot}825V \text{ kg/sec.}$$

Power in the jet $= \dfrac{WH}{75}$

or $\quad 10 = \dfrac{3{\cdot}825VH}{75}$

Now, $V = C_v\sqrt{2gH} = 0{\cdot}95\sqrt{19{\cdot}62 \times H} = 4{\cdot}1\sqrt{H}$

$$\therefore \quad 10 = \frac{3{\cdot}825 \times 4{\cdot}1\sqrt{H} \times H}{75}$$

$$\therefore \quad H^{3/2} = \frac{10 \times 75}{3{\cdot}825 \times 4{\cdot}1}$$

$$\therefore \quad H = \left(\frac{750}{15{\cdot}6825}\right)^{2/3} = 13{\cdot}2 \text{ m of oil.}$$

$$\therefore \quad \frac{p_a}{w} = 13{\cdot}2 - 4 = 9{\cdot}2 \text{ m of oil}$$

$$\therefore \quad p_a = \frac{9{\cdot}2 \times 0{\cdot}75 \times 1000}{10^4} = \underline{0{\cdot}69 \text{ kg/cm}^2}$$

Problem-11 : *A tank has two similar orifices one 1 m vertically above the other as shown in fig. 4-8. The constant depth of water above the centre of lower orifice is 3 m. The jet of water coming out from the orifice intersect each other at 4·8 m horizontal distance from vena contracta. Determine the coefficient of velocity assuming it to be the same for both the orifices.*

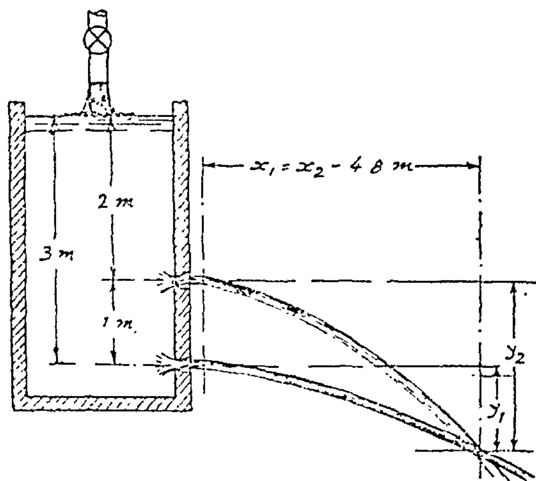


Fig. 4-8

Let H_1 and H_2 *m* be the head of water above the centre of the bottom and top orifices respectively.

Then $H_1 = 3$ m and $H_2 = 2$ m

$$\text{Now } C_{v1} = \sqrt{\frac{x_1^2}{4y_1H_1}}; \qquad C_{v2} = \sqrt{\frac{x_2^2}{4y_2H_2}}$$

But, $C_{v1} = C_{v2}$

$$\therefore \quad \frac{x_1^2}{4y_1H_1} = \frac{x_2^2}{4y_2H_2}$$

Also $\quad x_1 = x_2$

$$\therefore \quad y_1H_1 = y_2H_2$$

$$\therefore \quad y_1 = \frac{H_2}{H_1} \times y_2 = \frac{2}{3} \times y_2$$

Now $y_2 - y_1 = 1$ m

$\therefore \quad y_2 - \frac{2}{3} y_2 = 1$

$y_2 = 3$m

$\therefore \quad y_1 = 3 - 1 = 2$ m.

$$C_v = \sqrt{\frac{x_1^2}{4y_1 H_1}} = \sqrt{\frac{4{\cdot}8 \times 4{\cdot}8}{4 \times 2 \times 3}} = \underline{0{\cdot}98}$$

Problem-12 : *A closed tank contains water to a depth of 6 m and air above it as shown in fig 4-9. The tank is provided with an orifice of 5 cm diameter near its bottom. If the reading of the pressure gauge A is 0·5 kg/cm², determine the discharge through the orifice in litres/min. What will be the increase in pressure of the air if the discharge instantaneously is to be increased by 50%. Take $C_v = 0{\cdot}98$ and $C_c = 0{\cdot}62$. If the bottom of the tank is 4 m above the ground, find the horizontal distance where the jet will strike the ground in the first case.*

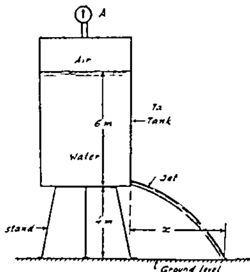


Fig. 4-9

Effective head of water H_1 above the centre of orifice is the actual head of water plus the equivalent head of water for air pressure.

$$\therefore \quad H_1 = 6 + \frac{10^4 p}{w} = 6 + \frac{10^4 \times 0.5}{1{,}000} = 11 \text{ m}$$

Also $C_d = C_v \times C_c = 0.98 \times 0.62 = 0.608$

$$\therefore \quad \text{Discharge,} \quad Q_1 = C_d \cdot a\sqrt{2gH_1}$$

$$= 0.608 \times \frac{\pi}{4}\left(\frac{5}{100}\right)^2 \times \sqrt{19.62 \times 11}$$

$$= 0.0176 \text{ m}^3/\text{sec.}$$

$$= 0.0176 \times 1{,}000 \times 60 = 1{,}050 \text{ litres/min.}$$

New discharge, $Q_2 = 1.5Q_1$

$$\therefore \quad Q_2 = 1.5 \times 0.0176 = 0.0264 \text{ m}^3/\text{sec}$$

Now $\dfrac{Q_2}{Q_1} = \dfrac{C_d a\sqrt{2gH_2}}{C_d a\sqrt{2gH_1}}$ $\qquad \therefore \quad \dfrac{0.0264}{0.0176} = \sqrt{\dfrac{H_2}{11}}$

$\therefore \quad (1.5)^2 = \dfrac{H_2}{11}$ $\qquad \therefore \quad H_2 = 11 \times (1.5)^2$

$$\therefore \quad H_2 = 24.75 \text{ m}$$

$$\therefore \quad \text{New pressure of air,} \quad p_2 = \frac{(24.75 - 6) \times 1{,}000}{10^4} = 1.875 \text{ kg/cm}^2$$

Hence increase of pressure is $1.875 - 0.5 = 1.375$ kg/cm^2

$$C_v = \sqrt{\frac{x^2}{4yH_1}}$$

$$0.98 = \sqrt{\frac{x^2}{4 \times 4 \times 11}}$$

$$(0.98)^2 \times 16 \times 11 = x^2$$

$$\therefore \quad x = 0.98\sqrt{176} = \underline{13 \text{ m}}$$

Time of Flow through an Orifice

The study of orifice involves the estimation of time required either to charge the level of the fluid by a certain height or to empty the entire tank. We first undertake the study of changing the levels of fluid in the tanks.

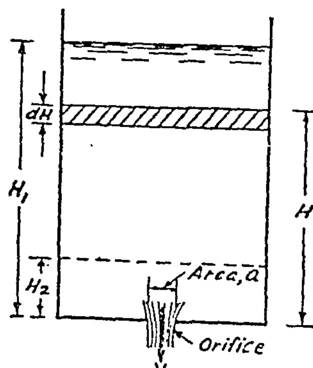


Fig. 4–10

Vessel of uniform cross section : A vessel of this description is shown in fig. 4-10.

Let, A – area of cross-section of the tank,

H_1 – initial head of water above the centre of the orifice,

H_2 – final head of water above the centre of the orifice,

T – time to change the level, and

a – area of cross-section of the orifice.

Let water level at any instant be H above the centre of the orifice and let it fall by an amount dH in small interval of time dT. Let dQ be the quantity of water discharged during this time. As dH is measured in the downward direction *i e.* against the direction of head, it will be negative. The volume of water discharged through an orifice equals the volume of water displaced from tank. Hence,

$$dQ = -A \cdot dH = C_d \cdot a \cdot V \cdot dT$$

But, $V = \sqrt{2gH}$

$$\therefore \quad -A \cdot dH = C_d \cdot a \cdot \sqrt{2gH} \cdot dT$$

$$\text{or } dT = -\frac{AdH}{C_d a \sqrt{2gH}} = \frac{AdH \cdot H^{-\frac{1}{2}}}{C_d a \sqrt{2g}}$$

Hence, the total time taken for lowering the level from H_1 to H_2 is

$$\int_O^T dT = -\frac{A}{C_d a \sqrt{2g}} \int_{H_1}^{H_2} H^{-\frac{1}{2}} dH$$

$$\therefore \quad T = -\frac{2A}{C_d a \sqrt{2g}} \left(H^{\frac{1}{2}} \right)_{H_1}^{H_2}$$

$$= \frac{2A}{C_d a \sqrt{2g}} \left(H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}} \right) \qquad \text{.. (4·12)}$$

If the tank is to be emptied upto the bottom edge of the orifice, $H_2 = 0$ and the total time required is

$$T = \frac{2A\sqrt{H_1}}{C_d a \sqrt{2g}} \qquad \text{..(4·12a)}$$

Here H_1 is the total head above the orifice.

Problem – 13 : *A swimming pool 10 m long and 6 m wide holds water to a depth of 1·25 m. If water is discharged through an opening, at the bottom of the pool, of area 0·23 m², find the*

time taken to empty it. Take the coefficient of discharge for the opening as 0·62.

The area of swimming pool, $A = 10 \times 6 = 60 \text{ cm}^2$.

$$\text{Time} \quad T = \frac{2A\sqrt{H}}{C_d a\sqrt{2g}} = \frac{2 \times 60 \times \sqrt{1{\cdot}25}}{0{\cdot}62 \times 0{\cdot}23 \times \sqrt{19{\cdot}62}}$$

$$= 212 \text{ sec} = \underline{3 \text{min. } 32 \text{ sec}}.$$

∧ ✓ **Problem – 14 :** *A cistern of cross section area of 1 m². contains water 4 m deep. An orifice of 6 cm diameter is provided at its bottom. Find the quantity of water flowing out from the cistern in 2 minutes, when the orifice is opened. The coefficient of discharge may be taken as 0·6.*

We have, area of cistern, $A = 1 \text{ m}^2$

and area of orifice, $a = \frac{\pi}{4}\left(\frac{6}{100}\right)^2 = 0{\cdot}00282 \text{ m}^2$

and time for which the orifice remains open, $T = 120$ sec.

$$\text{As } T = \frac{2A\,(\sqrt{H_1} - \sqrt{H_2})}{C_d a\sqrt{2g}} \qquad \text{[ref. eqn. (4·12)]}$$

$$\text{we have, } 120 = \frac{2 \times 1\,(\sqrt{4} - \sqrt{H_2})}{0{\cdot}6 \times 0{\cdot}00282 \times \sqrt{2 \times 9{\cdot}81}}$$

$$\therefore \quad H_2 = 2{\cdot}4 \text{ m}$$

Volume of water discharged $= A\,(H_1 - H_2) = 1 \times (4 - 2{\cdot}4)$
$= 1{\cdot}6 \text{ m}^3$

Problem-15 : *A vertical tank 60 cm in diameter and 2·5 m high is full of water. It has two orifices each of 13 cm² area. One orifice is at the bottom of the tank and the other one 1·25 m above the bottom as shown in fig. 4-11. Find the time taken to empty the tank. Take coefficient of discharge for the orifice as 0·62.*

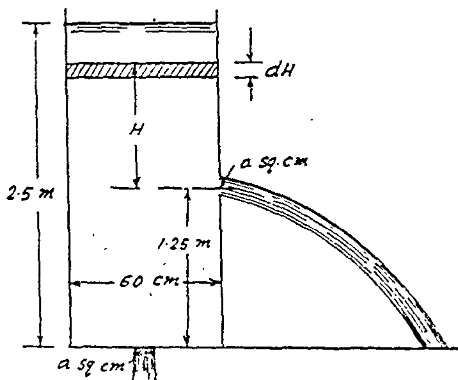


Fig. 4-11

We have, area of the tank as

$$A = \frac{\pi}{4}\left\{\frac{60}{100}\right\}^2 = 0.282 \text{ m}^2$$

and area of each orifice, $a = \dfrac{13}{100 \times 100} = 0.0013 \text{ m}^2$

Water will discharge through two orifices so long as its level reaches the centre of the top orifice, and then it will flow through only one orifice in the bottom. This shows that the problem is to be divided in two parts. Let H be the head in m of water above the centre of the top orifice at a particular instant, then the head over the bottom orifice will be $(H + 1.25)$ m. Let dH be the height of fall of water in time dT.

Discharge, $dQ = -AdH$

and also $dQ = C_d a\sqrt{2gH}.\, dT + C_d a\sqrt{2g(H+1.25)}\, dT$

$$\therefore \quad dT = \frac{-AdH}{C_d a\sqrt{2g}\{\sqrt{H+1.25} + \sqrt{H}\}}$$

If we integrate the above expression, we get

$$T_1 = -\frac{A}{C_d a\sqrt{2g}}\int_{H_1}^{H_2} \frac{dH}{\sqrt{H+1.25} + \sqrt{H}}$$

To integrate this, we first rationalize the expression by multiplying the denominator and numerator by $\sqrt{H+1.25} - \sqrt{H}$ and then integrate it.

$$\therefore \quad T_1 = \frac{A}{C_d a\sqrt{2g}}\int_{H_2}^{H_1} \frac{\sqrt{H+1.25} - \sqrt{H}}{1.25}\, dH$$

$$= \frac{A}{C_d a\sqrt{2g}} \times \frac{1}{1.25}\left\{\tfrac{2}{3}(H+1.25)^{\frac{3}{2}} - \tfrac{2}{3}(H)^{\frac{3}{2}}\right\}_{H_2}^{H_1}$$

$$= \frac{A}{C_d a\sqrt{2g}} \times \frac{8}{15}\left\{(H+1.25)^{\frac{3}{2}} - (H)^{\frac{3}{2}}\right\}_{H_2}^{H_1}$$

When depth of the water is 2.5m, $H = 1.25$m, and when it is 1.25m, $H = 0$.

$\therefore \quad H_1 = 1{\cdot}25$ and $H_2 = 0.$

$$\therefore \quad T_1 = \frac{0{\cdot}282}{0{\cdot}62 \times 0{\cdot}0013 \times 4{\cdot}43} \times \frac{8}{15} \times \left[\left\{ (1{\cdot}25 + 1{\cdot}25)^{\frac{3}{2}} - (1{\cdot}25)^{\frac{3}{2}} \right\} - (1{\cdot}25)^{\frac{3}{2}} \right]$$

$$= 48{\cdot}5 \text{ sec.}$$

$$\text{Time, } T_2 = \frac{2A\,(\sqrt{H_1} - \sqrt{H_2})}{C_d a \sqrt{2g}}$$

$$= \frac{2 \times 0{\cdot}282\,(\sqrt{1{\cdot}25} - \sqrt{0})}{0{\cdot}62 \times 0{\cdot}0013 \times 4{\cdot}43}$$

$$= 179 \text{ sec.}$$

$\therefore$ Total time $= T_1 + T_2 = 48{\cdot}5 + 179 = 227{\cdot}5$ sec.

✓Problem – 16 : *Compensation water is to be discharged by two circular orifices of the same diameter, situated at the bottom of a vessel having a constant head of 2 m. If the demand is 18,000,000 litres/day, what diameter will be required for each orifice ? Take coefficients of contraction and velocity as 0·62 and 0·98 respectively.*

$$\text{Discharge, } Q = \frac{18{,}000{,}000}{1{,}000 \times 3{,}600 \times 24} = 0{\cdot}208 \text{ m}^3\text{/sec.}$$

Coefficient of discharge is

$$C_d = C_c \times C_v = 0{\cdot}62 \times 0{\cdot}98 = 0{\cdot}608$$

If d is the diameter of each orifice in cm, we have,

$$Q = C_d \cdot a \sqrt{2gH}$$

$$\text{or } 0{\cdot}208 = 0{\cdot}608 \times 2 \times \frac{\pi}{4}\left(\frac{d}{100}\right)^2 \times \sqrt{2 \times 9{\cdot}81 \times 2}$$

$$\therefore \quad d^2 = 350$$

$$\therefore \quad d = \underline{18{\cdot}7 \text{ cm}}$$

Vessel of non-uniform cross-section : In this case, as the water level falls, the area of cross-section of the vessel at water surface changes and hence we come across two variables instead

of one variable as in case of vessels of uniform cross-section. Thus this method requires help of Calculus after establishing the relation between two variables for calculating the time of flow of water. The problem has to be tackled from first principles as no formula that could be applied to vessels of different cross-sections can be developed. This will be clear from problems that are following.

Problem-17 : *A tank 1·5 m deep is rectangular in cross section. It is 12 m × 6 m at its top and 9 m × 6 m at its bottom. The tank is full with water. If water is discharged through a short pipe 30 cm in diameter and fitted at its bottom as shown in fig. 4-12 (a), find the time taken to empty it. Take coefficient of discharge as 0 62.*

From fig. 4-12 (b); $\frac{\delta l}{H} = \frac{1{\cdot}5}{1{\cdot}5}$

$\therefore \quad \delta l = H$

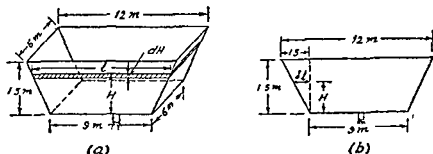


Fig. 4 – 12

Taking a thin strip of thickness dH, of water at height H from the bottom of tank and having dimensions of l m × 6 m,

$$l = 9 + 2 \times \delta l = 9 + 2H$$

Area of this strip, $A = l \times 6 = (9 + 2H) \times 6 = 54 + 12H$

Discharge of this strip is

$$dQ = -AdH = C_d a \cdot \sqrt{2gH} \cdot dT$$

$$\therefore \quad dT = \frac{-AdH\, H^{-\frac{1}{2}}}{C_d\, a\, \sqrt{2g}} \qquad .. (4{\cdot}13)$$

$$= \frac{-(54 + 12H)\, H^{-\frac{1}{2}}\, dH}{C_d \cdot a\, \sqrt{2g}}$$

$$\therefore \int_0^T dT = \frac{-1}{C_d \cdot a\sqrt{2g}} \int_{H_1}^{H_2} \left(54\, H^{-\frac{1}{2}} + 12\, H^{\frac{1}{2}} \right) dH$$

$$\therefore \quad T = \frac{+1}{C_d \cdot a\sqrt{2g}} \left[108\, H^{\frac{1}{2}} + \frac{24}{3}\, H^{\frac{3}{2}} \right]_{H_2}^{H_1}$$

$$= \frac{\left[108\,(\sqrt{1 \cdot 5} - \sqrt{0}) + \frac{24}{3} \left\{ (1 \cdot 5)^{3/2} - 0^{3/2} \right\} \right]}{0 \cdot 62 \times \frac{\pi}{4} \left(\frac{30}{100} \right)^2 \times 4 \cdot 43}$$

$= \underline{760 \text{ sec.}}$

Problem – 18 : *Find the time required to empty a swimming pool through an opening provided in the bottom of the deep end as well as near the bottom of the shallow end as shown in fig. 4–13 (a).*

Depth of water at deep end	*= 3 m*
Depth of water at shallow end	*= 1 m*
Length of bath, L	*= 30 m*
Width of bath, B	*= 10 m*
Area of each opening, a	*= 0·25 sq. m.*

Take discharge coefficient for each opening as 0·6.

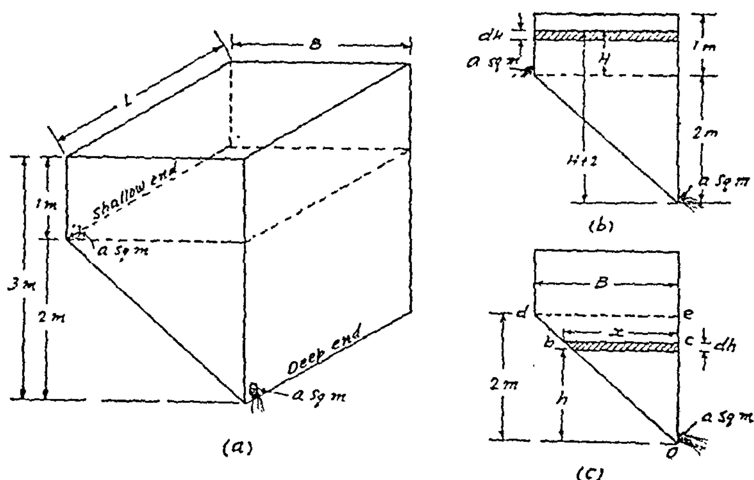


Fig. 4–13

The pool is uniform in cross-sectional area upto 1 m height from the top and water is discharged through two orifices as shown in fig. 4–13 (b).

Let the water level in the pool be H m above the centre of the top orifice and let it fall by dH in a small interval of time dT.

The volume of water discharged from the pool is

$$dQ = -A \times dH.$$

Discharge through the top opening in time dT is

$$\therefore \quad dQ_1 = C_d \times \text{area} \times \text{velocity} \times \text{time} = C_d \times a \times \sqrt{2gH} \times dT$$

Discharge through the bottom opening in time dT is

$$dQ_2 = C_d \times \text{area} \times \text{velocity} \times \text{time} = C_d \times a\ \sqrt{2g\,(H+2)} \times dT$$

Now total discharge through two orifice in the time dT is

$$dQ = dQ_1 + dQ_2$$

$$= C_d \cdot a \cdot \sqrt{2g\,H} \times dT + C_d \cdot a \cdot \sqrt{2g\,(H+2)} \times dT$$

$$\therefore \quad -A\,dH = \left\{ C_d \cdot a\ \sqrt{2gH} + C_d \cdot a\ \sqrt{2g\,(H+2)} \right\} dT$$

$$\therefore \quad dT = \frac{-A\,dH}{C_d \cdot a\,\sqrt{2gH} + C_d \cdot a \cdot \sqrt{2g\,(H+2)}}$$

$$\therefore \quad dT = -\frac{A \cdot dH}{C_d\,a\,\sqrt{2g}} \left\{ \frac{1}{\sqrt{H} + \sqrt{H+2}} \right\}$$

$$= \frac{-A \cdot dH}{C_d \cdot a\,\sqrt{2g}} \left\{ \frac{\sqrt{H+2} - \sqrt{H}}{(\sqrt{H+2} - \sqrt{H})\,(\sqrt{H+2} + \sqrt{H})} \right\}$$

$$= \frac{-A\,dH}{C_d \cdot a\,\sqrt{2g}} \left\{ \frac{\sqrt{H+2} - \sqrt{H}}{2} \right\}$$

$$\therefore \quad \int_0^{T_1} dT = \frac{-A}{2 \cdot C_d \cdot a \cdot \sqrt{2g}} \int_{H_1}^{H_2} (\sqrt{H+2} - \sqrt{H})\,dH$$

$$\therefore \quad T_1 = \frac{+A}{2 \cdot C_d \cdot a \cdot \sqrt{2g}} \left[\tfrac{2}{3}\,(H+2)^{\frac{3}{2}} - \tfrac{2}{3}\,H^{\frac{3}{2}} \right]_{H_2}^{H_1}$$

$$= \frac{A}{3 \cdot C_d \cdot a\,\sqrt{2g}} \left[(H+2)^{\frac{3}{2}} - H^{\frac{3}{2}} \right]_{H_2}^{H_1}$$

Now, $A = L \times B = 30 \times 10 = 300$ m²

$a = 0{\cdot}25$ m² $C_d = 0{\cdot}6$, $H_1 = 1$ m and $H_2 = 0$ m

$$\therefore \quad T_1 = \frac{300}{3 \times 0{\cdot}6 \times 0{\cdot}25 \times \sqrt{19{\cdot}62}}\left[\{(3)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\} - \{(1)^{\frac{3}{2}} - (0)^{\frac{3}{2}}\}\right]$$

$$= \frac{300 \times 1{\cdot}37}{0{\cdot}45 \times 4{\cdot}43} = 207 \text{ sec.}$$

When the water level falls below 1 m depth, the cross sectional area of the water level changes and the water is discharged through only one opening situated near the bottom of the deep end.

Let h be the head of water above the centre of the opening at a particular time and let it fall by 'dh' in a small interval of time dT. Let x m be the width of water level at a particular time.

$\therefore$ Area of water level, $A = x \times L = 30x$ m²

Volume of water flowing out from the pool in a time dT is

$$dQ = -30\,xdh$$

Also $dQ = C_d \times$ area $\times$ velocity $\times$ time

$$= C_d \times a \times \sqrt{2gh} \times dT$$

$$\therefore \quad -30xdh = C_d \cdot a\,\sqrt{2gh} \times dT$$

$$dT = \frac{-30\,x\,dh}{C_d \cdot a\,\sqrt{2gh}}$$

From similar tringles obc, and ode shown in fig. 4–13 (c) We have,

$$\frac{x}{B} = \frac{oc}{oe}$$

$$\therefore \quad x = B \times \frac{oc}{oe} = 10 \times \frac{h}{2} = 5h$$

$$\therefore \quad dT = \frac{-150\,hdh}{C_d \cdot a\,\sqrt{2gh}}$$

$$\therefore \quad \int_0^{T_2} dT = \frac{-150}{C_d \cdot a\,\sqrt{2g}} \int_{H_1}^{H_2} h^{\frac{1}{2}}dh$$

$$\therefore \quad T_2 = \frac{+150}{C_d \cdot a\sqrt{2g}} \left[\tfrac{2}{3} h^{\frac{3}{2}} \right]_{H_2 = 0}^{H_1 = 2}$$

$$\text{or} \quad T_2 = \frac{150 \times \frac{2}{3}}{0 \cdot 6 \times 0 \cdot 25 \times 4{\cdot}43} \left[(2)^{\frac{3}{2}} \right]$$

$$= \frac{400 \times 2{\cdot}83}{0{\cdot}6 \times 4{\cdot}43}$$

$$= 426 \text{ sec.}$$

$\therefore$ Total time taken to empty the pool is

$$T = T_1 + T_2$$

$$= 207 + 426 = \underline{633 \text{ sec.}}$$

Problem – 19 : *A tank 3 m deep is rectangular in cross-section, 20 m × 12 m at the top and 10 m × 6 m at the bottom. Water is discharged through a short vertical pipe, 45 cm diameter fitted at its bottom. If coefficient of discharge, C_d is 0·64, determine the time taken to empty the tank.*

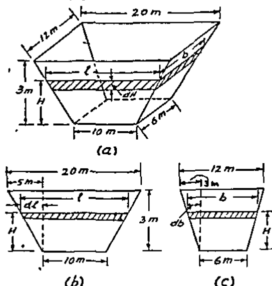


Fig. 4-14

Fig.4-14 (a) shows the overall view of tank, (b) shows the lengthwise section and (c) shows the breadthwise section.

Taking a small strip of water of thickness dH at height H from the bottom, from fig. 4-14 (b), we have $\frac{dl}{H} = \frac{5}{3}$ from which $dl = 1{\cdot}66H$ and hence $l = 10 + 2 \times dl = (10 + 3{\cdot}32H)$ m. From fig. 4-14 (c), we have, $\frac{db}{H} = \frac{3}{3}$ or $db = H$; then,

$b = (6 + 2db) = (6 + 2H)$ m. Area of cross section of water level at a height H above the bottom is

$$A = l \times b$$

$$= (10 + 3{\cdot}32\,H) \times (6 + 2H) = (60 + 39{\cdot}92H + 6{\cdot}44\,H^2)\ \text{m}^2$$

Volume of strip of area A and thickness dH is

$$dQ = -AdH = C_d{\cdot}a\sqrt{2gH} \times dT$$

This is so because this volume of water passes through the orifice in time dT when the head on the orifice is H.

$$\therefore \quad dT = \frac{-AdH{\cdot}H^{-\frac{1}{2}}}{C_d{\cdot}a\sqrt{2g}}$$

$$= \frac{-(60 + 39{\cdot}92\,H + 6{\cdot}44H^2)\,H^{-\frac{1}{2}}\,dH}{C_d{\cdot}a\sqrt{2g}}$$

$$\int_0^T dT = \frac{-1}{C_d{\cdot}a\sqrt{2g}} \int_{H_1}^{H_2} (60H^{-\frac{1}{2}} + 39{\cdot}92\,H^{\frac{1}{2}} + 6{\cdot}44H^{\frac{3}{2}})\,dH$$

$$\therefore \quad T = \frac{+1}{C_d a\sqrt{2g}} \left[120H^{\frac{1}{2}} + \frac{2}{3}\,39{\cdot}92H^{\frac{3}{2}} + \frac{2}{5}6{\cdot}44H^{\frac{5}{2}}\right]_{H_2}^{H_1}$$

Now, here, we have $C_d = 0{\cdot}64$, $a = \frac{\pi}{4}\left(\frac{45}{100}\right)^2 = 0{\cdot}159$ m^2

$H_1 = 3$ m and $H_2 = 0$ m.

$$\therefore \quad T = \frac{1}{0{\cdot}64 \times 0{\cdot}159 \times \sqrt{2 \times 9{\cdot}81}}\Big[120 \times 3^{\frac{1}{2}} + 26{\cdot}62 \times 3^{\frac{3}{2}}$$

$$+ 2{\cdot}58 \times 3^{\frac{5}{2}}\Big]$$

$= 854$ sec. $= 14$ min. and 14 sec.

Problem – 20 : *A triangular trough has each side 1·5 m and the length of the trough is 3 m. This trough is full of water and it is to be emptied through a circular hole 20 cm diameter at its bottom*

as shown in fig. 4-15. Find the time taken to empty it. Take coefficient of discharge $C_d = 0.64$.

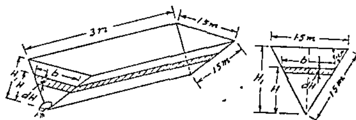


Fig. 4-15

Here, we have $\sin 60° = \frac{H_1}{1.5}$

$$\therefore \quad H_1 = 1.5 \sin 60° = 1.5 \times 0.866 = 1.3 \text{ m}$$

and $\frac{b}{1.5} = \frac{H}{H_1}$

$$\therefore \quad b = \frac{1.5H}{H_1} = \frac{1.5}{1.3} H = 1.155\, H$$

Area of cross section of the water strip at a height H from bottom is

$$A = b \times 3 = 1.155H \times 3 = 3.465H$$

Volume of this strip of water is

$$dQ = -AdH = C_d a \sqrt{2gH} \cdot dT$$

$$\therefore \quad dT = \frac{-AdH \cdot H^{-\frac{1}{2}}}{C_d a \sqrt{2g}}$$

$$= \frac{-3.465H^{\frac{1}{2}}dH}{C_d a \sqrt{2g}} \text{ (substituting for } A)$$

$$\therefore \quad \int_0^T dT = \frac{-3.465}{C_d \cdot a \sqrt{2g}} \int_{H_1}^{H_2} H^{\frac{1}{2}} dH$$

$$\therefore \quad T = \frac{+3{\cdot}465}{C_d a\sqrt{2g}}\left[\frac{2}{3} H^{\frac{3}{2}}\right]_{H_2}^{H_1}$$

Since, $C_d = 0{\cdot}64$, $a = \frac{\pi}{4}\left(\frac{20}{100}\right)^2 = 0{\cdot}0314$ m², $H_1 = 1{\cdot}3$ m and $H_2 = 0$

$$\therefore \quad T = \frac{3{\cdot}465 \times \frac{2}{3}(1{\cdot}3)^{\frac{3}{2}}}{0{\cdot}64 \times 0{\cdot}0314 \times 4{\cdot}43} = 38{\cdot}6 \text{ sec.}$$

Problem-21 : *A vertical tank 2 m deep is square in cross-section. Its sides at the top are each 1 m long and at the bottom each 0·3 m long. It is provided with a sharp edged hole 15 cm in diameter at the bottom. If the tank is full in the beginning, find the time taken to change the water level by 0·7 m. The coefficient of discharge $C_d = 0{\cdot}6$.*

Let l be the length of each side at a height H m above the orifice. Then, area of cross section of the water strip at a height, H m is, $A = l^2$ m².

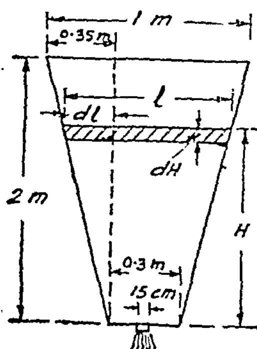


Fig. 4-16

$$\text{and} \quad \frac{dl}{2} = \frac{0{\cdot}35}{2} = 0{\cdot}175$$

$$\therefore \quad dl = 0{\cdot}175\ H$$

$$\therefore \quad l = 0{\cdot}3 + 2dl = 0{\cdot}3 + 0{\cdot}35H$$

$$\text{and area } A = l^2 = (0{\cdot}3 + 0{\cdot}35H)^2$$

$$= 0{\cdot}09 + 0{\cdot}21H + 0{\cdot}122H^2$$

Volume of the strip of water of thickness dH is

$$dQ = -AdH C_d a\sqrt{2gH} \cdot dT$$

$$\therefore \quad dT = \frac{-AdH \cdot H^{-\frac{1}{2}}}{C_d a\sqrt{2g}}$$

$$\frac{-\left(0{\cdot}09H^{-\frac{1}{2}} + 0{\cdot}21H^{\frac{1}{2}} + 0{\cdot}122H^{\frac{3}{2}}\right)dH}{C_d a\sqrt{2g}}$$

$$\text{or} \int_0^T dT = \frac{-1}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} (0.09H^{-\frac{1}{2}} + 0.21H^{\frac{1}{2}} + 0.122H^{\frac{3}{2}})\, dH$$

$$\text{or} \quad T = \frac{1}{C_d a\sqrt{2g}} \left[0.18H^{\frac{1}{2}} + 0.14H^{\frac{3}{2}} + 0.049H^{\frac{5}{2}}\right]_{H_2}^{H_1}$$

Here, we have $C_d = 0.6$, $a = \frac{\pi}{4}\left(\frac{15}{100}\right)^2 = 0.0176 \text{ m}^2$

$H_1 = 2$ m and $H_2 = 2 - 0.7 = 1.3$ m

$$\therefore \quad T = \frac{1}{0.6 \times 0.0176 \times \sqrt{2 \times 9.81}} \left[0.18\,(\sqrt{2} - \sqrt{1.3}) + 0.14 \times \left(2^{\frac{3}{2}} - 1.3^{\frac{3}{2}}\right) + 0.049\left(2^{\frac{5}{2}} - 1.3^{\frac{5}{2}}\right)\right]$$

$= 9.0$ sec.

Problem–22 : *A tank, circular in plan, is 15 m in diameter at the top and 3 m in diameter at the bottom. The height of tank is 3 m. It is emptied through a short vertical pipe 22·5 cm in diameter, fitted at the base of the tank. Determine the time required to empty it, if coefficient of discharge C_d is 0.62.*

From fig. 4-17, we have,

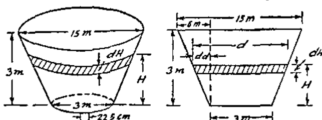


Fig. 4–17

$$\frac{dd}{H} = \frac{6}{3}$$

$$\therefore \quad dd = 2H$$

$$\text{and } d = 3 + 2dd = 3 + 4H$$

$$\text{Area of strip, } A = \frac{\pi}{4}(3 + 4H)^2 = \frac{\pi}{4}(9 + 24H + 16H^2)$$

Volume of water of this strip is

$$dQ = -AdH = C_d \cdot a\sqrt{2gH}\,dT$$

$$\therefore \quad dT = \frac{-AdH \cdot H^{-\frac{1}{2}}}{C_d \cdot a \cdot \sqrt{2g}}$$

$$= -\frac{\frac{\pi}{4}\left(9H^{-\frac{1}{2}} + 24H^{\frac{1}{2}} + 16H^{\frac{3}{2}}\right)dH}{C_d \cdot a \cdot \sqrt{2g}}$$

$$\therefore \quad \int_0^T dT = \frac{-\frac{\pi}{4}}{C_d \cdot a\sqrt{2g}} \int_H^0 (9H^{-\frac{1}{2}} + 24H^{\frac{1}{2}} + 16H^{\frac{3}{2}})\,dH$$

$$\therefore \quad T = \frac{\pi}{4 \times 0.62 \times \frac{\pi}{4}\left(\frac{22.5}{100}\right)^2 \times 4.43}\left[18H^{\frac{1}{2}} + \frac{48}{3}H^{\frac{3}{2}} + \frac{32}{5}H^{\frac{5}{2}}\right]_{H=0}^{H=3}$$

$$= \frac{\left[18\sqrt{3} + \frac{48}{3}(3)^{\frac{3}{2}} + \frac{32}{5}(3)^{\frac{5}{2}}\right]}{0.62 \times 0.0504 \times 4.43}$$

$= 1{,}540$ sec.

The hemispherical vessel : In this case, as the level of water falls, the area of cross-section of the water surface changes. The orifice is provided at the bottom of the vessel at the lowest point.

Let, R – radius of the hemispherical vessel,
H_1 – initial head of water,
H_2 – final head of water, and
T – time taken to change the water level from H_1 to H_2.

Further let r be the radius of the water level cross-section at H_m above the orifice and let dH be thickness of the strip.

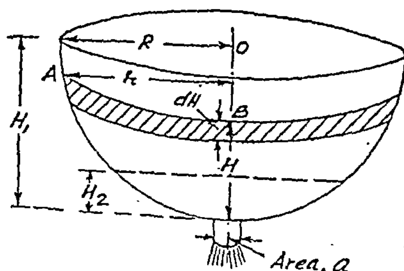


Fig. 4–18

Then, $OA^2 = AB^2 + BO^2$

or $\quad R^2 = r^2 + (R - H)^2$

$$\therefore \quad r^2 = R^2 - (R-H)^2 = 2RH - H^2$$

Area of the water surface at height, H is

$$A = \pi r^2 = \pi(2RH - H^2)$$

and volume of strip of water of area A and thickness dH is

$$dQ = -A dH = C_d a \sqrt{2gH}\, dT$$

$$\therefore \quad dT = \frac{-A dH \cdot H^{-\frac{1}{2}}}{C_d a \sqrt{2g}} = \frac{-\pi(2RH - H^2) H^{-\frac{1}{2}} dH}{C_d a \sqrt{2g}} \quad \text{.. (4·14)}$$

$$\int_0^T dT = -\frac{\pi}{C_d a \sqrt{2g}} \int_{H_1}^{H_2} \left(2RH^{\frac{1}{2}} - H^{\frac{3}{2}}\right) dH$$

$$\therefore \quad T = +\frac{\pi}{C_d a \sqrt{2g}} \left[\tfrac{4}{3} R H_1^{\frac{3}{2}} - \tfrac{2}{5} H^{\frac{5}{2}}\right]_{H_2}^{H_1}$$

$$= \frac{\pi}{C_d a \sqrt{2g}} \left\{\tfrac{4}{3} R\left(H_1^{\frac{3}{2}} - H_2^{\frac{3}{2}}\right) - \tfrac{2}{5}\left(H_1^{\frac{5}{2}} - H_2^{\frac{5}{2}}\right)\right\} \quad (4·15)$$

If the vessel were full at the commencement and is completely emptied, then,

$$H_1 = R \text{ and } H_2 = 0$$

$$\therefore \quad \text{time, } T = \frac{\pi\left(\tfrac{4}{3} R^{\frac{5}{2}} - \tfrac{2}{5} R^{\frac{5}{2}}\right)}{C_d a \sqrt{2g}}$$

$$= \frac{14 \pi R^{\frac{5}{2}}}{15 C_d a \sqrt{2g}}$$

Problem-23 : *A hemispherical tank is 4 m in diameter and is full of water, level of which is changed to 0·5 m in 80 sec, when water is discharged through a hole at the bottom of the tank. If coefficient of discharge of the hole is 0·6, determine its diameter. Prove the formula you may use.*

Referring to eqn. (4·15),

$$T = \frac{\pi}{C_d a \sqrt{2g}} \left\{\tfrac{4}{3} R\left(H_1^{\frac{3}{2}} - H_2^{\frac{3}{2}}\right) - \tfrac{2}{5}\left(H_1^{\frac{5}{2}} - H_2^{\frac{5}{2}}\right)\right\}$$

$$\text{or } 80 = \frac{\pi}{0.6 \times a \times \sqrt{2 \times 9.81}} \left[\tfrac{4}{3} \times 2\left\{2^{\frac{3}{2}} - (0.5)^{\frac{3}{2}}\right\} - \tfrac{2}{5}\left\{2^{\frac{5}{2}} - (0.5)^{\frac{5}{2}}\right\}\right]$$

$\therefore \quad a = 0{\cdot}066 \text{ m}^2$

i.e. $\frac{\pi}{4} d^2 = 0{\cdot}066$

$\therefore \quad d = \sqrt{\frac{4 \times 0{\cdot}066}{\pi}} = 0{\cdot}29 \text{ m}$

or $\quad d = \underline{29 \text{ cm}}$

Problem-24 : *A horizontal boiler 3 m in diameter and 10 m long contains water to a height of 2·5 m. Find the time of emptying the boiler through a short vertical pipe 15 cm in diameter and attached to the bottom of the boiler. Coefficient of discharge of the short pipe may be taken as 0·8.*

Fig. 4-19

The views of the boiler are shown in fig. 4-19. From it, we have,

$$OA^2 = AB^2 + BO^2$$

$$\text{or } R^2 = r^2 + (R - H)^2$$

$$\therefore \quad r^2 = 2RH - H^2$$

$$\text{or } r = \sqrt{2RH - H^2}$$

$$\text{Further, } AD = 2r = 2\sqrt{2RH - H^2}$$

The area of the strip of water of thickness dH is

$A = AD \times L = 2L\sqrt{2RH - H^2}$, where L is length of the boiler. and volume of that strip of water is

$$dQ = -AdH = C_d{\cdot}a\sqrt{2gH}\, dT$$

$$\therefore \; dT = \frac{-AdH{\cdot}H^{-\frac{1}{2}}}{C_d{\cdot}a\sqrt{2g}}$$

$$= \frac{-2L(\sqrt{2R - H})dH}{C_d a\sqrt{2g}}$$

$$\therefore \int_{O}^{T} dT = \frac{-2L}{C_d \cdot a\sqrt{2g}} \int_{H_1}^{H_2} \sqrt{2R-H} \cdot dH$$

$$\therefore \quad = \frac{+\frac{4}{3}L}{C_d \cdot a\sqrt{2g}} \left[(2R-H)^{\frac{3}{2}} \right]_{H_1}^{H_2}$$

$$= \frac{+\frac{4}{3}L}{C_d\, a\sqrt{2g}} \left[(2R-H_2)^{\frac{3}{2}} - (2R-H_1)^{\frac{3}{2}} \right]$$

Here, we have got,

$L = 10$ m; $a = \frac{\pi}{4}\left(\frac{15}{100}\right)^2 = 0.0177$ m²; $H_2 = 0$; $H_1 = 2.5$ m;

$C_d = 0.8$ and $R = 1.5$ m.

Hence, $T = \frac{\frac{4}{3} \times 10}{0.8 \times 0.0177 \times 4.43} [(3.0)^{1.5} - (0.5)^{1.5}]$

$= 1{,}030$ sec.

Flow of Water from Composite Vessels

Problem-25 : *A reservoir which is 2 m deep is circular in plan, the diameters at the top and bottom are 100 m and 80 m respectively. The mouth of the outlet pipe, which is 60 cm diameter, is 4 m below top water level. How long will it take to empty the reservoir ?*

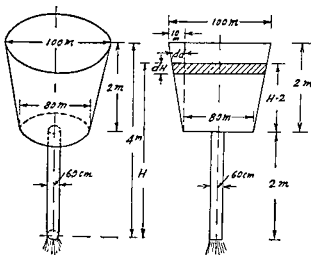


Fig. 4-20

Take coefficient of discharge, $C_d = 0{\cdot}8$ for the pipe. Neglect friction in the pipe.

From fig. 4–20, we have

$$\frac{10}{2} = \frac{dd}{H-2}$$

$$\therefore \quad dd = 5\,(H-2)$$

$$\text{and } d = 80 + 2dd$$
$$= 80 + 10\,(H-2)$$
$$= 10\,(6+H)$$

Area of the water surface at height H is

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4} \times 100\,(6+H)^2$$
$$= 25\pi\,(36 + 12H + H^2)$$

The volume of water of this strip of thickness dH is

$$dQ = -AdH = C_d{\cdot}a\sqrt{2gH}\,dT$$

$$\therefore \quad dT = -\frac{AdH{\cdot}H^{-\frac{1}{2}}}{C_d a\sqrt{2g}}$$

$$= -\frac{25\pi\,(36 + 12H + H^2)\,H^{-\frac{1}{2}}dH}{C_d a\sqrt{2g}}$$

$$\int_O^T dT = -\frac{25\pi}{C_d a\sqrt{2g}} \int_{H_1}^{H_2} \left(36H^{-\frac{1}{2}} + 12H^{\frac{1}{2}} + H^{\frac{3}{2}}\right)$$

$$\therefore \quad T = \frac{+25\pi}{C_d a\sqrt{2g}} \left[36 \times 2H^{\frac{1}{2}} + \frac{12 \times 2}{3}H^{\frac{3}{2}} + \frac{2}{5}H^{\frac{5}{2}}\right]_{H_2}^{H_1}$$

Here, we have, $H_1 = 4$ m; $H_2 = 2$ m; $a = \frac{\pi}{4}\left(\frac{60}{100}\right)^2 = 0{\cdot}283$ m²

$C_d = 0{\cdot}8$, Hence,

$$T = \frac{25\pi}{0{\cdot}8 \times 0{\cdot}283 \times 4{\cdot}43}\,[72\,(\sqrt{4} - \sqrt{2}) + 8\{4^{1\cdot5} - 2^{1\cdot5}\} + 0{\cdot}4\,\{4^{2\cdot5} - 2^{2\cdot5}\}]$$

$= 7{,}500$ sec. i.e. 2 hours and 5 min.

Problem – 26 : *A tank square in cross section (in plan) is 3 m deep. It has the top sides 6 m long and bottom sides 3 m long and is full of water. It is to be emptied through a vertical pipe, 15 cm diameter and 3 m long fitted at its bottom. Neglecting friction in the pipe, determine the time taken to empty it completely. Take coefficient of discharge as 0·8.*

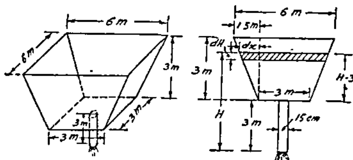


Fig. 4-21

The tank is shown in fig. 4-21. From it,

$$\frac{dx}{1 \cdot 5} = \frac{H-3}{3}$$

$$\therefore \quad dx = 0 \cdot 5\,(H-3)$$

$$\text{and } x = 3 + 2dx = 3 + 2 \times 0 \cdot 5\,(H-3) = H$$

Area of the strip of water is

$$A = x^2 = H^2$$

The volume of water strip of area A and thickness dH is

$$dQ = -AdH = C_d a \sqrt{2gH}\, dT$$

$$\therefore \quad dT = \frac{-AdH \cdot H^{-\frac{1}{2}}}{C_d a \sqrt{2g}}$$

$$= \frac{-H^2 dH \cdot H^{-\frac{1}{2}}}{C_d a \sqrt{2g}} \quad \text{As } A = H^2$$

$$\therefore \quad \int_0^T dT = -\frac{1}{C_d a \sqrt{2g}} \int_{H_1}^{H_2} H^{\frac{3}{2}} dH$$

$$\therefore \quad T = + \frac{1}{C_d a \sqrt{2g}} \left[\frac{2}{5} H^{\frac{5}{2}} \right]_{H_2}^{H_1}$$

Here, we have, $H_1 = 6$ m, $H_2 = 3$ m, and $a = \frac{\pi}{4}\left(\frac{15}{100}\right)^2 = 0.0177$ m^2

$$\text{Hence,} \quad T = \frac{1}{0.8 \times 0.0177 \times \sqrt{2 \times 9.81}} \times \frac{2}{5} \left[6^{2.5} - 3^{2.5} \right]$$

$$\approx \underline{460 \text{ sec. or } 7 \text{ min. } 40 \text{ sec.}}$$

Problem – 27 : *A tank is 3 m in diameter, and the height of water in it is 6 m. The bottom of the tank is hemispherical. It is to be emptied through a sharp edged circular hole, 22 cm diameter at its bottom. If coefficient of discharge of the orifice is 0·64, determine the time taken to empty the tank completely.*

This tank is divided in two parts, one cylindrical of 3 m diameter and 4·5 m high, and the other hemispherical of 3 m diameter.

Let, T_1 – time taken to empty the cylindrical portion, and

T_2 – time taken to empty the hemispherical portion.

$$\text{Then,} \quad T_1 = \frac{2A(\sqrt{H_1} - \sqrt{H_2})}{C_d a \sqrt{2g}} \quad \text{(ref. problem-10)}$$

Area of cross section of the cylindrical vessel is

$A = \frac{\pi}{4}(3)^2 = 7.08$ m^2 and heads H_1 and H_2 are 6 m, and 1·5 m respectively.

$$\text{Area of orifice, } a = \frac{\pi}{4}\left(\frac{22}{100}\right)^2 = 0.038 \text{ m}^2$$

$$\text{Hence,} \quad T_1 = \frac{2 \times 7.08 (\sqrt{6} - \sqrt{1.5})}{0.64 \times 0.038 \times 4.43}$$

$$= 161 \text{ sec.}$$

$$\text{Further,} \quad T_2 = \frac{14 \pi R^{\frac{5}{2}}}{15 C_d a \sqrt{2g}} \quad \text{(ref. eqn. 4.15 a)}$$

Here, $H = R = 1{\cdot}5$ m and a has same value,

$$\therefore \quad T_2 = \frac{14 \times \pi \times (1{\cdot}5)^{2.5}}{15 \times 0{\cdot}74 \times 0{\cdot}038 \times 4{\cdot}43}$$

$$= 65 \text{ sec.}$$

Hence, total time is

$$T = T_1 + T_2 = 161 + 65 = 226 \text{ sec. or 3 min. 46 sec.}$$

Problem-28 : *A tank 10 m deep is rectangular in cross section in plan and it is full with water. Its top is 20 m × 15 m and its bottom is 8 m × 3 m in cross section. The tank has constant cross section from the top upto 7 m depth and then it tapers upto its bottom. It is fitted with a pipe 30 cm in diameter and 3 m long at its bottom. How much time it will take to empty the tank completely ? Take pipe's discharge coefficient as 0·8.*

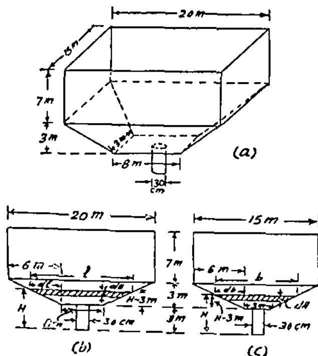


Fig. 4-22

This tank is to be divided in two parts, one part being of top constant cross-section and another part being of lower varying closs-section.

Let, T_1 - time taken to empty the top portion of the tank, and

T_2 - time taken to empty the lower portion of the tank.

$$T_1 = \frac{2A\,(\sqrt{H_1} - \sqrt{H_2})}{C_d a \sqrt{2g}}$$

Since, $A = 20 \times 15 = 300\ m^2$; $H_1 = 13$ m., $H_2 = 6$ m and area of orifice, $a = \frac{\pi}{4}\left(\frac{30}{100}\right)^2 = 0{\cdot}0706\ m^2$, we have

$$\text{time, } T_1 = \frac{2 \times 300 \times (\sqrt{13} - \sqrt{6})}{0{\cdot}8 \times 0{\cdot}0706 \times 4{\cdot}43}$$

$$= 2{,}760 \text{ sec.}$$

Further, referring to fig. 4–22 (b), we have

$$l = 8 + 2\,dl \text{ and } b = 3 + 2db$$

$$\text{and } \frac{dl}{6} = \frac{H-3}{3}$$

$$\therefore \quad dl = 2\,(H - 3)$$

$$\therefore \quad l = 8 + 4\,(H - 3) = (4H - 4)$$

$$\text{Similarly, } \frac{db}{6} = \frac{H-3}{3}$$

$$\therefore \quad db = 2\,(H - 3)$$

$$\therefore \quad b = 3 + 4\,(H - 3) = (4H - 9)$$

Area of strip, $A = (4H - 4)(4H - 9) = 16H^2 - 52H + 36$

Volume of strip of water is

$$dQ = -A dH = C_d a \sqrt{2gH} \cdot dT$$

$$\therefore \quad dT = \frac{-A dH^{-\frac{1}{2}}}{C_d a \sqrt{2g}}$$

$$= \frac{-(16H^2 - 52H + 36)\,H^{-\frac{1}{2}} dH}{C_d a \sqrt{2g}}$$

$$= -\frac{1}{C_d a \sqrt{2g}}\left(16H^{\frac{3}{2}} - 52H^{\frac{1}{2}} + 36H^{-\frac{1}{2}}\right) dH$$

$$\therefore \int_0^{T_2} dT = -\frac{1}{C_d a\sqrt{2g}} \int_{H_2}^{H_3} \left(16H^{\frac{3}{2}} - 52H^{\frac{1}{2}} + 36H^{-\frac{1}{2}}\right) dH$$

$$\therefore T_2 = \frac{+1}{0.8 \times 0.0706 \times 4.43} \left\{16 \times \tfrac{2}{5}H^{\frac{5}{2}} - \tfrac{2}{3} \times 52\,H^{\frac{3}{2}} + 36 \times 2H^{\frac{1}{2}}\right\}_{H_3}^{H_2}$$

where, $H_2 = 6$ m and $H_3 = 3$ m

$$= 4\left\{\frac{32}{5}\left(6^{\frac{5}{2}} - 3^{\frac{5}{2}}\right) - \frac{104}{3}\left(6^{\frac{3}{2}} - 3^{\frac{3}{2}}\right) + 72\left(6^{\frac{1}{2}} - 3^{\frac{1}{2}}\right)\right\}$$

$= 748$ sec.

$\therefore$ total time required is

$T = T_1 + T_2 = 2{,}760 + 748 = 3{,}508$ sec.

$= \underline{58 \text{ min } 28 \text{ sec.}}$

Problem - 29 : *A tank circular in plan is 7 m in diameter at the top and 3·5 m in diameter at the bottom. The depth of the tank is 8 m. The tank is constant in cross section from the top upto 5 m and then it tapers upto its bottom. It is fitted with a short vertical pipe 45 cm in diameter at the bottom. How much time it will take to empty it completely, if discharge coefficient is 0·64 ?*

Let T_1 - time taken to empty the portion of the tank with constant cross-sectional area, and

T_2 - time taken to empty the rest of the tank of varying cross sectional area.

Then, $T_1 = \dfrac{2A\,(\sqrt{H_1} - \sqrt{H_2})}{C_d a\sqrt{2g}}$

Here, $A = \frac{\pi}{4}\,7^2 = 38.5\text{ m}^2$, $H_1 = 8$ m, $H_2 = 3$ m,

Orifice area, $a = \frac{\pi}{4}\left(\frac{45}{100}\right)^2 = 0.159\text{ m}^2$.

$$T_1 = \frac{2 \times 38.5\left(8^{\frac{1}{2}} - 3^{\frac{1}{2}}\right)}{0.64 \times 0.159 \times \sqrt{2 \times 9.81}}$$

$= 188$ sec.

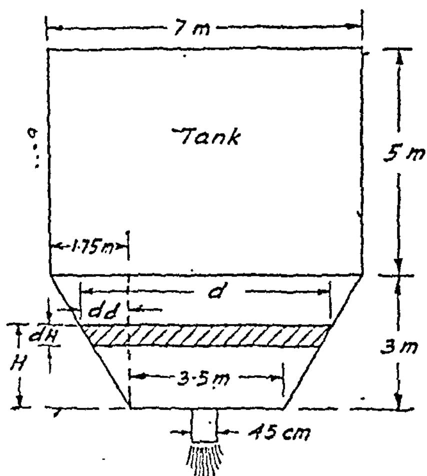


Fig. 4-23

Again referring to the variable size portion of the tank (fig. 4-23),

$$\frac{1{\cdot}75}{3} = \frac{dd}{H}$$

$\therefore$ $dd = 0{\cdot}583H$

$\therefore$ $d = 3{\cdot}5 + 2dd = 3{\cdot}5 + 2 \times 0{\cdot}583\,H = 3{\cdot}5 + 1{\cdot}166H$

Area of cross-section of the strip is

$$A = \frac{\pi}{4}(3{\cdot}5 + 1{\cdot}166H)^2$$
$$= 9{\cdot}6 + 6{\cdot}4H + 1{\cdot}07H^2$$

Volume of strip of water is

$$dQ = -\,AdH = C_d{\cdot}a\,\sqrt{2g{\cdot}H}\,dT$$

$$\therefore \quad dT = \frac{-\,AdH{\cdot}H^{-\frac{1}{2}}}{C_d a\,\sqrt{2g}} = \frac{-(9{\cdot}6 + 6{\cdot}4H + 1{\cdot}07H^2)\,H^{-\frac{1}{2}}dH}{C_d a\,\sqrt{2g}}$$

$$\therefore \quad \int_0^{T_2} dT = -\frac{1}{C_d{\cdot}a\,\sqrt{2g}} \int_{H_2}^{H_3} \left\{9.6\,H^{-\frac{1}{2}} + 6{\cdot}4\,H^{\frac{1}{2}} + 1{\cdot}07H^{\frac{3}{2}}\right\} dH$$

$$T_2 = \frac{+\,1}{0{\cdot}64 \times 0{\cdot}159 \times \sqrt{2 \times 9{\cdot}81}} \left[19{\cdot}2H^{\frac{1}{2}} + 4{\cdot}26H^{\frac{3}{2}} + 0{\cdot}43H^{\frac{5}{2}}\right]_{H_3}^{H_2}$$

Since, $H_2 = 3$ m, $H_3 = 0$,

$$T_2 = \frac{1}{0{\cdot}45}\left[19{\cdot}2 \times 3^{\frac{1}{2}} + 4{\cdot}26 \times 3^{\frac{3}{2}} + 0{\cdot}43 \times 3^{\frac{5}{2}}\right]$$
$$= 138 \text{ sec.}$$

$\therefore$ total time, $T = T_1 + T_2 = 188 + 138 = \underline{326 \text{ sec.}}$

Time of Flow between Vessels

The liquid flows from one tank to another tank through a sharp edged hole in partition wall as shown in fig. 4–24.

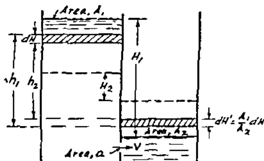


Fig. 4–24

Let, A_1, A_2 – areas of cross-section of the tanks,
a – area of the orifice,
H_1 – initial difference of levels in the tanks,
H_2 – final difference of the levels.
h_1 – difference in liquid levels at a given instant,
h_2 – difference in levels after a small interval of time dT,
dH – height of fall in the liquid level in the first tank in time dT.
dH' – rise in liquid level in the second tank, in same time dT.

The quantity of liquid leaving the first tank is the same as that which enters the second tank through the orifice.

Hence, $A_1 dH = A_2 dH'$

$$\therefore \quad dH' = \frac{A_1}{A_2} dH$$

Let dh be the amount by which the difference of levels in two tanks changes in small interval of time dT. Then,

$$dh = h_1 - h_2$$

Again, $h_1 = h_2 + dH + dH' = h_2 + dH + dH \times \frac{A_1}{A_2}$

$$= h_2 + dH\left(1 + \frac{A_1}{A_2}\right)$$

Hence, $dh = h_2 + dH\left(1 + \frac{A_1}{A_2}\right) - h_2 = dH\left(1 + \frac{A_1}{A_2}\right)$

$$\therefore \quad dH = \frac{dh}{1 + \frac{A_1}{A_2}}$$

The quantity of the liquid flowing through the orifice is (in time, dT).

$$dQ = -A_1 dH = -\frac{A_1 dh}{1 + \frac{A_1}{A_2}}$$

And, $dQ = C_d \cdot a\sqrt{2gh}\, dT$ (where h is the effective head at the given instant)

$$\therefore \quad dT = \frac{-A_1 dh \cdot h^{-\frac{1}{2}}}{\left(1 + \frac{A_1}{A_2}\right) C_d a\sqrt{2g}}$$

$$\therefore \int_0^T dT = -\frac{A_1}{\left(1 + \frac{A_1}{A_2}\right) C_d \cdot a\sqrt{2g}} \int_{H_1}^{H_2} h^{-\frac{1}{2}} \cdot dh$$

$$\text{or } T = \frac{+2A_1}{\left(1 + \frac{A_1}{A_2}\right) C_d \cdot a\sqrt{2g}} \left[h^{\frac{1}{2}}\right]_{H_2}^{H_1}$$

$$= \frac{2A_1\left(\sqrt{H_1} - \sqrt{H_2}\right)}{\left(1 + \frac{A_1}{A_2}\right) C_d \cdot a\sqrt{2g}} \qquad \text{.. (4.16)}$$

Problem-30 : *Two tanks of sizes 6 × 1·5 m and 1·5 × 1·5 m are connected at their bottom by a short pipe 20 cm in diameter. The difference of water levels in the tanks is 3 m when flow just starts. Find the time taken to bring the difference of water level down to 1·2 m, if coefficient of discharge for the pipe is 0·8.*

area of one tank, $A_1 = 6 \times 1.5 = 9\ m^2$

Area of another tank, $A_2 = 1.5 \times 1.5 = 2.25\ m^2$.

and area of pipe, $a = \frac{\pi}{4}\left(\frac{20}{100}\right)^2 = 0.0314\ m^2$.

$$\text{Time, } T = \frac{2A_1(\sqrt{H_1} - \sqrt{H_2})}{C_d a \sqrt{2g}\left(1 + \frac{A_1}{A_2}\right)}$$

$$= \frac{2 \times 9(\sqrt{3} - \sqrt{1.2})}{0.8 \times 0.0314 \times 4.43\left(1 + \frac{9}{2.25}\right)}$$

$= 20.6$ sec.

Problem-31 : *A tank, 12 m long and 6 m wide, is divided in two parts by a partition so that the area of one part is three times the area of the other. The partition contains a circular orifice 15 cm in diameter. Water level in the larger portion is 3 m higher than that in the smaller. Find the time taken to transfer 25,000 litres of water from the bigger portion to the smaller one. Coefficient of discharge of the orifice is 0.64.*

Area A of the whole tank is made of areas A_1 and A_2 of the two portions.

Hence, $A = A_1 + A_2$ and $A_1 = 3\ A_2$

$\therefore\ 12 \times 6 = 3A_2 + A_2$

$\therefore\ A_2 = 18\ m^2$ and $A_1 = 3 \times 18 = 54\ m^2$

Fall of level in the large portion $= \frac{\text{discharge}}{\text{area}}$

$$= \frac{25,000}{1,000 \times 54} = 0.464\ m$$

Rise of level in the smaller tank $= \frac{25,000}{1,000 \times 18} = 1.39\ m$

(1 cu. m = 1,000 litres)

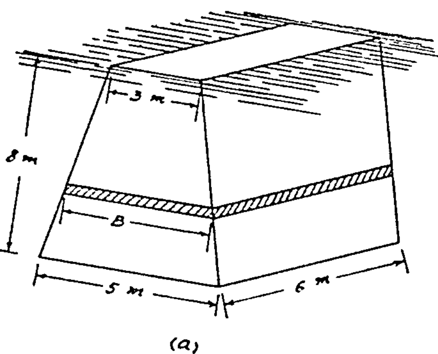
Final difference of water level, $H_2 = 3 - 0.464 - 1.39 = 1.15\ m$

Hence, time, $T = \dfrac{2A_1(\sqrt{H_1} - \sqrt{H_2})}{C_d a\sqrt{2g}\left(1 + \dfrac{A_1}{A_2}\right)}$

$$= \frac{2 \times 54\,(\sqrt{3} - \sqrt{1{\cdot}15})}{0{\cdot}64 \times \frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times \sqrt{2 \times 9{\cdot}81}\left(1 + \frac{54}{18}\right)}$$

$= 358$ sec. or 5 min. 58 sec.

Problem-32 : *An empty tank with vertical ends of trapezoidal section and plane sides 6 m long and 8 m deep is submerged and held with its top just in level with the water surface in a large reservoir. The tank is 3 m wide at the top and 5 m wide at the bottom. If the tank has an orifice of 5 cm diameter at its bottom, determine the time taken to fill the tank completely. Take coefficient of discharge for orifice as 0·62.*



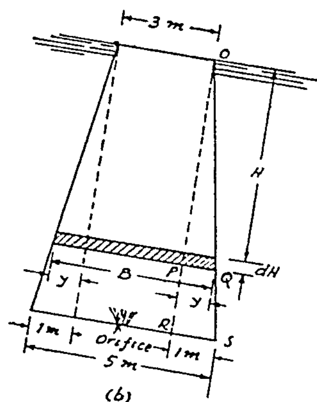


Fig. 4-25

At any instant, let H be the difference of water level in the and reservoir. The flow of water in the tank is the result of difference of water level. Aftet a time dT, the water level in nk rises by dH.

rom similar triangles OPQ and ORS we have

$$\frac{PQ}{RS} = \frac{OP}{OR}$$

$$\therefore \quad \frac{y}{1} = \frac{H}{8} \qquad \therefore \quad y = \frac{H}{8} \text{ m.}$$

$$\therefore \quad B = 3 + 2y = 3 + 2 \times \frac{H}{8} = 3 + \frac{H}{4} \text{ m.}$$

Area of cross-section of water level at the time considered is,

$$A = B \times 6 = \left(3 + \frac{H}{4}\right) 6 = (18 + 1.5H) \text{ m}^2$$

Area of orifice, $a = \frac{\pi}{4}\left(\frac{5}{100}\right)^2 = 0.00196 \text{ m}^2$

The flow of water into the tank under head H in time dT is dq.

$$\therefore \quad dq = C_d \times \text{area} \times \text{velocity} \times \text{time} = C_d \times a \times \sqrt{2gH}\, dT.$$

Volume of water entered into the tank is $dq = -AdH$.

$$\therefore \quad -AdH = C_d \cdot a \cdot \sqrt{2gH} \cdot dT$$

$$\therefore \qquad dT = \frac{-AdH}{C_d \cdot a\sqrt{2gH}} = \frac{-(18 + 1.5H)\, dH}{C_d \cdot a\sqrt{2gH}}$$

$$\therefore \quad \int_0^T dT = -\frac{1}{C_d \cdot a \cdot \sqrt{2g}} \int_8^0 (18H^{-\frac{1}{2}} + 1.5H^{\frac{1}{2}})\, dH$$

$$\therefore \quad T = + \frac{1}{C_d \cdot a \sqrt{2g}}\left[18 \times 2H^{\frac{1}{2}} + 1.5 \times \tfrac{2}{3} H^{\frac{3}{2}}\right]_0^8$$

$$= \frac{36(8)^{\frac{1}{2}} + (8)^{\frac{3}{2}}}{0.62 \times 0.00196 \times 4.43}$$

$$= \underline{23060 \text{ sec.}}$$

$$= \underline{6 \text{ hr. } 24 \text{ min. } 20 \text{ sec}}$$

Problem-33 : *A tank 3 m high has a form of a frustum of a cone with smaller end pointing downwards. The upper end is 2.4 m diameter and lower end 1.2 m diameter. The bottom end contains a central orifice whose coefficient of discharge is 0.6. This tank is*

submerged in a large reservoir so that its larger top is just in level with water surface in the reservoir. If this tank is to be filled in 20 minutes, find the size of the orifice.

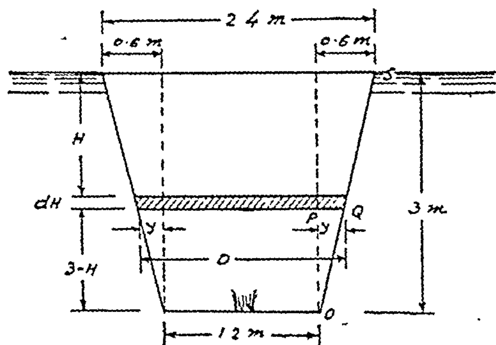


Fig. 4-26

Let H m be the difference in water level at a particular time which will change by dH m in time dT

Let D be the diameter of the circular cross section of the water level at a particular instant considered. From similar triangles (fig. 4-26) OPQ and ORS we have

$$\frac{OP}{OR}=\frac{PQ}{RS} \qquad \therefore \quad PQ = RS \times \frac{OP}{OR} = 0{\cdot}6 \times \frac{3-H}{3}$$

$$= 0{\cdot}2\,(3 - H) = 0{\cdot}6 - 0{\cdot}2H$$

$$\therefore \quad D = 1{\cdot}2 + 2y = 1{\cdot}2 + 2\,(0{\cdot}6 - 0{\cdot}2H) = (2{\cdot}4 - 0{\cdot}4\,H)\text{m}$$

Area of cross-section of water level, $A = \frac{\pi}{4} D^2$

$$\therefore \quad A = \frac{\pi}{4}(2{\cdot}4 - 0{\cdot}4H)^2 = \frac{\pi}{4}(5{\cdot}76 - 1{\cdot}92H + 0{\cdot}16H^2)$$

Volume of tank filled by water in time dT, is $dq = -AdH$

Discharge through the orifice in time dT is

$$dq = C_d \times \text{area} \times \text{velocity} \times \text{time}$$

$$\therefore \quad dq = C_d \times a \times \sqrt{2gH} \times dT$$

$$\therefore \quad C_d \cdot a\sqrt{2gH}\, dT = -AdH$$

$$\therefore \quad dT = \frac{-AdH}{C_d \cdot a\sqrt{2gH}}$$

$$\therefore \ dT = \frac{-\frac{\pi}{4}\,(5{\cdot}76 - 1{\cdot}92H + 0{\cdot}16\,H^2)\,dH}{C_d \cdot a\,\sqrt{2gH}}$$

$$\int_0^T dT = \frac{-\frac{\pi}{4}}{C_d \cdot a\sqrt{2g}} \int_{H_1}^{H_2} (5{\cdot}76H^{-\frac{1}{2}} - 1{\cdot}92H^{\frac{1}{2}} + 0{\cdot}16H^{\frac{3}{2}})\,dH$$

$$\therefore \ T = \frac{+\frac{\pi}{4}}{C_d \cdot a\sqrt{2g}}\left[5{\cdot}76 \times 2H^{\frac{1}{2}} - 1{\cdot}92 \times \tfrac{2}{3}\,H^{\frac{3}{2}} + 0{\cdot}16 \times \tfrac{2}{5}H^{\frac{5}{2}}\right]_{H_2=O}^{H_1=3}$$

$$20 \times 60 = \frac{+\frac{\pi}{4}}{0{\cdot}6 \times \frac{\pi}{4}d^2 \times 4{\cdot}43}\left[11{\cdot}52\,(3^{\frac{1}{2}} - 1{\cdot}28\,(3)^{\frac{3}{2}} + 0{\cdot}064\,(3)^{\frac{5}{2}}\right]$$

$$\therefore \quad d^2 = \frac{[\,19{\cdot}8 - 6\,65 + 0{\cdot}998\,]}{0{\cdot}6 \times 1{,}200 \times 4{\cdot}43} = \frac{14{\cdot}2}{3{,}200}$$

$$\therefore \quad d = 0\,0665 \text{ m}$$

$$d = \underline{6{\cdot}65 \text{ cm}}$$

Time of Emptying or Filling Canal Locks

This is a practical application of the flow of water from one vessel to another vessel. A lock is a chamber constructed in a river or canal which is navigable. This is constructed at a place where there is a difference in water levels as on the two sides of a dam. It is used to transfer the ship from one level of water to another. The level of water on the upper reach or lower reach side is assumed constant while we consider the time of emptying or filling the lock. This assumption is correct since there is abundance of water on the either side of the lock and the level of water does not change by taking away or adding small quantity of water. The lift of the lock is defined as the difference in water level between the upper reach and lower reach.

When the ship is to be transferred from lower reach to upper reach or vice versa, the time taken will be equal to either the time of filling the lock or the time of emptying the lock.

Let us consider a ship being transferred from lower reach to upper reach. First the lower gates are opened and ship enters the lock. The lower sluices or openings are now closed and upper sluices or openings are opened. Water enters the lock and water level rises as long as the level in the lock does not become equal to that in the upper reach. Now the upper gates are opened and the ship passes into the upper reach. The process is reversed when the ship proceeds down the lock.

Time of emptying the lock : A representative lock is shown in fig. 4-27.

Let, A – area of cross-section of the lock,

H_2 – lift of the lock, and

a_2 – area of cross section of lower sluices.

As water level in the lower reach does not change, its surface area is $A_2 = \infty$ (infinity).

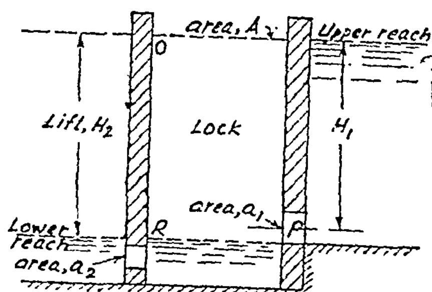


Fig. 4-27 Canal lock

As water level changes from O to R its initial head is H_2 and final head is zero.

$$\therefore \quad \text{time, } T = \frac{2A\sqrt{H_2}}{C_d \cdot a\sqrt{2g}}$$

(b) Time of filling the lock : Let H_1 be the head of water on the upper reach side above the centre of sluices in the upper gates and a_1 the area of cross-section of the sluices in the upper gates. Water flows from the upper reach under constant head, H_1 till the water level in the lock changes from R to P; while the flow

of water from P to O occurs under the head which is varying. If T_1 is time taken to fill the lock from R level,

$$T_1 = \frac{\text{volume}}{\text{rate of flow}}$$

Rate of flow = $C_d \times$ area $\times$ velocity

$$= C_d \times a_1\sqrt{2gH_1}$$

and volume = area $\times$ height = $A\ (H_2 - H_1)$

$$\therefore \quad T_1 = \frac{A\ (H_2 - H_1)}{C_d a_1\ \sqrt{2gH_1}} \qquad \text{... (4.17)}$$

If T_2 is time taken to fill the lock raising level from P to O,

$$T_2 = \frac{2A\ \sqrt{H_1}}{C_d a_1 \sqrt{2g}} \qquad \{\text{ref. eqn. (4.12)}\}$$

$$= \frac{2AH_1}{C_d a_1\ \sqrt{2gH_1}}$$

$\therefore$ total time for filling the lock is

$$T = T_1 + T_2$$

$$= \frac{A\ (H_2 - H_1)}{C_d a_1\ \sqrt{2gH_1}} + \frac{2AH_1}{C_d a_1\ \sqrt{2gH_1}}$$

$$= \frac{A\ (H_1 + H_2)}{C_d a_1\ \sqrt{2gH_1}} \qquad \text{.. (4.18)}$$

Problem-27 : *A lock 20 m × 3 m has a lift of 4 m. The lock is provided with two sluices each 0·6 m deep × 0·9 m wide, whose centres are 2·5 m below the water level of the upper reach; and two submerged sluices each 0·6 m deep × 0·45 m wide. If the coefficient of discharge for each sluice is 0·62, determine the time required to fill and empty the lock.*

Area of tha lock, $A = 20 \times 3 = 60\ m^2$

Area of upper sluices, $a_1 = 2(0.6 \times 0.9) = 1.08\ m^2$

and area of lower sluices, $a_2 = 2\ (0.6 \times 0.45) = 0.54\ m^2$

$$\text{Time of filling} = \frac{A\ (H_1 + H_2)}{C_d a_1 \sqrt{2gH_1}}$$

$$= \frac{60 \times (2 \cdot 5 + 4)}{0 \cdot 62 \times 1 \cdot 08 \sqrt{2 \times 9 \cdot 81 \times 2 \cdot 5}}$$

$$= \underline{83 \text{ sec.}}$$

$$\text{Time of emptying} = \frac{2A\sqrt{H_2}}{C_d \cdot a_2 \sqrt{2g}}$$

$$= \frac{2 \times 60 \times \sqrt{4}}{0 \cdot 62 \times 0 \cdot 54 \times 4 \cdot 43}$$

$$= \underline{161 \text{ sec.}}$$

Problem-28 : *Find the diameter of a submerged sluice to empty a lock 30 m long and 4·5 m wide, with a lift of 5 m in 15 minutes. Take a discharge coefficient of the sluice as 0·64.*

Area of the lock, $A = 30 \times 4 \cdot 5 = 135 \text{ m}^2$

$$\text{Time, } T = \frac{2A\sqrt{H_2}}{C_d \cdot a_2 \sqrt{2g}}$$

$$\text{or } \quad 15 \times 60 = \frac{2 \times 135 \times \sqrt{5}}{0 \cdot 64 \times a_2 \times \sqrt{2 \times 9 \cdot 81}}$$

$$\therefore \quad a_2 = 0 \cdot 237 \text{ m}^2$$

$$\text{i.e.} \quad \frac{\pi}{4} d^2 = 0 \cdot 237$$

$$\therefore \quad d = \sqrt{\frac{0 \cdot 37 \times 4}{\pi}} = 0 \cdot 55 \text{ m or } 55\text{cm.}$$

4·8 Large Vertical Orifice

In case of a large orifice, the head of water is not constant between its lower edge and upper edge and hence the velocity of water particles is different at every point in the vertical direction. If this orifice is located in horizontal base this will not happen. Consider a large vertical orifice, shown in fig. 4–28 through which the liquid is flowing.

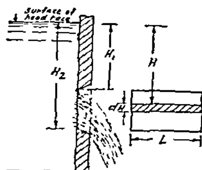


Fig. 4-28 Large vertical orifice

Let, L — breadth of the orifice,

H_1 — head of water above the top edge of the orifice,

H_2 — head of water above the lower edge of the orifice,

dH — thickness of the horizontal strip of liquid, and

H — depth of the strip below the surface of water in the tank.

Then, area of the strip, $dA = L \times dH$

Velocity of the liquid, which may be taken as constant, the strip being considered very small, is given by the expression $\sqrt{2gH}$. The discharge through the strip is :

$$dQ = C_d \times \text{area of the strip} \times \text{velocity}$$
$$= C_d \times L \times dH \times \sqrt{2gH}$$

The total discharge through the orifice is

$$\int_0^Q dQ = C_d \cdot L \cdot \sqrt{2g} \int_{H_1}^{H_2} H^{\frac{1}{2}} dH$$

$$\text{i.e.} \quad Q = \tfrac{2}{3} C_d \cdot L \cdot \sqrt{2g} \left[H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}} \right] \quad \text{.. (4·19)}$$

Problem-29: *A large vertical orifice is 1 m wide and 0·3 m deep. The surface of the water is 15 cm above the upper edge of the orifice. If the coefficient of discharge for the orifice is 0·62, determine the discharge in litres/sec.*

Here, we have got the breadth of the orifice, $L = 1$ m, and the heads above the upper and lower edges of the orifice as $H_1 = 0{\cdot}15$ m and $H_2 = 0{\cdot}45$ m respectively.

The discharge, $Q = \frac{2}{3} C_d L \sqrt{2g} \left[H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}\right]$

$= \frac{2}{3} \times 0{\cdot}62 \times 1 \times 4{\cdot}43 \left[0{\cdot}45^{\frac{3}{2}} - 0{\cdot}15^{\frac{3}{2}}\right]$

$= 0{\cdot}447 \text{ m}^3/\text{sec.}$

or $= 447$ litres/sec.

Submerged Orifice

When water discharges from the upstream side to the down steam side in such a way that water level on the downstream side is higher than the upper edge of the orifice, the orifice is said to be a submerged orifice. The flow of liquid which takes place through such an orifice is due to the difference in heads that exists between the water levels on the upstream and downstream sides. Thus the liquid particles at every section will move with the same velocity.

Let H – difference of heads on the upstream and downstream sides,
a – area of cross-section of the large vertical orifice, and
C_d – coefficient of discharge.

The discharge through the orifice is

$$Q = C_d \times \text{area of flow} \times \text{velocity of flow}$$

$$= C_d \times a \sqrt{2gH} \qquad .. (4{\cdot}20)$$

If the liquid level on the downstream side of the orifice is above the lower edge and below the top edge of the orifice, the

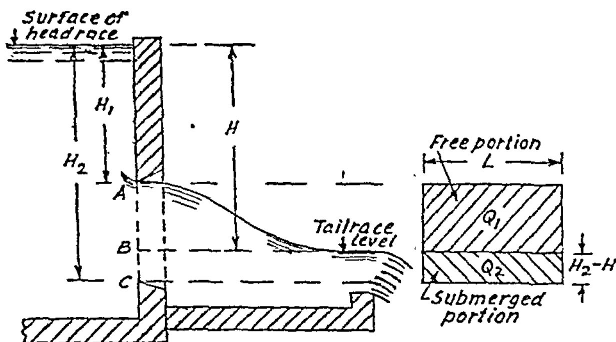


Fig. 4-29 Partially submerged orifice

orifice is said to be *partially submerged orifice*. In this case the flow of liquid has constant velocity produced under the head, H which is the difference of liquid levels on the upstream and down-stream sides for the portion of the orifice which is below the free surface of the tail race. The liquid moves with variable velocity for the portion of the orifice which is above the free surface of the tail race. Thus discharge can be divided into two parts, one occuring into the atmosphere, the other under the liquid. The flow pattern is presented in fig. 4–29. For the free portion AB of the orifice,

discharge, $Q_1 = \frac{2}{3} C_d L \sqrt{2g} \, (H^{\frac{3}{2}} - H_1^{\frac{3}{2}},)$ {ref eqn. (4·19),}

For the submerged portion, BC of orifice,

discharge, $Q_2 = C_d \times \text{area} \times \text{velocity}$

$$= C_d \times L \, (H_2 - H) \, \sqrt{2gH}$$

Therefore, the total discharge for the partially submerged orifice is

$$Q = Q_1 + Q_2$$

$$= \frac{2}{3} C_d L \sqrt{2g} \, (H^{\frac{3}{2}} - H_1^{\frac{3}{2}}) + C_d L \, (H_2 - H) \sqrt{2gH} \quad (4{\cdot}21)$$

Problem – 30 : *An orifice in the side of a large tank is rectangular in shape, 1·5 m broad and 1 m deep. The water level on one side of the orifice is 1 m above the top edge; the water level on the other side of the orifice being 0·5 m below the top edge. Find the discharge per second if coefficient of discharge is 0·62.*

Here, we have,

$H = 1 + 0{\cdot}5 = 1{\cdot}5$ m, $H_1 = 1$ m, $H_2 = 1 + 1 = 2$ m, and breadth $L = 1{\cdot}5$ m. Discharge through the free portion of the orifice is

$$Q_1 = \frac{2}{3} C_d L \sqrt{2g} \, (H^{\frac{3}{2}} - H_1^{\frac{3}{2}})$$

$$= \frac{2}{3} \times 0{\cdot}62 \times 1{\cdot}5 \times 4{\cdot}43 \, (1{\cdot}5^{1{\cdot}5} - 1^{1{\cdot}5})$$

$$= 2{\cdot}3 \text{ m}^3/\text{sec.}$$

For the submerged portion of the orifice,

$$H_2 - H = 2 - 1{\cdot}5 = 0{\cdot}5 \text{ m}$$

$$\therefore \quad Q_2 = C_d{\cdot}L\,(H_2 - H)\sqrt{2gH}$$
$$= 0{\cdot}62 \times 1{\cdot}5 \times 0{\cdot}5 \times 4{\cdot}43 \times \sqrt{1{\cdot}5}$$
$$= 2{\cdot}52 \text{ m}^3/\text{sec.}$$

Now, $Q = Q_1 + Q_2$
$= 2{\cdot}3 + 2{\cdot}52 = 4{\cdot}82 \text{ m}^3/\text{sec.}$
or $= 4{,}820$ litres/sec.

TUTORIAL - 4

Orifice coefficients

1. In making an experiment to find different coefficients of an orifice a jet of water is made to flow through a circular orifice of 4 cm diameter. The orifice is fitted in the vertical side of a tank. The head of water above the centre of the orifice is 1 metre. The jet is passed through a ring whose centre is found to be 38 cm horizontally away and 4 cm vertically down from the centre of vena contracta. The actual discharge found by measuring with collecting tank is 200 kg per minute. Determine the coefficients of the orifice.

[$C_v = 0{\cdot}95$; $C_c = 0{\cdot}63$, $C_d = 0{\cdot}60$, $C_r = 0{\cdot}11$]

2. The centre of an orifice is situated 15 cm above the bottom of a vessel containing water to depth of 75 cm. Assuming a coefficient of velocity of 0·975, estimate the fall in the height of jet for a horizontal travel of 65 cm from vena contracta when the orifice is fitted in the side of vessel.

[$y = 17{\cdot}6$ cm]

3. Water issues vertically up from an orifice in a horizontal plane of a tank (ref. fig. 4-5). The jet issues under a pressure of 0·6 kg/cm^2 gauge. If the coefficient of velocity of the orifice is 0·96, find the height through which the jet rises, the actual velocity of the jet at the vena contracta and the head lost due to friction.

[5·53 m; 10·4 m/sec; 0·47 m of water]

4. A 5 cm diameter orifice is provided in a tank containing water to a height of 1·5 m above the centre of the orifice. If the coefficients of contraction and velocity for the orifice are 0·62 and 0·98 respectively, determine, (i) the coefficient of discharge, (ii) the theoretical discharge, (iii) actual discharge, (iv) loss of head, and (v) coefficient of resistance

[(i) 0·608, (ii) 10·651 litres/sec.
(iii) 6·47 litres/sec (iv) 0·0594
m of water, (v) 0·04]

5. A large vertical plate is held perpendicular to a jet issuing from a sharp edged orifice 36 mm in diameter, in the vertical side of a tank. The force exerted on the plate is 2 kg. The measured discharge is 218 kg per minute. The constant head of water on the orifice is 1·6 m. Determine the coefficients of the orifice.

[C_c = 0·65, C_v = 0·965, C_d = [illegible], C_r = [illegible]]

6. A vessel contains water and is fitted with an orifice on one side and of 24 mm in the diameter. The orifice is at first plugged and the vessel is suspended so that the [illegible] force could be measured. On removal of plug, the horizontal force required to keep the vessel in the place was found to be [illegible] kg. This force was applied on the side of vessel opposite to the orifice. The constant level of water above the centre of the orifice is [illegible] m. The discharge collected in the measuring tank was [illegible] kg in [illegible] minutes. Determine the coefficients of the orifice.

[C_c = [illegible], C_v = [illegible], C_d = [illegible], C_r = [illegible]]

7. A [illegible] mm diameter orifice is provided in a closed tank containing water at a constant height of [illegible] m above the orifice. The coefficients of contraction and velocity for the orifice are [illegible] and 0·97 respectively. The pressure of air above water surface is 0·3 kg / cm² which is maintained constant. Calculate, (i) the actual discharge, (ii) the theoretical discharge, (iii) coefficient of discharge, (iv) loss of head, and (v) coefficient of resistance.

[(i) [illegible] (ii) [illegible] litres/sec., (iii) [illegible],
(iv) [illegible] m of water, (v) [illegible]]

8. In order to determine the different coefficients of an orifice, a tank in suspended on knife edges 2·4 m above the centre line of the orifice which is 6·5 cm^2 in area. When the head of water in the tank is 1·25 m, water is discharged at the rate of 120 kg per minute and a weight of 2·35 kg at a leverage of 1 m is required to keep the tank vertical. Find the coefficients of velocity, contraction and discharge.

[0·975, 0·635, 0·62]

Time of emptying

9. A swimming pool 11 m long and 7 m wide holds water to a depth of 4 m. If water is discharged through the opening, at the bottom of the pool, of area 0·3 m^2, find the time taken to empty it. Take the coefficient of discharge for the opening as 0.62.

[8 minutes]

10. A cistern of cross sectional area of 1 m square contains water 4 m deep. An orifice of 6·5 cm diameter is provided at its bottom. Find the quantity of water flowing out from the cistern in 2 minutes when the orifice is opened. The coefficient of discharge may be taken as 0·6.

[1·68 m^3]

11. A vertical tank 1 m in diameter is 3 m high. It has two orifices each of 12·5 cm^2 area. One orifice is at the bottom of the tank and the other one in the side 1·5 m above the bottom. Find the time taken to empty the tank. Take coefficient of discharge for the orifices as 0·62.

[9 min. 31·5 sec.]

12. Compensation water is to be discharged by two circular orifices of the same diameter situated at the bottom of a vessel and having a constant head of 1·5 m. If the demand is $1{\cdot}728 \times 10^7$ litres/day, what diameter will be required for each orifice. Take coefficient of contraction and velocity as 0·62 and 0·98 respectively.

[19·6 cm]

13. A tank 2 m deep is rectangular in cross section. It is 14 m × 7 m at its top and 10 m × 7 m at its bottom. The tank is full with water. If water is discharged through an orifice 30 cm in diameter and fitted at the bottom, find the time taken to empty it. Take coefficient of discharge as 0 62.

[116·5 sec]

14. A tank 3 m deep is rectangular in cross section; 22 m × 12 m at the top and 10 m × 6 m at the bottom Water is discharged through a short vertical pipe, 45 cm diameter, fitted at its bottom If coefficient of discharge, C_d is 0·64, determine the time taken to empty the tank.

[15 min. 10·6 sec.]

15. A triangular trough has each side 1·5 m and the length of the trough is 3 m. This trough is full of water and it is to be emptied through a circular hole 25 cm in diameter at its bottom. Find the time taken to empty it. Take coefficient of discharge as 0·62

[25·4 sec]

16. A vertical tank 3·5 m deep is square in cross section. Its sides at the top are each 3 m long and at the bottom each 1 m long. It is provided with a sharp edged hole 15 cm in diameter at the bottom. If the tank is full in the beginning, find the time taken to change the water level by 1·5 m. Take coefficient of discharge, C_d = 0·6.

[81·7 sec.]

17. A tank, circular in plan, is 10 m in diameter at the top and 2 m in diameter at the bottom. The height of the tank is 4 m. It is emptied through a short vertical pipe 20 cm in diameter, fitted at the base of the tank. Determine the time required to empty it, if coefficient of discharge is 0·64.

[16 min. 5 sec.]

18. A hemispherical tank is 4 m in diameter and is full of water. Its level is changed to 1·5 m in 80 sec. when water is discharged through a hole at the bottom of the tank. If the coefficient of discharge of the hole is 0·6, determine the diameter. Prove the formula you may use.

[16·7 cm]

19. A horizontal boiler 3 m in diameter and 10 m long contains water to a height of 2 m. Find the time taken to empty the boiler through a short vertical pipe 15 cm in diameter and attached to the bottom of the boiler. Take coefficient of discharge of the short pipe as 0·8.

[14 min. 57 sec.]

20. A reservoir which is 4 m deep is circular in plan, the diameters at the top and bottom are 80 m and 60 m respectively. The mouth of the outlet pipe which is 50 cm in diameter is 6 m below top water level. How long will it take to empty the reservoir ? Take coefficient of discharge, C_d = 0·8 for the pipe. Neglect frictional effect in the pipe.

[18 min. 40 sec]

21. A tank, square in cross section (in plan) is 3 m deep. It has top sides 6 m long, and bottom sides 3 m long, and is full of water. It is to be emptied through a vertical pipe 15 cm diameter and 2 m long, fitted at its bottom. Neglecting friction in the pipe, determine the time taken to empty it completely. Take coefficient of discharge as 0·8.

[8 min. 58 sec.]

22. A tank is 4 m in diameter and the height of water in it is 6 m. The bottom of the tank is hemispherical. It is to be emptied through a sharp edged circular hole, 30 cm in diameter, at its bottom. If coefficient of discharge of the orifice is 0·62, determine the time taken to empty the tank completely.

[3 min. 14 sec.]

23. A tank 5 m deep is rectangular in cross section in plan and it is full with water. Its top is 12 × 8 m and its bottom is 8 × 4 m in cross section. The tank has constant cross section from the top upto 3 m depth and then it tapers upto its bottom. It is fitted with a pipe 30 cm in diameter and 3 m long at its bottom. How much time it will take to empty the tank completely ? Take pipe discharge coefficient as 0·8.

[11 min. 34 sec.]

24. A tank circular in plan is 6 m in diameter at the top and 3 m in diameter at the bottom. The depth of the tank is 9 m. The tank is constant in cross section from the top upto 6 m, and then it tapers upto its bottom. How much time it will take to empty it completely if discharge coefficient is 0·64 for an orifice in bottom of 30 cm diameter.

[5 min. 30 sec.]

25. A tank circular in plan, 5 m high, is hemispherical at its top with a flat circular bottom of 6 m diameter. It is provided with a sharp edged orifice 30 cm in diameter at its bottom. If C_d = 0·62, determine the time taken to empty tank completely.

[9 min. 30 sec.]

26. A tank full of water is 6 m high and circular in plan. The diameter at the top of the tank is 12 m and it tapers to a diameter of 8 m in the depth of 4 m from the top. The rest of the portion of the tank is cylindrical. The tank is fitted with a small vertical pipe 50 cm in diameter at its bottom. Determine the time taken to empty the tank completely if C_d = 0·8.

[7 min. 14 sec]

Flow between vessels

27. A tank 15 m long and 8 m wide is divided into two parts by a partition so that the area of one part is three times the area of the other. The partition contains a circular-orifice 15 cm in diameter. Water level in the larger portion is 3 m higher than that in the smaller. Find the time taken to transfer 20,000 litres of water from the bigger portion to the smaller one. Coefficient of discharge of the orifice is 0·64.

[4 min. 10 sec.]

28. Two tanks of sizes 7 m × 2 m and 3 m × 3 m are connected at their bottom by a short pipe 25 cm in diameter. The difference of water levels in tank is 4 m when flow just starts. Find the time taken to flow 14 m^3 of water from the larger tank to the smaller tank if coefficient of discharge of the pipe is 0·8.

[50·5 sec.]

29. Find the diameter of a submerged sluice to empty a lock 40 m long and 5 m wide with a lift of 5 m in 15 min. Take discharge coefficient of the sluice as 0·64.

[66 cm]

Time for lock filling

30. A lock 20 m × 4 m has a lift of 4 m. The lock is provided with sluices each 0·75 m deep × 1 m wide, whose centres are 3 m below the water level of the upper reach; and two submerged sluices each 0·75 m × 0·5 m wide. If the coefficient of discharge for each sluice is 0·64, determine the time required to fill and empty the lock.

[76·4 sec. 151 sec.]

Large orifice

31. A large vertical orifice is 1 m wide and 0·25 m deep. The surface of water is 15 cm above the upper edge of the orifice. If the coefficient of discharge for the orifice is 0·62, determine the discharge in litres per second.

[302]

Submerged orifice

32. An orifice in the side of a large tank is square in shape 1·5 m wide and 1·5 m deep. The water level on one side of the orifice is 1 m above the top edge, the level on the other side of the orifice being 0·5 m below the top edge. Find the discharge per second if coefficient of discharge is 0·64.

[12·76 m^3/sec.]

5
FLOW THROUGH MOUTHPIECES

Introduction

Water flowing along passages having straight uniform and perfectly smooth walls, suffers no loss of energy except that due to viscous resistance. It is not possible to attain this condition of no loss in practice. Flow will suffer from loss of energy due to the roughness of the sides of the passage, changes in section, changes of direction and obstructions. All these losses are expressed in terms of the velocity head. Loss of head on account of roughness of sides of the passage walls is dealt with in chapter 7. The evaluation of other losses mentioned is dealt with here This is necessary before taking up the study of flow of water through mouthpieces.

Loss of Head in Fluid Flow

In a flow of fluid if the velocity changes in magnitude due to enlargement, contraction, obstruction etc , the fluid loses some energy. This loss of energy results in loss of some pressure head. These losses are termed minor secondary losses In comparison with these losses there exists loss due to friction and as this loss depends on the length and velocity, its value is comparatively greater and it cannot be neglected in practice.

Loss of head due to sudden enlargement : The cross section of a pipe line is suddenly changed at EF, and water will flow in this pipe line as shown in fig. 5-1. It is found from experimental results that the *pressure* p_o *of water in annular spaces ' G ' where* the bigger pipe joins the smaller pipe, is the same as in the smaller pipe and is equal to p_1.

Let, A_1—cross-sectional area of the smaller pipe,
A_2—cross-sectional area of the bigger pipe,
p_1—pressure in the smaller pipe,
p_2—pressure in the bigger pipe,
V_1—velocity of water at section AB, and
V_2—velocity of water at section CD.

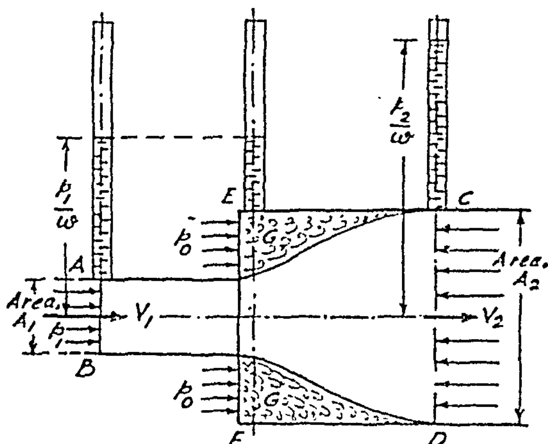


Fig. 5-1 Sudden enlargement

Rate of flow at section AB = wA_1V_1

Rate of flow at section CD = wA_2V_2

Consider the quantity of water between AB and CD. The resultant force acting on this mass of water is

$$p_2A_2 - p_1A_1 - p_0(A_2 - A_1)$$

But $p_0 = p_1$

$\therefore$ net total force $= A_2(p_2 - p_1)$ ....(5·1)

The change of momentum per second of this mass of water between the sections AB and CD is the difference between the momentum at AB and the momentum at CD which is equal to

$$\frac{wA_1V_1}{g}V_1 - \frac{wA_2V_2}{g}V_2 = \frac{wA_1V_1^2}{g} - \frac{wA_2V_2^2}{g}$$

But, $A_1V_1 = A_2V_2$

$\therefore$ change of momentum per second $= \frac{wA_2V_2V_1}{g} - \frac{wA_2V_2^2}{g}$

Now the force causing the flow of water is equal to change of momentum per second.

$$\therefore \quad A_2 (p_2 - p_1) = \frac{wA_2V_2V_1}{g} - \frac{wA_2V_2^2}{g}$$

$$\therefore \quad \frac{p_2}{w} - \frac{p_1}{w} = \frac{V_1V_2}{g} - \frac{V_2^2}{g} \quad \quad \ldots.(5{\cdot}2)$$

Now, applying Bernoulli's equation to sections AB and CD and considering the loss due to enlargement which is equivalent to head h_e,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2 + h_e \quad \quad \ldots.(5{\cdot}3)$$

For the horizontal pipe, here $Z_1 = Z_2$.

$$\therefore \quad \frac{p_2}{w} - \frac{p_1}{w} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

From eqns. (5·2) and (5·3),

$$\frac{V_1V_2}{g} - \frac{V_2^2}{g} = \frac{V_1^2}{2g} - \frac{V_2^2}{g} - h_e$$

$$\therefore \quad h_e = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{V_1V_2}{g} + \frac{V_2^2}{g}$$

$$= \frac{V_1 - 2V_1V_2 + V_2^2}{2g}$$

$$= \frac{(V_1 - V_2)^2}{2g} \quad \quad \ldots.(5{\cdot}4)$$

Loss of head due to sudden contraction : When water flows along a pipe line and if it meets with a section which suddenly contracts, as shown in fig. 5-2, there will occur a loss of head. Water flowing in the larger portion of the pipe enters into a smaller pipe having an area of cross section A. Water after entering the smaller pipe will contract as shown, at section AB, and then will enlarge to fill the pipe at section EF Thus due to sudden enlargement taking place between the sections. AB and EF, there will be a loss of head. Thus loss of head is not due to sudden contraction but due to *the sudden enlargement following the contraction.*

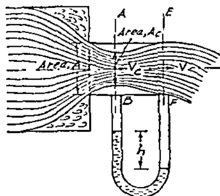


Fig. 5-2 Sudden contraction

$$\therefore \text{ loss of head, } h_c = \frac{(V_c - V)^2}{2g}$$

where, V_c – velocity of water at section AB,
and V – velocity of water at section EF,
Let, A_c – area of cross-section of flow at section AB, and
A – area of cross-section of flow at section EF,
(which is equal to cross-sectional area of smaller pipe.)

The ratio of the area of cross-section at the contracted section to the area of smaller pipe is known as coefficient of contraction, C_c.

$$\therefore \quad C_c = \frac{A_c}{A} \quad \therefore \quad A_c = C_c A$$

Now, according to law of mass continuity (law of conservation of mass), we have,

$$A_c V_c = AV$$

$$\therefore \quad V_c = \frac{A}{A_c} \times V = \frac{AV}{AC_c} = \frac{V}{C_c}$$

$\therefore$ loss of head due to sudden contraction is

$$h_c = \frac{\left(\frac{V}{C_c} - V\right)^2}{2g} = \left\{\frac{1}{C_c} - 1\right\}^2 \frac{V^2}{2g} \qquad ..(5\cdot5)$$

Generally the value of C_c is 0·62.

$$\therefore \quad h_c = \left(\frac{1}{0\cdot62} - 1\right)^2 \frac{V^2}{2g} \doteq 0\cdot38 \frac{V^2}{2g} \qquad ..(5\cdot5a)$$

If the value of C_c is not given it is usual to assume

$$h_c = 0\cdot5 \frac{V^2}{2g}$$

Loss of head at entrance to pipe : When water enters from a tank or large container, there occurs a sudden contraction to the flow of water and this results in the loss of head. If V is velocity of water in the pipe, loss of head at entrance,

$$h_e = 0\cdot5 \frac{V^2}{2g} \qquad ..(5\cdot6)$$

The loss is very small compared to frictional loss in the pipe and hence it is usually neglected.

Loss of head due to obstruction to flow : When water flowing in a pipe line, meets with an obstruction, a change in direction of

by sudden enlargement. Area of flow at the contracted section, known as vena-contracta, is $C_0 a_0$.

$$\text{Hence,} \quad V_c \times C_0 a_0 = VA$$

$$V_c = \frac{AV}{C_0 a_0}$$

$$\text{and loss of head, } h_0 = \left[\frac{A}{C_0 a_0} - 1\right]^2 \frac{V^2}{2g} \quad .. (5\cdot7a)$$

Loss of head due to friction: There will be loss of head due to friction when water flows in the pipe. Friction is caused by resistance between water and the sides of the pipe. This will be dealt with in detail in chapter 7 on flow through pipe lines. However, we write down an expression for the head lost on friction.

$$\text{Friction head, } h_f = \frac{4fl}{d} \frac{V^2}{2g} \quad .. (5\cdot8)$$

where, l – length of the pipe line,
d – diameter of the pipe line,
V – velocity of water in the pipe line, and
f – coefficient of friction for the pipe.

Over and above the losses considered here, there are few other minor losses due to bend, elbow, tapering section, etc. Since they are very small in magnitude, they are not discussed here and in actual practice they are disregarded.

Problem-1: *A pipe 10 cm in diameter is suddenly enlarged to 20 cm in diameter. Determine the loss of energy per kg of water when the discharge is 0·06 m³/sec.*

Since discharge, $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1}$$

$$= \frac{0\cdot06}{\frac{\pi}{4}\left(\frac{10}{100}\right)^2} = 7\cdot64 \text{ m/sec.}$$

$$\text{and } V_2 = \frac{Q}{A_2} = \frac{0\cdot06}{\frac{\pi}{4}\left(\frac{20}{100}\right)^2} = 1\cdot91 \text{ m/sec}$$

Now, loss of head due to enlargement is

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7{\cdot}64 - 1{\cdot}91)^2}{2 \times 9{\cdot}81} = 1{\cdot}67 \text{ m of water}$$

$\therefore$ loss of energy per kg of water = 1·67 kg-m

Problem-2: *A horizontal pipe 15 cm in diameter is suddenly reduced to 7·5 cm in diameter. Water is flowing from the larger pipe to smaller pipe at the rate of 10 litres/sec. What is the loss of energy at the sudden contraction in kg-m per minute? Assume coefficient of contraction as 0·64.*

Velocity of water in larger pipe is

$$V_1 = \frac{10 \times 1{,}000}{100^3} \times \frac{1}{\frac{\pi}{4}\left(\frac{15}{100}\right)^2} = 0{\cdot}568 \text{ m/sec}$$

(Here 1 litre = 1,000 cm³)

$$\text{and} \quad V_2 = V_1 \cdot \frac{A_1}{A_2} = 0{\cdot}568 \times \frac{15^2}{7{\cdot}5^2} \; 2{\cdot}272 \text{ m/sec.}$$

Now, loss of head due to contraction is (ref. eq. 5·5)

$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$$

$$= \left(\frac{1}{0{\cdot}64} - 1\right)^2 \times \frac{2{\cdot}272^2}{2 \times 9{\cdot}81} = 0{\cdot}083 \text{ m of water}$$

and loss of energy is 10 × 60 × 0 083 = 49 8 kg-m/min.

Problem-3: *A pipe 15 cm in diameter is suddenly enlarged to 30 cm in diameter. Find the difference in the two limbs of a U-tube containing mercury. The limbs are connected at two points on each side of the enlargements where the flow has settled down. Water flows at the rate of 56 litres/sec. from the smaller to the larger section. Specific gravity of mercury is 13·6.*

Since, $Q = A_1 V_1 = A_2 V_2$

$$V_1 = \frac{Q}{A_1} = \frac{56 \times 1000}{100^3} \times \frac{1}{\frac{\pi}{4}\left(\frac{15}{100}\right)^2} = 3{\cdot}17 \text{ m/sec.}$$

$$V_2 = V_1 \frac{A_1}{A_2} = 3{\cdot}17 \times \left(\frac{15}{30}\right)^2 = 0{\cdot}79 \text{ m/sec}\cdot$$

Now, head lost due to sudden enlargement is

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(3{\cdot}17 - 0{\cdot}79)^2}{2 \times 9{\cdot}81} = 0{\cdot}29 \text{ m of water}$$

Applying Bernoulli's equation to the two sections,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{p_2}{w} + \frac{V_2^2}{2g} + h_e$$

$$\therefore \quad \text{difference of head, } H = \frac{p_2}{w} - \frac{p_1}{w} = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{3{\cdot}17^2 - 0{\cdot}79^2}{2 \times 9{\cdot}81} - 0{\cdot}29$$

$$= 0{\cdot}187 \text{ m of water}$$

This head of water is converted into equivalent head of mercury, h by the expression,

$$H = \frac{h}{100}(13{\cdot}6 - 1)$$

$$\text{or } 0{\cdot}187 = \frac{h}{100} \times 12{\cdot}6$$

$$\therefore \quad h = \underline{1{\cdot}49 \text{ cm}}$$

Problem-4 : *A short (30 cm long) pipe is fitted to one side of the tank. The pipe is 10 cm in diameter at both ends but the central part is enlarged to 20 cm diameter for 10 cm length. The discharge from the pipe is 1,360 kg/minute. Determine the head lost due to sudden enlargement, sudden contraction and at the entrance to the pipe. Take contraction coefficient as 0·64.*

$$\text{Discharge, } Q = \frac{1{,}360}{1{,}000 \times 60} = 0{\cdot}02266 \text{ m}^3\text{/sec.}$$

Velocity in 10 cm diameter section (before enlargement) is

$$V_1 = \frac{0{\cdot}02266}{\frac{\pi}{4}\left(\frac{10}{100}\right)^2} = 2{\cdot}9 \text{ m/sec.}$$

and velocity in the enlargement is

$$V_2 = \frac{Q}{A_2} = \frac{0.02266}{\frac{1}{4}\pi\left(\frac{20}{100}\right)^2} = 0.725 \text{ m/sec}$$

Now, the head lost at entrance is

$$h_{ent} = \frac{0.5\, V_1^2}{2g} = \frac{0.5 \times 2.9^2}{2 \times 9.81} = \underline{0.214 \text{ m of water}}$$

Head lost due to enlargement of pipe is

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(2.9 - 0.725)^2}{2 \times 9.81} = \underline{0.242 \text{ m of water}}$$

Head lost due to contraction of pipe is

$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \frac{V^2}{2g} = \left(\frac{1}{0.64} - 1\right)^2 \times \frac{2.9^2}{19.62} = \underline{0.134 \text{ m of water}}$$

Mouthpieces

The flow through orifices suffers from various drawbacks such as loss of effective area, velocity, and discharge of the jet. One method of improving upon these defects is to attach a short pipe of length about 2 to 3 times the diameter of the orifice. This extension of the orifice is called a tube or mouthpiece. The mouthpiece may have different shapes like cylindrical, convergent or divergent. It may entirely be outside the tank or part of it may remain inside the tank. The study of flow through various mouthpieces is important to understand the behaviour of liquid under such special circumstances.

External Cylindrical Mouthpiece

This type of mouthpiece has uniform cross-section and its length is about 2·5 times the diameter. The flow through it is shown in fig 5·4. The vessel contains water to a height H above the centre line of the pipe which discharges it to the outside. Water on entering the pipe first contracts and then expands suddenly between sections A-A and B-B. Due to high velocity at the vena contracta, the pressure head falls below the atmospheric

pressure acting at the outlet end. Thus the discharge of water is increased due to sucking action of the vacuum pressure at section A – A and as it is forced under a head H, the effective head under which the flow takes place is comparatively increased.

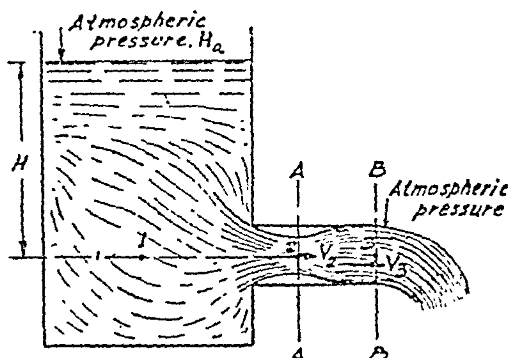


Fig. 5–4 External cylindrical mouthpiece

As the pipe is flowing full at outlet, the coefficient of contraction will be unity. The coefficient of velocity may be calculated by applying Bernoulli's equation, and the coefficient of discharge will then be equal to the coefficient of velocity.

Let, V_2 – velocity of water at the vena contracta, *i.e.* at section A–A,
V_3 – velocity of water at exit. *i.e.* at section B–B,
A_2 – area of cross section at the vena-contracta,
A_3 – area of cross-section of the mouthpiece,
C_c – coefficient of contraction of orifice part say 0·62 and

$$C_c = \frac{A_2}{A_3}$$

H_2 – pressure head at the vena contracta $\left(= \frac{p_2}{w} \right)$

H_a – pressure head at exit which is atmospheric $\left(= \frac{p_a}{w} \right)$

From mass continuity law,

$$A_2 V_2 = A_3 V_3$$

$$\therefore \quad V_2 = \frac{A_3}{A_2} V_3 = \frac{V_3}{C_c} = \frac{V_3}{0.62}$$

Applying Bernoulli's equation to sections A–A and B–B, we have,

$$\frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2 = \frac{p_3}{w} + \frac{V_3^2}{2g} + Z_3 + \left\{ \begin{array}{l} \text{loss of head between} \\ \text{sections AA and BB} \end{array} \right\} \text{ and}$$

loss of head due to sudden enlargement between the sections is,

$$h_e = \frac{(V_2 - V_3)^2}{2g} = \frac{\left\{ \frac{V_3}{0.62} - V_3 \right\}^2}{2g} = 0.375 \frac{V_3^2}{2g}$$

and $Z_2 = Z_3$

$$\therefore \quad H_2 + \frac{V_2^2}{2g} = H_a + \frac{V_3^2}{2g} + 0.375 \frac{V_3^2}{2g}$$

$$\therefore \quad H_2 = H_a + \frac{V_3^2}{2g} - \frac{V_2^2}{2g} + 0.375 \frac{V_3^2}{2g}$$

$$= H_a + \frac{V_3^2}{2g} - 2.6 \frac{V_3^2}{2g} + 0.375 \frac{V_3^2}{2g} \text{ as } V_2 = \frac{V_3}{0.62}$$

$$= H_a - 1.225 \frac{V_3^2}{2g}$$

Applying Bernoulli's equation to points 1 and 3, we have

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{p_3}{w} + \frac{V_3^2}{2g} + Z_3 + \text{loss of head}$$

Here, $Z_1 = Z_3$; $\frac{p_1}{w} = H + H_a$; $V_1 = 0$; $\frac{p_3}{w} = H_a$,

and loss of head $= 0.375 \frac{V_3^2}{2g}$

$$\therefore \quad H + H_a = H_a + \frac{V_3^2}{2g} + 0.375 \frac{V_3^2}{2g}$$

$$\therefore \quad H = 1.375 \frac{V_3^2}{2g}$$

$$\therefore \quad \frac{V_3^2}{2g} = \frac{H}{1.375}$$

$$\therefore \quad V_3 = \frac{\sqrt{2gH}}{\sqrt{1.375}}$$

The theoretical velocity at exit $= \sqrt{2gH}$

$$\therefore \quad C_v = \frac{V_3}{\text{theoretical velocity}} = \frac{\sqrt{2gH}}{\sqrt{1{\cdot}375}} \times \frac{1}{\sqrt{2gH}} = \frac{1}{\sqrt{1{\cdot}375}}$$

$$= 0{\cdot}855 \qquad ..(5{\cdot}8)$$

$$\text{and } C_d = C_v \times C_c = 0{\cdot}855 \qquad (\text{as } C_c = 1) \qquad ..(5{\cdot}9)$$

The pressure head at vena contracta is

$$H_2 = H_a - 1{\cdot}225\frac{V_3^2}{2g}, \text{ But } \frac{V_3^2}{2g} = \frac{H}{1{\cdot}375}$$

$$\therefore \quad H_2 = H_a - \frac{1{\cdot}225 \times H}{1{\cdot}375}$$

$$= H_a - 0{\cdot}89H \qquad ..(5{\cdot}10)$$

The effect of friction of the sides of the mouthpiece is to reduce the coefficient of discharge from 0·855 to 0·813, and the pressure at vena contracta from $0{\cdot}89H$ vacuum to $0{\cdot}74H$ vacuum.

The maximum vacuum pressure allowed at the vena contracta is 8 m of water. If it falls below this, the flow will become unsteady. Hence, $0{\cdot}74H = 8$

$$\therefore \quad H = \frac{8}{0{\cdot}74} = 10{\cdot}8 \text{ m}$$

Thus for a cylinderical mouthpiece the maximum satisfactory head allowed in the tank is 10·8 m.

Problem–5 : *An external cylindrical mouthpiece, 4 cm diameter and 12·5 cm long is fitted to the side of a vertical tank containing water to 2 m height above the centre of the mouthpiece. Find the discharge through the mouthpiece. Take coefficient of contraction as 0·62.*

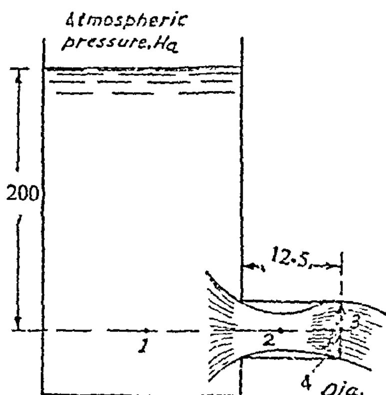


Fig. 5–5

The flow is represented in fig. 5–5 where important sections 1, 2, 3 are shown.

Area, $A_2 = 0{\cdot}62A_3$

and discharge $= A_2V_2 = A_3V_3$

$$\therefore \quad V_2 = \frac{A_3}{A_2} \times V_3 = \frac{V_3}{0{\cdot}62}$$

Due to sudden enlargement, loss of head between sections 2 and 3 is

$$\frac{(V_2 - V_3)^2}{2g} = \left(\frac{1}{0{\cdot}62} - 1\right)^2 \frac{V_3^2}{2g} = 0{\cdot}375\ \frac{V_3^2}{2g}$$

Applying Bernoulli's equation to sections 1 and 3, we have,

$$H_a + H = H_a + \frac{V_3^2}{2g} + \text{loss of head}$$

$$= H_a + \frac{V_3^2}{2g} + 0{\cdot}375\ \frac{V_3^2}{2g}$$

$$\therefore \quad H = 1{\cdot}375\ \frac{V_3^2}{2g}$$

$$\therefore \quad V_3 = \sqrt{\frac{2gH}{1{\cdot}375}}$$

Now, coefficient of velocity $C_v = \dfrac{\text{actual velocity}}{\text{theoretical velocity}}$

$$= \frac{\sqrt{2gH}}{\sqrt{1{\cdot}375}} \times \frac{1}{\sqrt{2gH}} = 0{\cdot}855$$

Now, $C_d = C_c \times C_v = 1 \times 0{\cdot}855$

Area of mouthpice $= \dfrac{\pi}{4}\left(\dfrac{4}{100}\right)^2 = 0{\cdot}00126\ \text{m}^2$

$\therefore$ discharge $= C_d \times \text{area} \times \text{velocity}$

$= 0{\cdot}855 \times 0{\cdot}00126 \times \sqrt{19{\cdot}62 \times 2}$

$= 0{\cdot}0067\ \text{m}^3/\text{sec.}$

Problem-6 : *An external cylindrical mouthpiece fitted to the side of a vertical tank containing water upto 3 m constant height above its centre line is 4·5 cm in diameter and 8 cm long. Determine the discharge through the mouthpiece. The coefficient of contraction of the mouthpiece may be taken as 0·62.*

The flow with its important sections can be represented as in fig. 5-5. Following identical procedure of problem-5, it can be established that coefficient of discharge $C_d = 0.855$.

$$\text{Area of mouthpiece, } A_3 = \frac{\pi}{4}\left(\frac{4.5}{100}\right)^2 = 0.00159 \text{ m}^2$$

$$\text{and discharge} = C_d \times A_3 \times V_3$$

$$= 0.855 \times 0.00159 \times \sqrt{2 \times 9.81 \times 3}$$

$$= 0.0105 \text{ m}^3/\text{sec.}$$

$$\text{or} \quad = 0.0105 \times 1{,}000 = \underline{10.5 \text{ litres/sec.}}$$

Convergent-Divergent Mouthpiece or Bell-Mouthpiece

As is discussed above, loss of head takes place between the sections A-A and B-B due to sudden enlargement. If we make the *mouthpiece to converge first upto vena contracta and thus make the water follow the profile of the mouthpiece, and then we diverge the mouthpiece slowly, the loss due to enlargement is considerably reduced.* Due to divergence the area at exit is more than that at the vena contracta. As the divergence increases, the vacuum

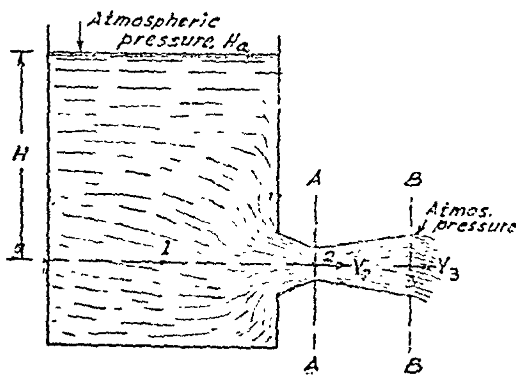


Fig. 5-6 Convergent-divergent mouthpiece

increases at the vena contracta; and, as this cannot be greater than 10·5 m theoretically, or 8 m *practically, there is a limit to the amount of divergence if steady flow is to be maintained.* This type of mouthpiece is shown in fig. 5-6.

Applying Bernoulli's equation to points 1, 2 and 3, we have,

$$H_a + H = H_2 + \frac{V_2^2}{2g} = H_a + \frac{V_3^2}{2g}$$

where, H_a – atmospheric pressure,

H_2 – pressure head at the vena contracta,

V_2 – velocity of water at the vena contracta,

V_3 – velocity of water at exit of mouthpiece,

H – head of water in the tank above the centre line of the mouthpiece.

Since $\frac{V_3^2}{2g} = H,\ V_3 = \sqrt{2gH}$

and $\frac{V_2^2}{2g} = H_a + H - H_2$

But maximum vacuum pressure allowed at the vena contracta is 8 m of water. Hence, $H_a - H_2 = 8$ m. where $H_a - H_2$ is the maximum vacuum that is permissible under practical conditions. Let this be represented by $H_{v\ max}$ whose value is 8 m of water.

$$\therefore \quad \frac{V_2^2}{2g} = H + H_{v\ max}$$

$$\therefore \quad V_2 = \sqrt{2g\,(H + H_{v\ max})} \text{ or } \sqrt{2g\,(H + H_a - H_2)}$$

Since, $A_2V_2 = A_3V_3$

$$\therefore \quad \frac{A_3}{A_2} = \frac{V_2}{V_3}$$

$$= \frac{\sqrt{2g\,(H + H_a - H_2)}}{\sqrt{2gH}} = \sqrt{1 + \frac{H_a - H_2}{H}} = \sqrt{1 + \frac{H_{v\ max}}{H}}$$

$$\therefore \quad \text{maximum value of } \frac{A_3}{A_2} = \sqrt{\frac{1 + H_{v\ max}}{H}} \qquad (5.11)$$

$$\text{and } H_2 = H_a + H - \frac{V_2^2}{2g} \qquad \text{.. } (5.12)$$

Problem-7 : *An external convergent-divergent mouthpiece has 5 cm diameter at the section where the convergent and divergent portions meet. Water is discharged through the mouthpiece under a head of 3 m above its centre line. Determine the maximum discharge and the corresponding diameter at outlet. The maximum vacuum head allowed is 8 m of water.*

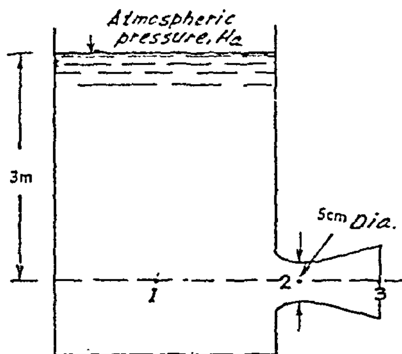


Fig. 5-7

Applying Bernoulli's equation to sections 1 & 2,

$$H_a + 3 = H_2 + \frac{V_2^2}{2g}$$

$$\frac{V_2^2}{2g} = H_a + 3 - H_2$$

$$= 8 + 3 = 11 \text{ m of water (as } H_a - H_2 = 8\ m)$$

$$\therefore \quad V_2 = 4{\cdot}43\sqrt{11} = 14{\cdot}7 \text{ m/sec.}$$

Discharge, $Q = A_2 \times V_2 = \dfrac{\pi}{4}\left(\dfrac{5}{100}\right)^2 \times 14{\cdot}7 = \underline{0{\cdot}029 \text{ m}^3\text{/sec.}}$

or $= 0{\cdot}029 \times 1{,}000 = 29$ litres/sec.

Applying again Bernoulli's equation to sections 1 & 3, we have

$$H_a + 3 = H_a + \frac{V_3^2}{2g}$$

$$\therefore \quad V_3 = 4{\cdot}43 \times \sqrt{3} = 7{\cdot}65 \text{ m/sec.}$$

Since discharge $= A_2V_2 = A_3V_3$

$$\therefore \quad \frac{A_3}{A_2} = \frac{V_2}{V_3} = \frac{14{\cdot}7}{7{\cdot}65} = 1{\cdot}92$$

$$\text{and } 1{\cdot}92 = \left(\frac{D_3}{D_2}\right)^2$$

$$\therefore \quad D_3 - D_2\sqrt{1{\cdot}92} = 5 \times \sqrt{1{\cdot}92} = \underline{6{\cdot}93 \text{ cm}}$$

Problem – 8 : *A nozzle (convergent mouthpiece) of 2·5 cm diameter is attached to a 7·0 cm diameter hose carrying water under pressure of 4 kg/cm². Calculate velocity of jet, and discharge through the nozzle. The coefficient of velocity for the nozzle may be taken as 0·98.*

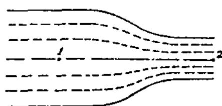


Fig. 5-8

The velocity through the nozzle is calculated by use of eqn. (4·9b, Chap. 4) and velocity in the hose is expressed in terms of velocity of jet.

$\therefore\ V_1 = V_2 \times$ ratio of areas of nozzle and hose

$$= V_2 \times \frac{\frac{\pi}{4}(2{\cdot}5)^2}{\frac{\pi}{4}\,7^2} = 0{\cdot}128 V_2 \text{ m/sec.}$$

Writing Bernoulli's equation in the manner of eqn. (4·9b) and treating the nozzle as an orifice,

$$\frac{p_1}{w} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} + \left(\frac{1}{C_v^2} - 1\right)\frac{V_2^2}{2g}$$

It may be noted here that the jet issues in the atmosphere and hence p_2 = atmospheric pressure, and pressure p_1 is the gauge pressure hence term $\frac{p_2}{w}$ does not appear on the R.H.S. of the above equation.

$$\text{Hence, } \frac{4 \times 100^2}{1{,}000} + \frac{(0{\cdot}128 V_2)^2}{2 \times 9{\cdot}81} = \frac{V_2^2}{2 \times 9{\cdot}81} + \left(\frac{1}{0{\cdot}98^2} - 1\right)\frac{V_2^2}{2 \times 9{\cdot}81}$$

$$\therefore\ V_2^2\,(1 + 0{\cdot}042 - 0{\cdot}0164) = 40 \times 19{\cdot}62$$

$$\therefore\ V_2 = \sqrt{\frac{40 \times 19{\cdot}62}{1{\cdot}026}} = \underline{27{\cdot}6 \text{ m/sec.}}$$

The discharge through the nozzle is

$$Q = A_2V_2 = \frac{\pi}{4}\left(\frac{2.5}{100}\right)^2 \times 27.6 = 0.0136 \text{ m}^3/\text{sec.}$$

or = 13·6 litres/sec.

Head loss in the nozzle is given by

$$h_n = \left(\frac{1}{C_v^2} - 1\right)\frac{V_2^2}{2g} = \left(\frac{1}{0.98^2} - 1\right)\frac{27.6^2}{2 \times 9.81}$$

$$= 1.64 \text{ m of water}$$

Problem – 9 : *A convergent-divergent mouthpiece is fitted to the side of a tank. If the diameter is 6 cm at the outlet and 3 cm at the vena contracta, find the maximum head in the tank at which steady flow through the mouthpiece can be obtained with the following assumptions :*

(i) Separation will occur at an absolute pressure of 2.5 m of water. (ii) Losses occur in the divergent portion of the mouthpiece and amount to 25% of the head lost at sudden enlargement for the same change in section.

Applying Bernoulli's equation to the point in the tank at level of the mouthpiece and the vena-contracta, we have,

$$H_a + H = H_2 + \frac{V_2^2}{2g}$$

$$\text{or } 10 + H = 2.5 + \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{V_2^2}{2g} = 7.5 + H \qquad \text{.. (i)}$$

Applying again Bernoulli's eqn. to the same point and the exit of the mouthpiece,

$$H_a + H = H_a + \frac{V_3^2}{2g} + \text{losses in divergent part}$$

$$= H_a + \frac{V_3^2}{2g} + 0.25\left\{\frac{(V_2 - V_3)^2}{2g}\right\}$$

$$\therefore \quad H = \frac{V_3^2}{2g} + \frac{0.25\,(V_2 - V_3)^2}{2g}$$

But, $V_2 = \frac{A_3 V_3}{A_2} = \left(\frac{d_3}{d_2}\right)^2 V_3 = (\frac{6}{3})^2 V_3 = 4V_3$

$\therefore \quad V_3 = 0{\cdot}25 V_2$

Hence, $H = \frac{(0{\cdot}25\, V_2)^2}{2g} + \frac{0{\cdot}25\,(1 - 0{\cdot}25)^2 V_2^2}{2g}$

$= \frac{0{\cdot}203\, V_2^2}{2g}$

$\therefore \quad \frac{V_2^2}{2g} = \frac{H}{0{\cdot}203} = 4{\cdot}93H$ ..(ii)

Combining eqns. (i) and (ii),

$7{\cdot}5 + H = 4{\cdot}93H$

$\therefore \quad 3{\cdot}93H = 7{\cdot}5$

$\therefore \quad H = 1{\cdot}91$ m of water

Re-entrant or Borda's Mouthpiece

This is an *internal cylindrical mouthpiece. If the length of the mouthpiece* is less than three times its diameter, the jet after contraction does not touch the sides of the mouthpiece as shown in fig. 5-9. This type of mouthpiece is known as Borda's mouthpiece

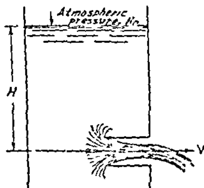


Fig. 5-9 Borda's mouthpiece running free

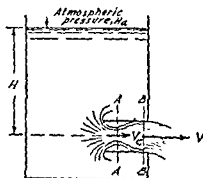


Fig. 5-10 Borda's mouthpiece running full

running free. If after contraction the jet expands and fills the mouthpiece as shown in fig. 5-10, it is said to be *running full.* Let us consider the mouthpiece running free.

Let, H – height of water surface above centre of mouthpiece,

A – area of mouthpiece,

V – velocity of water at exit of mouthpiece, and

A_c – contracted area of jet.

According to the second law of Newton, the force equals the rate of change of momentum.

$\therefore$ total pressure at entrance = change of momentum per second

$$i.e.\quad p \times A = \frac{(w \cdot A_c \cdot V)}{g} V$$

$$\text{But,}\quad p = wH$$

$$\therefore\quad wAH = \frac{wA_cV^2}{g}$$

$$\text{But,}\ H = \frac{V^2}{2g}$$

$$\therefore\quad A_c = \frac{A}{2} \qquad \text{.. (5·13)}$$

Thus the coefficient of contraction is 0·5. Now consider the mouthpiece running full. Consider the sections A–A and B–B as shown in fig. 5–10. In this case there will be loss of head between the two sections due to enlargement following the contraction.

$$\text{Loss of head} = \frac{(V_c - V)^2}{2g}$$

where V_c = velocity of water at the vena-contracta.

and, $A_cV_c = AV$.

$$\therefore\quad V = \frac{A}{A_c} \times V = \frac{V}{C_c}$$

$$\text{and loss of head} = \left(\frac{1}{C_c} - 1\right)^2 \frac{V^2}{2g}$$

But, $C_c = 0{\cdot}5$

$$\therefore\quad \text{loss of head} = \frac{V^2}{2g}$$

Applying Bernoulli's equation to sections at entrance and exit of the mouthpiece,

$$H_a + H = H_a + \frac{V^2}{2g} + \text{loss of head}$$

$$\text{or} \quad H = \frac{V^2}{2g} + \frac{V^2}{2g}$$

$$\therefore \quad H = \frac{V^2}{g}$$

$$\therefore \quad V = \sqrt{gH}$$

Thus it is seen that the discharge when running full is $A\sqrt{gH}$ *and when running free it is* $0{\cdot}5\ A\sqrt{2gH}$. *Therefore the discharge is* increased by $\frac{1}{0{\cdot}5\sqrt{2}}$ when running full.

The coefficient of discharge when running full is

$$C_d = \frac{1}{\sqrt{2}} = 0{\cdot}707 \qquad ..(5{\cdot}15)$$

In practice, the value of coefficient of discharge is found to be more than 0·707.

Applying Bernoulli's equation to sections A-A and B-B, we have, (fig. 5-10) (suffix c refers to vena contracta)

$$H_c + \frac{V_c^2}{2g} = H_a + \frac{V^2}{2g} + \text{loss of head}$$

$$\text{or } H_c + \frac{V_c^2}{2g} = H_a + \frac{V^2}{2g} + \frac{V^2}{2g} \qquad \left(\text{as loss of head} = \frac{V^2}{2g}\right)$$

As $C_c = 0{\cdot}5$ and $V_c = 2V$

$$H_c + \frac{4V^2}{2g} = H_a + \frac{2V^2}{2g}$$

$$\therefore \quad H_c = H_a - \frac{2V^2}{2g} = H_a - \frac{V^2}{g}$$

$$\text{But, } H = \frac{V^2}{g}$$

$$\therefore \quad H_c = H_a - H$$

Maximum allowable value for H_c is 3 m of water absolute and $H_a = 10{\cdot}5$ m of water

$$\therefore \quad 3 = 10{\cdot}5 - H$$

$$\therefore \quad H = 7{\cdot}5 \text{ m}$$

Thus the head in the tank cannot be increased beyond 8 m in this type of mouthpiece for satisfactory working.

Problem–10 : *Water under a constant head of 3 m discharges through an internal cylindrical mouthpiece of 3 cm diameter working as Borda's mouthpiece running full. Determine the discharge in litres/sec of the mouthpiece.*

Area of the mouthpiece, $A = \frac{\pi}{4}\left(\frac{5}{100}\right)^2 = 0{\cdot}00196 \text{ m}^2$

Taking $C_c = 0{\cdot}5$ in the case of Borda's mouthpiece,

the loss of head $= \left(\frac{1}{C_c} - 1\right)^2 \frac{V^2}{2g} = \frac{V^2}{2g}$

Applying Bernoulli's equation to the points on the centre line of mouthpiece one just at the entrance and other at outlet end of the mouthpiece, we have,

$$H_a + H = H_a + \frac{V^2}{2g} + \text{loss of head}$$

$$\text{or } 10 + 3 = 10 + \frac{V^2}{2g} + \frac{V^2}{2g}$$

$$\therefore \quad 3 = \frac{V^2}{g}$$

$$\therefore \quad V = \sqrt{9{\cdot}81 \times 3} = 5{\cdot}42 \text{ m}^3/\text{sec.}$$

Discharge, $Q = AV = 0{\cdot}00196 \times 5{\cdot}42 = 0{\cdot}01065 \text{ m}^3/\text{sec.}$

or $= 0{\cdot}01065 \times 1{,}000 = 10{\cdot}65$ litres/sec.

Problem–11 : *A Borda's mouthpiece 5 cm in diameter is provided to the side of a tank full of water upto the height of 3 m above the centre line of the mouthpiece. If the mouthpiece runs free and if its coefficient of velocity is 0·99, calculate the discharge through it and the head lost.*

The coefficient of contraction for this mouthpiece running free is 0·5

The discharge through it, $Q = C_c A C_v \sqrt{2gH}$

$$= 0{\cdot}5 \times \frac{\pi}{4}\left(\frac{5}{100}\right)^2 \times 0{\cdot}99\sqrt{2 \times 9{\cdot}81 \times 3}$$

$$= 0{\cdot}0075 \text{ m}^3/\text{sec.} = 7{\cdot}5 \text{ litres/sec.}$$

By Bernoulli's equation,

$$H = \frac{(C_v V)^2}{2g} + h_l$$

$$\text{or} \quad 3 = \frac{(0.99 \sqrt{2g} \sqrt{3})^2}{2g} + h_l$$

$$\therefore \quad h_l = 3 - (0.99 \sqrt{3})^2$$

$$= 3\,(1 - 0.99^2) = 3 \times 0.02 = 0.06 \text{ m of water}$$

TUTORIAL-5

Loss of head

1 A pipe 15 cm diameter is suddenly enlarged to 22·5 cm diameter. Determine the loss of H. P. when the discharge is 150 litres/sec.

[2·28 H. P.]

2 *A pipe is conveying 150 litres of water per second. If the* pipe changes suddenly from 20 cm to 60 cm diameter determine the loss of head.

[0·923 m of water]

3 Water flows in a 15 cm diameter pipe which enlarges suddenly to diameter, *d*. Loss of head at sudden enlargement is found to be one-fourth of the velocity head in 15 cm diameter pipe. Determine the value of *d*.

[20·1 cm]

4 Water flows in a 10 cm diameter pipe and at a sudden enlargement, the loss of head is found to be half of the velocity head in 10 cm diameter pipe. Determine the diameter of the enlarged portion.

[18·5 cm]

5 A pipe 15 cm diameter is suddenly enlarged to 30 cm diameter. Find the difference in the two limbs of a U-tube containing mercury which is connected to two points on each side of the enlargement where the flow has settled down, and if water flows at the rate of 60 litres per sec. from the smaller to the larger section. The sp. gr. of mercury is 13·6.

[1·77 cm]

6 A horizontal pipe 15 cm diameter is suddenly reduced to 7·5 cm diameter. Water is flowing from the larger to the smaller pipe at the rate of 20 litres/sec. What is the loss of energy at the sudden contraction in kg-m per minute ? Assume $C_c = 0{\cdot}64$.

[392 kg-m/min]

7 A pipe 22·5 cm diameter has an orifice plate with a hole of 10 cm diameter at its centre. Water flows in the pipe line with a velocity of 0·5 m/sec. If contraction coefficient is 0·62, find the loss of head due to obstruction.

[0·64 m of water]

8 The passage of water through a 15 cm pipe is restricted by diaphragm with 5 cm diameter hole at its centre. The loss of head at the diaphragm when the velocity in the pipe is 0·2 m/sec equals 0·4 m. Assuming the head lost = $\frac{KV^2}{2g}$ where V = the velocity of water in pipe, find C_c the coefficient of contraction of the stream passing through the diaphragm.

[$C_c - 0{\cdot}6$]

9 A short (60 cm long) pipe is fitted to one side of the tank. The pipe is 15 cm in diameter at both ends but the central part is enlarged to 30 cm diameter for 20 cm length. The discharge from the pipe is 1,600 kg/min. Determine the head lost due to sudden enlargement, sudden contraction, and at the entrance to pipe. Take contraction coefficient as 0·62.

[0·0654 m of water ; 0·0433 m of water, 0·0582 m of water]

Cylindrical mouthpieces

10 An external cylindrical mouthpiece 6 cm diameter and 20 cm long is fitted to the side of a vertical tank containing water to 4 m height above the centre of the mouthpiece. Find the discharge through the mouthpiece. Take coefficient of contraction as 0·64.

[21·9 litres/sec]

11 An external cylindrical mouthpiece fitted to the side of a vertical tank *containing water upto 3 m constant height above its* centre line is 6 cm in diameter and 25 cm long. Determine the discharge through the mouthpiece. The coefficient of contraction of the mouthpiece may be taken as 0·62.

[12·85 litres/sec]

Convergent-divergent mouthpiece

12 An external convergent-divergent mouthpiece has 8 cm diameter at the section where the convergent and divergent portions meet. Water is discharged through the mouthpiece under a head of 5 m above its centre line. Determine the maximum discharge and the corresponding diameter at outlet. The maximum vacuum head allowed is 8 m of water.

[80·5 litres/sec, 10·15 cm]

13 A nozzle (convergent-mouthpiece) of 4 cm diameter is attached to a 8 cm diameter hose carrying water under pressure of 2·8 kg/cm^2. Calculate velocity of jet, discharge through the nozzle, and head lost through the nozzle. The coefficient of velocity for the nozzle may be taken as 0·98.

[23·62 m/sec, 29·7 litres/sec, 1·2 m of water]

14 A convergent-divergent mouthpiece is fitted to the side of a tank. If the diameter is 5 cm at the outlet and 2 cm at the vena contracta, find the maximum head in the tank at which steady flow through the mouthpiece can be obtained, making the following assumptions :

(i) Separation will occur at an absolute pressure of 2·5 m of water. (ii) Losses occur in the divergent portion of mouthpiece and amount to 25 % of head lost at sudden enlargement for the same change in section.

[1·9 m]

Borda's mouthpiece

15 Water under a constant head of 2 m discharges through an internal cylindrical mouthpiece 9 cm diameter working as Borda's mouthpiece running full. Determine the discharge in litres/sec.

[5·57 litres/sec]

16 A Borda's mouthpiece 2·5 cm in diameter is provided to the side of a tank full of water upto the height of 5 m above the centre line of the mouthpiece. If the mouthpiece runs free and if its coefficient of velocity is 0·99, calculate the discharge through it and the head los

6
FLOW OVER NOTCHES AND WEIRS

Introduction

Sometimes it is necessary to measure the flow of water which is open to atmosphere such as the flow of water in rivers, streams and canals or discharge of condensate from the condenser etc. The fluid is passed over a particularly shaped structure built across its flow. This structure over which the fluid flows is termed a *weir*. A *notch* is used to measure the flow of water from a tank or condensate from the condenser. The difference between an orifice and a notch may be pointed out by the following considerations.

	Orifice		Notch
1	This is circular or rectangular opening in the side of tank, through which the fluid flows.	1	This is rectangular or V-shaped opening in the dam or tank, over which the fluid flows.
2	The upper edge of the orifice is below the free surface of water in the tank.	2	The water level is below the upper edge of the notch.
3	The stream of water flowing through an orifice is termed *jet*.	3	The sheet of water flowing over a notch or weir is termed *nappe* or *vein*
4	Head of water compared to the orifice dimension is large.	4	Head of water over the sill of the notch is less compared to its dimensions
5	Pressure on the upstream side of the orifice is more than the downstream side pressure which is atmospheric.	5	Pressure on the upstream as well as downstream s of the notch or weir atmospheric

Classification of Weirs

Weirs are classified as under :

(i) They are classified according to their shape. They may be rectangular, triangular, trapezoidal, stepped, etc. All of them are useful for the measurement of the flow of water in the laboratories. The rectangular weir is the one preferred in storage reservoirs for removing excess water. Triangular or V-shaped weirs are used for calibration of the flow of water in a stream.

(ii) Weirs can be classified on the basis of its edge or top surface with which the overflowing water comes in contact. This edge or top surface of a weir is called *crest* of the weir. There are sharp crested weirs, broad crested weirs and ogee or round crested weirs.

(a) **Sharp-crested weirs :** In sharp crested weir (fig. 6-1) the edge is sharp and water flowing over the crest comes in contact with the crest line. Water at the lower edge first springs up from the crest as shown and then falls as a trajectory. Thus the nappe or vein is free from the wall of the weir. In this case the thickness of the weir wall is kept less than half the head on the weir *i.e.* $b < 0{\cdot}5H$.

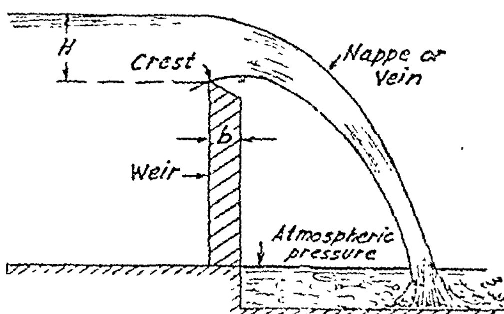


Fig. 6-1 Sharp-crested weir

(b) **Broad-crested weir :** The broad crested weir has broad crest so that the overflowing water comes in contact with the surface. Water springs up at entrance but it slides down in

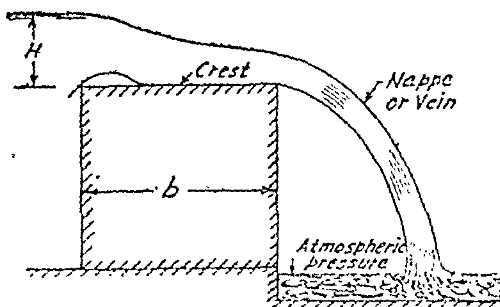


Fig. 6-2 Broad-crested weir

horizontal direction at discharge end. The free surface drops as water flows and minimum depth of flow occurs at the crest; after this flow is in parallel streamlines. In this case wall thickness b is greater than 0 66 times the head on weir.

(c) Ogee weir : This weir (fig. 6-3) is a typical one which is also called *spillway* and it is used on the structures over which surplus water cannot be stored in a reservoir passes. The nappe at its crest rises slightly and after reaching the maximum rise of $0{\cdot}115H'$ forms a trajectory. If the lower face of the nappe is now filled with the solid boundary of either masonary or cement concrete, the shape similar to the one in the above figure would result and it is known as Ogee weir. This type of weir has higher coefficient of discharge.

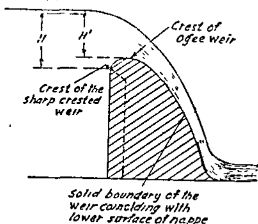


Fig. 6-3 Ogee weir

The discharge over this weir is obtained from the expression,

$$Q = CLH^{\frac{3}{2}} \qquad . (6\ 1)$$

where C – a constant of weir, (= 1·84)

L – length of the weir in metres, and

H – head of water on the upstream side above the crest of the weir in metres.

Flow over Notches

Different shapes (looking from the front of it) of the notches are now considered and expressions for flow over them are established in the following paragraphs.

Rectangular notch : This notch is shown in fig. 6–4. The breadth L and the head H on the weir are indicated thereon. This weir is constructed in the middle or the end of the dam.

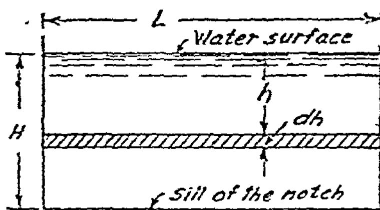


Fig. 6–4 Rectangular notch

Let, L – breadth of notch,

H – height of water surface above sill of the notch,

C_d – coefficient of discharge,

Q – discharge over the weir.

Consider a horizontal strip of water of thickness dh and at depth h of water. The theoretical velocity of water flowing through the strip will be $\sqrt{2gh}$.

Discharge through the strip, $dQ = C_d \times$ area of strip $\times$ velocity

$$= C_d \times L \times dh \times \sqrt{2gh}$$

$$\text{or} \quad \int_0^Q dQ = C_d\, L\sqrt{2g} \int_0^H h^{\frac{1}{2}}\, dh$$

$$\therefore \quad Q = \tfrac{2}{3}\, C_d L \sqrt{2g} \left[h^{\frac{3}{2}} \right]_0^H$$

$$= \tfrac{2}{3}\, C_d L \sqrt{2g} \cdot H^{\frac{3}{2}}$$

$$\text{or} \quad Q = KH^{\frac{3}{2}}$$

where $K = \frac{2}{3}\, C_d \sqrt{2g} \cdot L$ ✓

Triangular or V-notch : Consider the triangular notch shown in fig. 6–5.

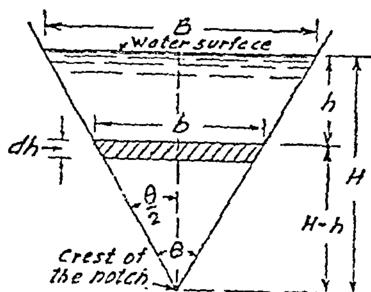


Fig. 6–5 V-notch

Let, H – height of water surface.

θ – angle of the notch,

B – width of the water surface,

b – width of the elementary strip,

dh – thickness of the strip.

$$\text{Then, } \tan\frac{\theta}{2} = \frac{\frac{b}{2}}{H-h} = \frac{\frac{B}{2}}{H}$$

$$\therefore \quad b = \frac{B}{H}(H-h)$$

$$\text{But, } 2\tan\frac{\theta}{2} = \frac{B}{H}$$

$$\therefore \quad b = 2\tan\frac{\theta}{2}(H-h)$$

Theoretical velocity of flow through the strip $= \sqrt{2gh}$

Discharge through the strip is

$$dQ = C_d \times \text{area of the strip} \times \text{velocity of flow}$$

$$= C_d \times b \times dh \times \sqrt{2gh}$$

$$= C_d\, 2\tan\frac{\theta}{2}(H-h)\sqrt{2gh}\, dh$$

$$\text{or} \int_0^Q dQ = C_d\, 2\tan\frac{\theta}{2}\sqrt{2g}\int_0^H (H-h)\, h^{\frac{1}{2}}\, dh$$

$$\therefore \quad Q = 2C_d\tan\frac{\theta}{2}\sqrt{2g}\left[\frac{2}{3}Hh^{\frac{3}{2}} - \frac{2}{5}h^{\frac{5}{2}}\right]_0^H$$

$$\text{or} \quad Q = \frac{8}{15} C_d\sqrt{2g}\tan\frac{\theta}{2}\, H^{\frac{5}{2}} \qquad \text{.. (6·3)}$$

Assuming average value of C_d as 0·6,

$$Q = 1{\cdot}418\tan\frac{\theta}{2}\, H^{\frac{5}{2}}$$

If a 90° notch is used then, $\tan\frac{\theta}{2} = 1$

$$\text{and } Q = 1{\cdot}418\, H^{\frac{5}{2}} \qquad \text{.. (6·3a)}$$

In the case of a rectangular notch, it will be noticed that the total wetted edge of the notch does not vary directly with the head, as the length of the base is the same for all heads. Therefore the coefficient of contraction, which depends on the length of wetted edge is not constant for all heads. But in the case of a triangular notch, there is no base to cause contraction, it will be due to the sides only. The coefficient of contraction will, therefore, be constant for all heads. For this reason, the triangular notch is the most satisfactory type for measuring the quantity of flowing water.

Trapezoidal notch : A notch of this type (fig. 6-6) is a combination of a rectangular notch and a V-notch, subtending an angle θ between its inclined edges.

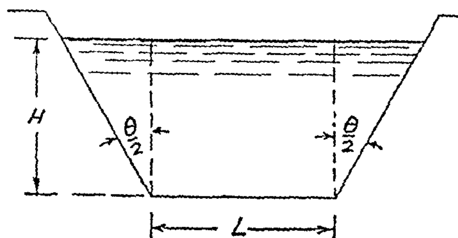


Fig. 6-6 Trapezoidal notch

The total discharge will be obtained by adding together the discharges from the rectanglar and the triangular portions. Hence,

$$Q = \frac{2}{3} C_{d_1} L \sqrt{2g}\, H^{\frac{3}{2}} + \frac{8}{15} C_{d_2} \sqrt{2g} \tan \frac{\theta}{2} H^{\frac{5}{2}} \quad \quad .. (6.4)$$

Cippoletti weir : It is a trapezoidal weir having a side slope of 1 horizontal to 4 vertical. This was invented by an Italian engineer Cippoletti. In this type of weir the end contraction effect is not required to be considered in Francis formula. It is mostly used for measuring the irrigation water. This is special type of weir such that its side slope is so arranged that it increases the discharge through the triangular portion of the weir which would have been decreased by having end contractions in case of rectangular weirs. Hence the advantage of Cippoletti weir is that Francis formula will not require a correction factor for each contraction.

Discharge through rectangular portion considering end contraction, $Q_1 = \frac{2}{3} C_d \sqrt{2g}\ (L - 0.2 H)\ H^{\frac{3}{2}}$ Discharge through triangular portion,

$$Q_2 = \frac{8}{15} C_d \sqrt{2g}\ \tan\frac{\theta}{2} H^{\frac{5}{2}}$$

∴ Total discharge through traperoidal weir,

$$Q = Q_1 + Q_2$$
$$= \frac{2}{3} C_d \sqrt{2g}\,(L - 0.2\ H)\ H^{\frac{3}{2}} + \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\ H$$

The discharge for Cippoletti weir is $Q = \frac{2}{3} C_d \sqrt{2g}\ LH^{\frac{3}{2}}$

$$\therefore\ \frac{2}{3} C_d \sqrt{2g}\ LH^{\frac{3}{2}} - \frac{2}{3} C_d \sqrt{2g}\,(L - 0\ 2H)\ H^{\frac{3}{2}} + \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\ {}^{\frac{5}{2}}$$

$$= \frac{2}{3} C_d \sqrt{2g}\ LH^{\frac{3}{2}} - \frac{2}{3} C_d \sqrt{2g} \times 0.2\ H^{\frac{5}{2}} + \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\ H^{\frac{5}{2}}$$

$$\therefore\ \frac{2}{3} C_d \sqrt{2g} \times 0.2 H^{\frac{5}{2}} = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\ H^{\frac{5}{2}}$$

$$\therefore\ \frac{0\ 4}{3} = \frac{8}{15}\ \tan\ \frac{\theta}{2}$$

$$\therefore\ \tan\frac{\theta}{2} = \frac{15}{8} \times \frac{0\ 4}{3} =$$

$$\therefore\ \frac{\theta}{2} = \tan^{-1}\tfrac{1}{4} \qquad \text{i.e. slop } 1:4$$

$$\therefore\ Q = \frac{2}{3} C_d \sqrt{2g}\ LH^{\frac{3}{2}} = 1{\cdot}84\ LH^{\frac{3}{2}} \text{ assuming } C_d = 0{\cdot}623$$

Law of comparison for notches : The notches or weirs which are similar in shape and which can be represented by *drawing of the same notch but to a different scale are known as similar weirs* or notches. The discharge through similar notches will depend *on* their linear dimensions raised to power $\frac{5}{2}$.

Consider two similar notches.

Discharge ∝ area x velocity

But, area ∝ H^2

and velocity $\propto \sqrt{H}$

then, discharge, $Q \propto H^2 \times \sqrt{H}$

$$\therefore \quad Q = KH^{\frac{5}{2}} \quad .. (6.5)$$

The constant K should be the same for all notches.

In the case of two similar rectangular notches, let the breadths of the notches be L and nL and the corresponding heads be H and nH respectively.

Then, discharge of one weir, $Q_1 = KnL \times nH \times \sqrt{nH}$

and discharge of other weir, $Q_2 = K \times L \times H \sqrt{H}$

$$\frac{Q_1}{Q_2} = \frac{KnLnH \sqrt{nH}}{KLH \sqrt{H}} = n^{\frac{5}{2}} \quad .. (6.5a)$$

Problem–1 : *A rectangular notch 2·5 m wide has constant head of 30 cm. Taking the coefficient of discharge as 0·62, calculate the discharge of the notch in litres/sec.*

Here, $L = 2.5$ m, $H = 30/100 = 0.3$ m, $C_d = 0.62$,

Discharge is given by eqn. (6·2) as

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{\frac{3}{2}}$$

$$= \frac{2}{3} \times 0.62 \times 2.5 \times 4.43 \times 0.3^{\frac{3}{2}}$$

$= 1.7$ m³/sec or 1,700 litres/sec.

Problem–2 : *A weir 7 m long is to be built across a rectangular channel to discharge 8,200 litres/sec of water. If the maximum depth of water on the upstream side of the weir is to be 2 m, what should be the height of weir ? Neglect end contractions and take discharge coefficient as 0·62.*

Let the height of the weir crest above the channel bed be Z m. Then head of water above the sill of the weir is $(2 - Z)$ m.

The discharge over the weir is given by

$$Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{\frac{3}{2}}$$

$$\text{or } \frac{8{,}200}{1{,}000} = \tfrac{2}{3} \times 0.62 \times 7 \times \sqrt{2 \times 9.81}\ (2 - Z)^{\frac{3}{2}}$$

$$\therefore \quad (2 - Z)^{\frac{3}{2}} = 0.64$$

$$\therefore \quad (2 - Z) = 0.64^{\frac{2}{3}} = 0.77$$

$$\therefore \; Z = \underline{1.23 \text{ m}}$$

Problem-3 : *A rectangular channel 2 m wide carries a maximum discharge of 300 litres/sec and a minimun of 150 litres/sec. It discharges through a V-notched weir. The angle of the notch is 120°. At what level from the channel bed, should the notch be placed to give the maximum depth of 1 m for the channel ? What is then the minimum depth of water in the channel ? Assume value of 0 6 for discharge coefficient. Neglect velocity of approach.*

Let the weir be placed Z m above the channel bed. Therefore, for maximum discharge, the head is $H = (1 - Z)$ m.

$$\text{Discharge, } Q = \tfrac{8}{15}\, C_d \sqrt{2g} \tan \frac{\theta}{2}\; H^{\frac{5}{2}}$$

$$i.e.\ 0.3 = \tfrac{8}{15} \times 0.6 \times 4.43 \tan 60°\ (1 - Z)^{\frac{5}{2}}$$

$$\therefore \quad (1 - Z)^{\frac{5}{2}} = 0.122$$

$$\therefore \quad Z = 1 - (0.122)^{\frac{2}{5}} = \underline{0.431 \text{ m}}$$

With the minimum discharge of 0.15 m³/sec, let the head be, H_1 m.

$$i.e.\ 0.15 = \tfrac{8}{15} \times 0.6 \times 4.43 \times \tan 60°\ H_1^{\frac{5}{2}}$$

$$\therefore \quad H_1^{\frac{5}{2}} = 0.061$$

$$\therefore \quad H_1 = 0.326 \text{ m}$$

Therefore, minimum water depth = 0.431 + 0.326 = 0.757 m

Problem-4 : *Water flows over a rectangular weir 1.25 m wide to a depth of 10 cm and afterwards passes over a right angled V-notch.*

Taking coefficients of discharge for the notches as 0·62 and 0·6 respectively, find the depth of water over the V-notch.

Since discharge over the two weirs is the same,

$$Q = \frac{2}{3} C_{d_1} L \sqrt{2g}\, H_1^{\frac{3}{2}} = \frac{8}{15} C_{d_2} \sqrt{2g} \tan\frac{\theta}{2} H_2^{\frac{5}{2}}$$

$$\text{or } \frac{2}{3} \times 0{\cdot}62 \times 1{\cdot}25 \times \left(\frac{10}{100}\right)^{\frac{3}{2}} = \frac{8}{15} \times 0{\cdot}6 \times \tan 45° H_2^{\frac{5}{2}}$$

$$\therefore \quad H_2^{\frac{5}{2}} = 0{\cdot}0515$$

$$\therefore \quad H_2 = 0{\cdot}33 \text{ m or } \underline{33 \text{ cm}}$$

Problem – 5 : *Find the quantity of water flowing through the stepped notch shown in fig. 6-7, assuming the coefficient of discharge to be 0·6 for all portions of the notch. The level of water coincides with the top of the notch.*

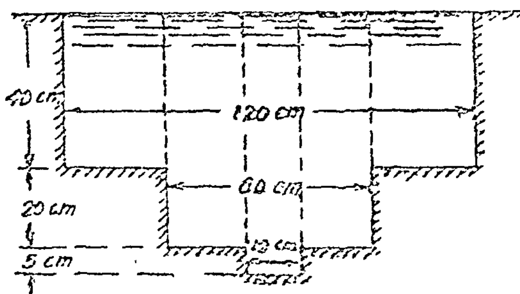


Fig. 6-7

The discharge can be found by dividing the water area into vertical strips as shown by the dotted lines in fig. 6-7 and then applying formula for discharge of the rectangular notch.

Discharge from the central vertical strip is

$$Q_1 = \frac{2}{3} C_d \sqrt{2g} L_1 H_1^{\frac{3}{2}} \qquad \text{.. (i)}$$

Discharge from the intermediate strips is

$$Q_2 = \frac{2}{3} C_d \sqrt{2g} L_2 H_2^{\frac{3}{2}} \qquad \text{.. (ii)}$$

Discharge from outer strips is

$$Q_3 = \frac{2}{3} C_d \sqrt{2g} L_3 H_3^{\frac{3}{2}} \quad \text{.. (iii)}$$

The total discharge is,

$$Q = Q_1 + Q_2 + Q_3$$

$$= \frac{2}{3} C_d \sqrt{2g} \left[L_1 H_1^{\frac{3}{2}} + L_2 H_2^{\frac{3}{2}} + L_3 H_3^{\frac{3}{2}} \right]$$

Here $L_1 = \frac{15}{100} = 0{\cdot}15$ m, $\quad H_1 = \frac{40 + 20 + 5}{100} = 0{\cdot}65$ m,

$L_2 = \frac{60 - 15}{100} = 0{\cdot}45$ m, $\quad H_2 = \frac{40 + 20}{100} = 0{\cdot}6$ m,

$L_3 = \frac{120 - 60}{100} = 0{\cdot}6$ m, $\quad H_3 = \frac{40}{100} = 0{\cdot}4$ m,

$$\therefore \quad Q = \frac{2}{3} \times 0{\cdot}62 \times 4{\cdot}43 \left[0{\cdot}15 \times 0{\cdot}65^{1{\cdot}5} + 0{\cdot}45 \times 0{\cdot}6^{1{\cdot}5} + 0{\cdot}6 \times 0{\cdot}4^{1{\cdot}5} \right]$$

$$= 0{\cdot}916 \text{ m}^3/\text{sec, or } 916 \text{ litres/sec.}$$

Problem - 6 *In measuring the discharge of 85 litres/sec of water, a rectangular notch 0·3 m wide, and then a right angled triangular notch are used. If in measuring the head of water over weir an error of 0·025 cm is made each time, determine the percentage error in finding out the discharge. Take discharge coefficient as 0·62 for notch.*

For right-angled V-notch,

$$Q = \frac{8}{15} C_d \tan \frac{\theta}{2} \sqrt{2g}\ H^{\frac{5}{2}}$$

and $$\frac{dQ}{dH} = \frac{8}{15} \times C_d \tan \frac{\theta}{2} \sqrt{2g}\ \frac{5}{2} H^{\frac{3}{2}}$$

$$\therefore \quad dQ = \frac{4}{3} C_d \tan \frac{\theta}{2} \sqrt{2g}\ H^{\frac{3}{2}}\ dH$$

Taking coefficients of discharge for the notches as 0·62 and 0·6 respectively, find the depth of water over the V-notch.

Since discharge over the two weirs is the same,

$$Q = \frac{2}{3}\, C_{d1} L \sqrt{2g}\, H_1^{\frac{3}{2}} = \frac{8}{15}\, C_{d2} \sqrt{2g}\, \tan\frac{\theta}{2} H_2^{\frac{5}{2}}$$

$$\text{or} \quad \frac{2}{3} \times 0{\cdot}62 \times 1{\cdot}25 \times \left(\frac{10}{100}\right)^{\frac{3}{2}} = \frac{8}{15} \times 0{\cdot}6 \times \tan 45^\circ H_2^{\frac{5}{2}}$$

$$\therefore \quad H_2^{\frac{5}{2}} = 0{\cdot}0515$$

$$\therefore \quad H_2 = 0{\cdot}33 \text{ m or } \underline{33 \text{ cm}}$$

Problem – 5 : *Find the quantity of water flowing through the stepped notch shown in fig. 6-7, assuming the coefficient of discharge to be 0·6 for all portions of the notch. The level of water coincides with the top of the notch.*

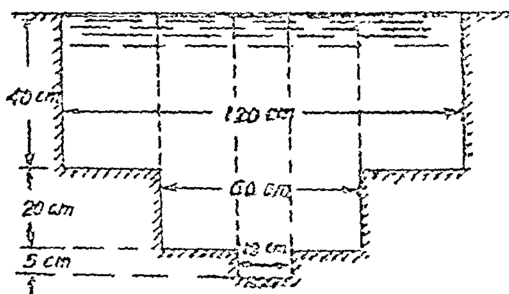


Fig. 6-7

The discharge can be found by dividing the water area into vertical strips as shown by the dotted lines in fig. 6-7 and then applying formula for discharge of the rectangular notch.

Discharge from the central vertical strip is

$$Q_1 = \frac{2}{3}\, C_d \sqrt{2g} L_1 H_1^{\frac{3}{2}} \qquad \text{.. (i)}$$

Discharge from the intermediate strips is

$$Q_2 = \frac{2}{3}\, C_d \sqrt{2g} L_2 H_2^{\frac{3}{2}} \qquad \text{.. (ii)}$$

Discharge from outer strips is

$$Q_3 = \frac{2}{3}\ C_d\ \sqrt{2g}\, L_3\ H_3^{\frac{3}{2}} \qquad \text{.. (iii)}$$

The total discharge is,

$$Q = Q_1 + Q_2 + Q_3$$

$$= \frac{2}{3}\ C_d \sqrt{2g} \left[L_1 H_1^{\frac{3}{2}} + L_2 H_2^{\frac{3}{2}} + L_3 H_3^{\frac{3}{2}} \right]$$

Here $L_1 = \dfrac{15}{100} = 0{\cdot}15$ m, $\quad H_1 = \dfrac{40 + 20 + 5}{100} = 0{\cdot}65$ m,

$L_2 = \dfrac{60 - 15}{100} = 0{\cdot}45$ m, $\quad H_2 = \dfrac{40 + 20}{100} = 0{\cdot}6$ m,

$L_3 = \dfrac{120 - 60}{100} = 0{\cdot}6$ m, $\quad H_3 = \dfrac{40}{100} = 0{\cdot}4$ m,

$$\therefore \quad Q = \frac{2}{3} \times 0{\cdot}62 \times 4{\cdot}43 \left[0{\cdot}15 \times 0{\cdot}65^{1{\cdot}5} + 0{\cdot}45 \times 0{\cdot}6^{1{\cdot}5} + 0{\cdot}6 \times 0{\cdot}4^{1{\cdot}5} \right]$$

$= 0{\cdot}916$ m³/sec, or 916 litres/sec.

Problem - 6 · *In measuring the discharge of 85 litres/sec of water, a rectangular notch 0·3 m wide, and then a right angled triangular notch are used. If in measuring the head of water over weir an error of 0·025 cm is made each time, determine the percentage error in finding out the discharge. Take discharge coefficient as 0·62 for notch.*

For right-angled V-notch,

$$Q = \frac{8}{15}\ C_d\ \tan\frac{\theta}{2}\sqrt{2g}\ H^{\frac{5}{2}}$$

$$\text{and } \frac{dQ}{dH} = \frac{8}{15} \times C_d\ \tan\frac{\theta}{2}\sqrt{2g}\ \frac{5}{2} H^{\frac{3}{2}}$$

$$\therefore \quad dQ = \frac{4}{3}\ C_d \tan\frac{\theta}{2}\sqrt{2g}\ H^{\frac{3}{2}}\, dH$$

$$\therefore \quad \frac{dQ}{Q} = \frac{\frac{4}{3} C_d \tan \frac{\theta}{2} \sqrt{2g}\, H^{\frac{3}{2}}\, dH}{\frac{8}{15} C_d \tan \frac{\theta}{2} \sqrt{2g}\, H^{\frac{5}{2}}}$$

$$\therefore \quad \frac{dQ}{Q} \times 100 = \frac{5}{2} \frac{dH}{H} \times 100$$

Again, $Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} H^{\frac{5}{2}}$ for V-notch

i.e $0{\cdot}085 = \frac{8}{15} \times 0{\cdot}62 \times 4{\cdot}43 \times 1 \times H^{\frac{5}{2}}$

or $H^{\frac{5}{2}} = 0{\cdot}058$

$\therefore \quad H = (0{\cdot}058)^{0{\cdot}4} = 0{\cdot}3187$ m.

$$\therefore \quad \frac{dQ}{Q} \times 100 = \frac{5}{2} \times \frac{0{\cdot}025}{100} \times \frac{100}{0{\cdot}3187} = \underline{0{\cdot}196\%}$$

For rectangular notch,

$Q = \frac{2}{3} C_d \sqrt{2g}\, L\, H^{\frac{3}{2}}$

or $0{\cdot}085 = \frac{2}{3} \times 0{\cdot}62 \times 4{\cdot}43 \times 0{\cdot}3 \times H^{\frac{3}{2}}$

$\therefore \quad H^{\frac{3}{2}} = 0{\cdot}155 \qquad \therefore \quad H = (0.155)^{\frac{2}{3}} = 0{\cdot}289$ m

In this case, $\frac{dQ}{Q} \times 100 = \frac{3}{2} \frac{dH}{H} \times 100$

$$\frac{dQ}{Q} \times 100 = 1{\cdot}5 \times \frac{0{\cdot}025}{100} \times \frac{100}{0{\cdot}28} = 0{\cdot}13\%$$

$$\frac{\%\text{ error in V-notch}}{\%\text{ error in rectangular notch}} = \frac{0{\cdot}196}{0{\cdot}13} = 1{\cdot}51$$

Problem-7 : *What length of Cippoletti weir is required for flow of 4·25 litres/sec, if the maximum head is limited to $\frac{1}{10}$th of its length.*

Here, head $H = \frac{1}{10} L$ where L is length of weir.

Discharge $Q = 1{\cdot}84\; LH^{\frac{5}{2}} = 1{\cdot}84 \times 10 \times H^{\frac{5}{2}}$

$\therefore \quad 0{\cdot}425 = 18{\cdot}4\; H^{\frac{5}{2}}$

$\therefore \quad H^{\frac{5}{2}} = \frac{0.425}{18.4} = 0.0231$

$\therefore \quad H = 0.0231^{\frac{2}{5}} = 0.2215$ m

$\therefore \quad L = 10H = 10 \times 0.2215 = 2.215$ m

Problem 8 : *Find the discharge through a triangular notch shown in fig. 6-8. Take coefficient of discharge as 0·62 for all portions.*

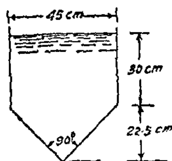


Fig. 6-8 Triangular notch

The above notch can be looked upon as the combination of two V-notches such that the discharge of the smaller V-notches formed on the sides is to be subtracted from the discharge of the bigger V-notch. The dimensions of the two V-notches are : (Imagine that the lines including 90° angle at the sill and the water level line extending and intersecting one another.)

Bigger notch : Width at water level = 45 + 2 × 30 × tan 45°
= 105 cm
depth = total head of water
= 30 + 22·5 = 52·5 cm

Smaller notches : Width at water level = 105 − 45 = 60 cm
depth = head of water = 30 cm

Then $Q = Q_1 - Q_2$

Using the equation (6·3),

discharge, $Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H_1^{\frac{5}{2}} - \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H_2^{\frac{5}{2}}$

Here, $H_1 = \frac{30 + 22.5}{100}$ m, and $H_2 = \frac{30}{100} = 0.3$ m; $\frac{\theta}{2} = 45°$

$\therefore \quad Q = \frac{8}{15} \times 0.62 \times 4.43 \times 1 \left[\left(\frac{52.5}{100}\right)^{\frac{5}{2}} - \left(\frac{30}{100}\right)^{\frac{5}{2}}\right]$

$= 1.47 \times (0.1995 - 0.0493)$

$= 1.47 \times 0.1502 = 0.221$ m³/sec. or 221 litres/s

Problem - 9 : *Find the discharge over the notch shown in fig. 6-9. Coefficient of discharge may be taken as 0·6 for it for its different portions.*

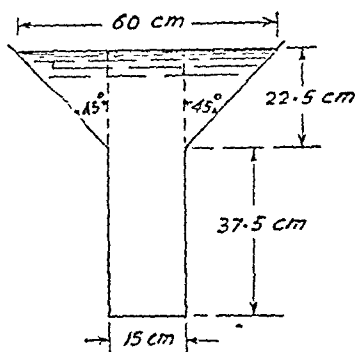


Fig. 6-9

By drawing vertical lines as shown in the fig. 6-9 a rectangular notch of 15 cm width with 60 cm head and a V-notch with 90° angle and 22·5 cm head are formed. The total discharge is the sum of discharges over these two notches.

Hence, $Q = Q_1 + Q_2 = \frac{2}{3} C_d L \sqrt{2g}\, H^{\frac{3}{2}} + \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} H^{\frac{5}{2}}$

$= (0{\cdot}6 \times \sqrt{2 \times 9{\cdot}81}) \left\{ \frac{2}{3} \times \frac{15}{100} \left(\frac{60}{100}\right)^{\frac{3}{2}} + \frac{8}{15} \times 1 \times \left(\frac{22{\cdot}5}{100}\right)^{\frac{5}{2}} \right\}$

$= 0{\cdot}1568$ m³/sec. or 156·8 litres/sec.

Problem-10 : *Find the discharge through the notch shown in fig. 6-10 (a). Take discharge coefficient C as 0·62 for all portions*

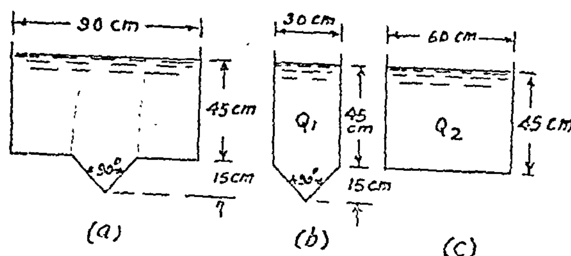


Fig. 6-10

The notch shown in fig. 6-10 (a) can be looked upon as the combination of two notches shown in (b) and (c) respectively. Let the discharge of notches of fig. 6-10 (b) & (c) be Q_1 and Q_2 respectively, Q_1 can be obtained as shown in problem-8 of this chapter.

$$= \tfrac{2}{3}\, C_d\, L\, \sqrt{2g} H^{\frac{3}{2}} + \tfrac{8}{15}\, C_d \sqrt{2g}\, \tan\frac{\theta}{2}\, H^{\frac{3}{2}}$$

$$= \tfrac{2}{3} \times 0{\cdot}6 \times 0{\cdot}45 \times 4{\cdot}43 \times \left(\frac{22{\cdot}5}{100}\right)^{1{\cdot}5}$$

$$+ \tfrac{8}{15} \times 0{\cdot}6 \times 4{\cdot}43 \times 1{\cdot}25 \times \left(\frac{22{\cdot}5}{100}\right)^{2{\cdot}5}$$

$$= 0{\cdot}0852 + 0{\cdot}0425 = \underline{0{\cdot}1277\ \text{m}^3/\text{sec.}} \text{ or } 127{\cdot}7 \text{ litres/sec.}$$

Experimentally Derived Emperical Formula for Rectangular Notches or Weirs

(a) **Francis' formula** : From fig. 6–12, it is clear that the vertical walls A and B of the notch are projecting into the stream. This results in diverting the flow of the stream, and hence the effective length of water flow becomes less than the length of the notch sill as shown in fig. 6–12. Francis carried out experiments on weirs 2 to 3 m long and with heads varying from 0·2 m to 0·5 m and deduced that as water flows over the weir the vein is contracted at each side by an amount of $0{\cdot}1H$. Thus the effective length of the notch or vein would be $(L-0{\cdot}2H)$ m. Substituting the effective length in the equation for discharge for the rectangular weir *viz.*

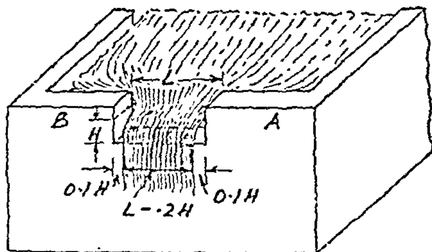


Fig. 6–12

$$Q = \tfrac{2}{3}\, C_d\, L\, \sqrt{2g} H^{\frac{3}{2}} \text{ we have,}$$

$$Q = \tfrac{2}{3}\, C_d \sqrt{2g}\, (L - 0{\cdot}2H)\, H^{\frac{3}{2}}$$

$$Q = K\,(L - 0{\cdot}2H)\, H^{\frac{3}{2}} \qquad \text{.. (6·6)}$$

Francis found out experimentally the value of K as 1·84. assuming $C_d = 0{\cdot}623$

$$Q = 1{\cdot}84\,(L - 0{\cdot}2H)\, H^{\frac{3}{2}} \qquad \text{.. (6·6a)}$$

For a large weir which is split up into a number of bays by vertical posts, number of side contractions, n will depend on the number of bays into which the weir is divided.

$$\therefore \quad Q = 1{\cdot}84\,(L - 0{\cdot}1nH)\,H^{\frac{3}{2}} \qquad \ldots \text{[illegible]}$$

If the weir has the same length as the width of the channel from which water approaches the weir, then such a weir is known as the *suppressed weir* and $n = 0$.

$$\text{Then, } Q = 1{\cdot}84\,LH^{\frac{3}{2}} \qquad \ldots \text{[illegible]}$$

(b) Bazin's formula : Bazin, after carrying out experiments on suppressed weirs, deduced the following formula :

$$Q = m\,\sqrt{2g}\,LH^{\frac{3}{2}}$$

Problem – 13 : *A sharp-crested rectangular contracted weir 2 m long is provided in the channel. The channel has a flow of 1,150 litres/sec. Determine the head of water above the sill of the weir. Use Francis' formula.*

$$\text{Since, } Q = 1{\cdot}84\,(L - 0{\cdot}1nH)\,H^{\frac{3}{2}}$$

$$\text{i.e. } 1{\cdot}15 = 1{\cdot}84\,(2 - 0{\cdot}2H)\,H^{\frac{3}{2}}$$

$$\text{or } \frac{1{\cdot}15}{1{\cdot}84} = (2 - 0{\cdot}2H)\,H^{\frac{3}{2}}$$

$$\text{or } \frac{0{\cdot}625}{H^{\frac{3}{2}}} = 2 - 0{\cdot}2H$$

$$\text{or } \frac{0{\cdot}3125}{H^{\frac{3}{2}}} = 1 - 0{\cdot}1H$$

Let each side of the above equation be equal to y.

$$\text{Then, } y_1 = \frac{0{\cdot}3125}{H^{\frac{3}{2}}} \text{ and } y_2 = 1 - 0{\cdot}1H$$

The following table is now prepared.

Table – 1

1	H in m	0·3	0·4	0·5	0·6	0·7	0·8	0·9
2	$H^{\frac{3}{2}}$	0·1645	0·253	0·354	0·465	0·585	0·715	0·855
3	$y_1 = \frac{0{\cdot}3125}{H^{\frac{3}{2}}}$	1·9	1·19	0·885	0·672	0·535	0·437	0·366
4	$y_2 = 1 - 0{\cdot}1H$	0·97	0·96	0·95	0·94	0·93	0·92	0·91

From the above table it is clear that when $H = 0{\cdot}4$ m, $y_1 > y_2$ and when $H = 0{\cdot}5$ m, $y_1 < y_2$. Hence H lies between 0·4 m and 0·5 m. Hence we prepare another table to get correct value of H between the limits.

Table – 2

1	H, m	0·4	0 42	0 44	0·46	0 48	0·5
2	$H^{\frac{3}{2}}$	0 253	0·272	0·292	0·312	0·332	0·354
3	$y = \frac{0\ 3125}{H^{\frac{3}{2}}}$	1 19	1·15	1·07	1·001	0 943	0·885
4	$y = 1 - 0·1H$	0·96	0·958	0·956	0·954	0 952	0·95

Now H is taken as abscissa and y_1 and y_2 as ordinate and the graphs as shown in fig. 6-13 are plotted. The point P is the intersection of the two curves and gives the correct value. A perpendicular from point P is drawn on the abscissa and the value of H is read. From the graph the value of H is 0·475 m. Thus the head of water above the sill of the weir is 47·5 cm.

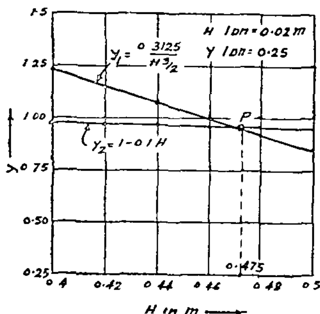


Fig. 6-13

Problem-14 : *The maximum flood discharge of $1{\cdot}4 \times 10^6$ litres/sec passes over a weir built across a river. A road bridge is provided with 1 m wide pillars over the weir which is divided into a number of openings each of 6 m span. Find the number of openings required if the head over the crest is not to exceed 2 m. Neglect the velocity of approach and use Francis formula. What is the total length of bridge ?*

Let Z be the number of openings.

Then, number of end contractions, $n = 2Z$

and weir length, $L = 6Z$ m

Now discharge, $Q = 1{\cdot}84\ (L - 0{\cdot}1\ nH)\ H^{\frac{3}{2}}$

or $1{,}400 = 1{\cdot}84\ \{6Z - 0{\cdot}1 \times 2Z \times 2\} \times 2^{\frac{3}{2}}$

$\therefore \quad 4{\cdot}8Z = 269$

$\therefore \quad Z = 55{\cdot}9$

Therefore, the number of openings should be 56.

The number of pillars will be 56 + 1 = 57

Length of the bridge = 6 × 56 + 57 = 393 m

Probelm-15 : *A reservoir has a catchment area of 5×10^7 m². What length of weir will it require to carry away a rainfall of 10 cm in 24 hours, if 50% of the rainfall reaches the reservoir with a head of 1·2 m above the weir ?*

Catchment area = 5×10^7 m²

Volume of water reaching the reservoir per second is

$$\frac{5 \times 10^7}{24 \times 3{,}600} \times \frac{10}{100} \times \frac{50}{100} = 29 \text{ m}^3/\text{sec.}$$

Discharge, $Q = 1{\cdot}84\ (L - 0{\cdot}1 nH)\ H^{\frac{3}{2}}$

or $29 = 1{\cdot}84\ (L - 0{\cdot}1 \times 2 \times 1{\cdot}2)\ 1{\cdot}2^{\frac{3}{2}}$

$\therefore \quad L = 12{\cdot}19$ m

Problem-16 : *Show that a rational formula for the flow Q over a rectangular weir of width B can be expressed as $Q = A\ (B - CH)H^{\frac{3}{2}}$ where H is the depth of water at a paint near the weir which is not affected by the curvature of the surface.*

In rectangular weir 2 m in breadth the discharge is 300 litres/sec when the head of water is 20 cm; and 830 litres/sec when the head is 40 cm. Find the values of the constants in the above expression and estimate the discharge when the head is 25 cm.

Discharge, $Q = 1{\cdot}84\,(L - 0{\cdot}1nH)\,H^{\frac{3}{2}}$

Let, $1{\cdot}84 = A$, $L = B$ and $0{\cdot}1n = C$

Then, $Q = A\,(B - CH)\,H^{\frac{3}{2}}$

Now, we have $Q_1 = 0{\cdot}3$ m³/sec. when $H_1 = 0{\cdot}2$ m.
and $Q_2 = 0{\cdot}83$ m³/sec, when $H_2 = 0{\cdot}4$ m; and width $B = 2$ m.

$$\therefore \quad 0{\cdot}3 = A\,(2 - C \times 0{\cdot}2)\,(0{\cdot}2)^{\frac{3}{2}} \qquad \text{.. (i)}$$

$$\text{and} \quad 0{\cdot}83 = A\,(2 - C \times 0{\cdot}4)\,(0{\cdot}4)^{\frac{3}{2}} \qquad \text{.. (ii)}$$

$$\therefore \quad 3{\cdot}35 = 2A - 0{\cdot}2\,AC \quad \{\text{from (ii)}\} \qquad \text{.. (iii)}$$

$$\text{and} \quad 3{\cdot}28 = 2A - 0{\cdot}4 \times AC \quad \{\text{from (i)}\} \qquad \text{.. (iv)}$$

Subtracting (iv) from (iii), we have,

$$0{\cdot}07 = 0{\cdot}2\,AC$$

$$\therefore \quad C = \frac{0{\cdot}07}{0{\cdot}2A}$$

Now, Substituting this valve of C in (iv),

$$3{\cdot}28 = 2A - \frac{0{\cdot}4A \times 0{\cdot}07}{0{\cdot}2A}$$

$$\therefore \quad A = 1{\cdot}71$$

$$\text{and} \quad C = \frac{0{\cdot}07}{0{\cdot}2 \times 1{\cdot}71} = \underline{0{\cdot}205}$$

Hence, $Q = 1{\cdot}71\,(2 - 0{\cdot}205H)\,H^{\frac{3}{2}}$

When $H = 0{\cdot}25$ m.

$$Q = 1{\cdot}71\,(2 - 0{\cdot}205 \times 0{\cdot}25)\,(0{\cdot}25)^{1{\cdot}5} = \underline{0{\cdot}42 \text{ m}^3/\text{sec.}}$$

Problem-17 : *Find the discharge, using Bazin's formula, for a supressed rectangular weir when the head is 22·5 cm and the length is 1·22 m.*

$$\text{Constant,} \quad m = 0{\cdot}405 + \frac{0{\cdot}00984}{H} = 0{\cdot}405 + \frac{0{\cdot}00984}{0{\cdot}225}$$

$$= 0{\cdot}4437$$

$$\text{Now } Q = m\sqrt{2g}\,LH^{\frac{3}{2}}$$
$$= 0{\cdot}4437 \times 4{\cdot}43 \times 1{\cdot}22 \times (0{\cdot}225)^{1{\cdot}5}$$
$$= 0{\cdot}265 \text{ m}^3/\text{sec or } 265 \text{ litres/sec.}$$

Problem - 18 : *During the test in a laboratory, water which has passed through a venturimeter flows over a right angled V–notch. The head at the notch is registered. The larger diameter of the venturimeter is 25 cm and the diameter of the throat is 10 cm. When water flows through the venturimeter, the difference of pressure heads between the entrance and the throat is found to be 0·6 m of water. The coefficient of the meter is 0·96, while that for the V - notch is 0·6. Determine the reading of the steady head over the V - notch.*

Discharge through the venturimeter is,

$$Q = \frac{C_d A\sqrt{2gH}}{\sqrt{K^2 - 1}}$$

Here, $C_d = 0{\cdot}96$, $A = \frac{\pi}{4}\left(\frac{25}{100}\right)^2 = 0{\cdot}0491 \text{ m}^2$, $K^2 = \left(\frac{d_1}{d_2}\right)^4 = \left(\frac{25}{10}\right)^4 = 36{\cdot}$

$$\therefore \quad Q = \frac{0{\cdot}96 \times 0.491 \times 4{\cdot}43\sqrt{0{\cdot}6}}{\sqrt{36{\cdot}6 - 1}}$$
$$= 0{\cdot}0269 \text{ m}^3/\text{sec.}$$

The discharge over the V-notch is

$$Q = \tfrac{8}{15} C_d \sqrt{2g}\,\tan\frac{\theta}{2}\,H^{\frac{5}{2}}$$

$$\text{or } 0{\cdot}0269 = \tfrac{8}{15} \times 0{\cdot}6 \times 4{\cdot}43 \times 1 \times H^{\frac{5}{2}}$$
$$\therefore \quad H = 0{\cdot}2 \text{ m or } 20 \text{ cm.}$$

If the area of the channel which carries water to the weir is larger than the weir itself, water will have some velocity on reaching the weir and which is known as the velocity of approach. This velocity may be assumed to be uniform over the whole weir. In all the previous cases we have considered the head of water H over the weir, on assumption that water is at rest. Now since the velocity of approach tends to increase the discharge through the notch, some correction is called for in the above mentioned formulae.

Let, A – cross sectional area of channel behind weir,
V_a – velocity of approach, and
Q – discharge over weir in cu. m. per second.

Then, $V_a = \frac{Q}{A}$

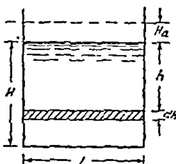


Fig. 6-14

The quantity Q is determined as a first approximation, from the ordinary weir equation, ignoring the velocity of approach. Hence the additional head H_a required to produce this velocity will be $H_a = \frac{V_a^2}{2g}$. Now imagine a small strip of water of thickness dh at depth h below free surface of water.

Area of the strip $= L \times dh$

Velocity of water through the strip $= \sqrt{2gh}$

Head of water at the bottom $= H + H_a$

Head of water at the top $= H_a$

Discharge through the strip, $dQ = C_d \times$ area of the strip $\times$ velocity

$$\text{or } dQ = C_d \times L dh \sqrt{2gh}$$

$$\text{or } \int_0^Q dQ = C_d L \sqrt{2g} \int_{H_a}^{H+H_a} h^{\frac{1}{2}} dh$$

$$\therefore \quad Q = \tfrac{2}{3} C_d L \sqrt{2g} \left[h^{\frac{3}{2}} \right]_{H_a}^{H+H_a}$$

$$= \tfrac{2}{3} C_d L \sqrt{2g} \left[(H + H_a)^{\frac{3}{2}} - H_a^{\frac{3}{2}} \right] \quad .. \text{ (6.8)}$$

As the V_a was obtained only approximately in the first case, it should be corrected to suit the new discharge obtained

from eqn. (6·8). Then with this new discharge new value of velocity of approach is found and then by substituting it in eqn. (6·8), the discharge is calculated. This process is repeated so long as the difference between the two consequtive discharges is very small.

Similarly the expression for finding the discharge through a V-notch taking velocity of approach into account is

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\theta}{2} \left\{ (H + H_a)^{\frac{5}{2}} - (H_a)^{\frac{5}{2}} \right\} \quad ..(6·9)$$

Francis' formula after taking velocity of approach into account becomes,

$$Q = 1·84 \left[L - 0·1nH_1 \right] \times \left[H_1^{\frac{3}{2}} - H_a^{\frac{3}{2}} \right] \quad ..(6·10)$$

where $H_1 = H + H_a$

Bazin after carrying out experiments found that the discharge could be obtained by increasing the actual measured head, H by the amount $H_a = C \frac{V_a^2}{2g}$ where C is a constant having a mean value of 1·6. Bazin's formula after taking velocity of approach into account becomes

$$Q = m \sqrt{2g} L H_1^{\frac{3}{2}} \quad ..(6·11)$$

$$\text{where } H_1 = H + H_a = H + C \frac{V_a^2}{2g}$$

$$\text{and } C = 1·6 \text{ and } m = 0·402 + \frac{0·00984}{H_1}$$

Problem–19 : *Find the discharge over a weir 2·5 m long under a measured head of 60 cm, if the channel approaching the weir is 6 m wide and 1·2 m deep. Accounting for velocity of approach, find the discharge over the weir.*

Discharge, $Q = 1·84 \, (L - 0·1nH) \, H^{\frac{3}{2}}$

$= 1·84 \, (2·5 - 0·1 \times 2 \times 0·6) \, 0·6^{1·5} = 2·04 \text{ m}^3/\text{sec}$

Velocity of approach, $V_a = \frac{Q}{A} = \frac{2.04}{6 \times 1.2} = 0.2835$ m/sec.

Head, $H_1 = H + H_a = 0.6 + \frac{(0.2835)^2}{19.62} = 0.604$ m

$$\therefore \quad Q = 1.84\,(2.5 - 0.1 \times 2 \times 0.604) \times \{ (0.604)^{\frac{3}{2}} - (0.004)^{\frac{3}{2}} \}$$
$$= 2.062 \text{ m}^3/\text{sec}.$$

In this example value of V_a is so small that no second trial is necessary.

Problem – 20 : *The discharge of a rivulet or a stream is being measured by a sharp edged right angled V-notch. The depth of water above the apex of notch is 55 cm. The sill of the notch is 60 cm above the bottom of the approach channel which is 1.5 m wide. The water level 2 m behind the weir is found to be 5 cm above that of the weir. If discharge coefficient is 0.6, determine the discharge of the stream in litres/sec.*

The head above the sill of the notch at a short distance upstream of the weir is

$$H = 55 + 5 = 60 \text{ cm}$$

Depth of water in the approach channel

= height of the weir sill above the bed of the channel + head of water above the sill

= 0.6 + 0.6 = 1.2 m

Area of the approach channel = 1.5 × 1.2 = 1.8 m².

First neglect the velocity of approach and find the discharge. It is

$$Q = \tfrac{8}{15}\, C_d \sqrt{2g} \tan \frac{\theta}{2} H^{\frac{5}{2}}$$

$$= \tfrac{8}{15} \times 0.6 \times 4.43 \times 1 \times 0.6^{2.5} = 0.395 \text{ m}^3/\text{sec}.$$

Velocity of approach, $V_a = \frac{0.395}{1.8} = 0.22$ m/sec.

Equivalent head, $H_a = \frac{0.22 \times 0.22}{19.62} = 0.00252$ m

Now using the formula for discharge having velocity of approach considered,

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}\left[(H+H_a)^{\frac{5}{2}} - H_a^{\frac{5}{2}}\right]$$

$$= \frac{8}{15} \times 0.6 \times 4.43 \times 1 \left[(0.60252)^{2.5} - (0.00252)^{2.5}\right]$$

$$= 0.3975 \text{ m}^3/\text{sec.} = 397.5 \text{ litres/sec.}$$

As the value of V_a is very small, second trial is not necessary.

Problem - 21 : *A stream approaches the water fall having a fall of 30 m. Just before the fall the flow of the stream is measured by a weir built across the stream. The measured head over the sill of the weir is found to be 45 cm and the length of the weir is 2·5 m. The mean velocity of approach is 2 m/sec. Find the volume of water available in the stream and the horse power that could be developed from the fall if 55% of the energy could be utilized.*

Velocity of approach, $V_a = 2$ m/sec.

$$\text{Head}, H_a = \frac{V_a^2}{2g} = \frac{2^2}{2 \times 9.81} = 0.204 \text{ m}$$

$$H = \frac{45}{100} = 0.45 \text{ m}, \ L = 2.5 \text{ m}, \ H_1 = H + H_a = 0.45 + 0.204 = 0.654 \text{ m}$$

$$\text{Discharge, } Q = 1.84\,(L - 0.1nH_1)\left\{H_1^{\frac{3}{2}} - H_a^{\frac{3}{2}}\right\}$$

$$= 1.84\,(2.5 - 0.1 \times 2 \times 0.654)\left\{(0.654)^{1.5} - (0.204)^{1.5}\right\}$$

$$= 1.95 \text{ m}^3/\text{sec.}$$

Energy available from the fall is due to the weight of water falling through a height of 30 m, every second.

$$\text{Energy available} = WH = wQH$$

$$= 1{,}000 \times 1.95 \times 30$$

$$\text{Useful energy per second} = \frac{55}{100} \times 1{,}000 \times 1.95 \times 30$$

$$\text{H.P. available is, } \frac{55}{100} \times \frac{1{,}950 \times 30}{75} = 430$$

Time of Emptying

C. Rectangular weir : During the monsoon if there is heavy rain, there will be rise of water level in the reservoir, as all water will not pass over the weir immediately. It is therefore necessary to know the time in which the water level will again become normal after the fresh income of water due to rain in the reservoir

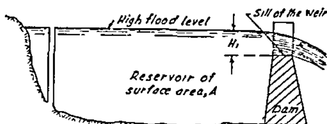


Fig. 6-15

has stopped. In finding the time we consider the reservoir so large that its area practically remains the same at all levels, *i.e.* the area is constant.

Let, A – area of the reservoir,

L – length of the weir,

H_1 – final height of water level over the weir sill,

T – time taken to change the water level from height H_1 to H_2,

h – head above the sill of the weir at any instant,

dT – time taken to change the water level by an amount dh.

Volume of water flowing from the reservoir in time dT is

$$dQ = -Adh \qquad \text{(since } dh \text{ is - ve as level falls)}$$

The discharge over the weir in time dT is

$$dQ = \tfrac{2}{3}\, C_d L \sqrt{2g}\;\; h^{\frac{3}{2}} dT$$

Hence, $$-Adh = \tfrac{2}{3}\, C_d L \sqrt{2g}\;\; h^{\frac{3}{2}} dT$$

$$\therefore \quad dT = \frac{-Adh}{\frac{2}{3} C_d L \sqrt{2g}\, h^{\frac{3}{2}}}$$

$$\therefore \quad \int_0^T dT = \frac{-A}{\frac{2}{3} C_d L \sqrt{2g}} \int_{H_1}^{H_2} h^{-\frac{3}{2}}\, dh$$

$$\text{or} \quad T = \frac{+2A}{\frac{2}{3} C_d L \sqrt{2g}} \left[h^{-\frac{1}{2}} \right]_{H_1}^{H_2}$$

$$= \frac{2A}{\frac{2}{3} C_d L \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right] \quad ..(6{\cdot}9)$$

If the end contraction effect is considered, the Francis formula should be used and the equation for finding the time assumes the form

$$T = \frac{2A}{1{\cdot}84\,(L - 0{\cdot}1nH)} \left\{ \frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right\} \quad ..(6{\cdot}10a)$$

where $H = \dfrac{H_1 + H_2}{2}$

Similarly, using Bazin's formula, the equation for time becomes

$$T = \frac{2A}{m\sqrt{2g}\,L} \left\{ \frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right\} \quad ..(6{\cdot}10b)$$

where $m = 0{\cdot}405 + \dfrac{0{\cdot}00984}{H}$ and $H = \dfrac{H_1 + H_2}{2}$

V-Notch : Consider that at a particular instant the head of water above the sill of the V-notch is h, and it reduces by an amount dh in a small interval of time dT allowing a small quantity of water dQ to flow out from the reservoir having an area A. Then from eqn. (6·3), we have,

$$dQ = \frac{8}{15} C_d \sqrt{2g}\ \tan\frac{\theta}{2} h^{\frac{5}{2}}\, dT$$

And $dQ = -Adh$ (as dh is – ve due to fall in level)

$$\therefore -Adh = \frac{8}{15} C_d \sqrt{2g}\ \tan\frac{\theta}{2}\ h^{\frac{5}{2}}\, dT$$

$$\therefore \quad dT = \frac{-A\,dh}{\frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \times h^{\frac{5}{2}}}$$

$$\int_0^T dT = \frac{-A}{\frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}} \int_{H_1}^{H_2} h^{-\frac{5}{2}}\, dh$$

$$\therefore \quad T = \frac{+\frac{2}{3}A}{\frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2}} \left[h^{-\frac{3}{2}} \right]_{H_1}^{H_2}$$

$$= \frac{1{\cdot}25}{C_d \sqrt{2g} \tan\frac{\theta}{2}} \left[\frac{1}{H_2^{\frac{3}{2}}} - \frac{1}{H_1^{\frac{3}{2}}} \right] \qquad ..(6{\cdot}11)$$

If angle of the notch is 90°, we have,

$$T = \frac{1{\cdot}25A}{C_d \sqrt{2g}} \left[\frac{1}{H_2^{\frac{3}{2}}} - \frac{1}{H_1^{\frac{3}{2}}} \right] \qquad ..(6{\cdot}11a)$$

Problem - 22 : *A pond having a water spread of 8×10^6 m^2 is provided with a narrow crested weir 10 m long which discharges with a head of 1 m on its crest. If head is to be lowered to 0·5 m, find the time taken. Take $C_d = 0{\cdot}62$.*

Area, $A = 8 \times 10^6$ m^2, $L = 10$ m, $H_1 = 1$ m, $H_2 = 0{\cdot}5$ m

$$\text{Time, } T = \frac{2A}{\frac{2}{3} C_d L \sqrt{2g}} \left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}} \right]$$

$$= \frac{2 \times 8 \times 10^6}{\frac{2}{3} \times 0{\cdot}62 \times 10 \sqrt{2 \times 9{\cdot}81}} \left[\frac{1}{\sqrt{0{\cdot}5}} - \frac{1}{\sqrt{1}} \right]$$

= 3,61,700 sec. or 100 hr. 28 min. 20 sec.

Problem - 23 : *A rectangular channel is 125 m long, and 3 m wide. The depth of water in the channel is 2 m. A right angled triangular notch is fitted at one end of the channel with apex 0·5 m above the bed of the channel. Find the time taken to change the water level by 1 m. Neglect velocity of approach and take C_d = 0·62*

Area, $A = 125 \times 3 = 375\ m^2$, $H_1 = 2 - 0.5 = 1.5$ m, $H_2 = 1.5 - 1 = 0.5$ m

$$\text{Time, } T = \frac{1.25A}{C_d \sqrt{2g}\ \tan\frac{\theta}{2}} \left[\frac{1}{H_2^{\frac{3}{2}}} - \frac{1}{H_1^{\frac{3}{2}}}\right]$$

$$= \frac{1.25 \times 375}{0.62 \times \sqrt{2 \times 9.81} \times 1} \left[\frac{1}{0.5^{\frac{3}{2}}} - \frac{1}{1.5^{\frac{3}{2}}}\right]$$

$$= 392 \text{ sec or } 6 \text{ min. } 32 \text{ sec}$$

Broad-crested Weir

As discussed earlier, a weir having a thick sill is known as a broad-crested weir. In this case, the crest has a thickness more than half the head of water on the weir. On the thick crest, the flow has the pattern shown in fig. 6–16. The head first falls sharply and then the streamlines become parallel to the crest and the velocity becomes uniform in the stream. The velocity over the crest is due to the fall in head by an amount H.

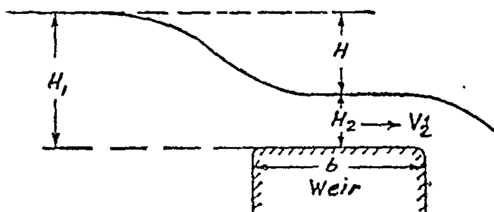


Fig. 6–16 Broad-crested weir (end view)

$$\therefore \quad V_2 = \sqrt{2gH} \text{ where } H = H_1 - H_2$$

The discharge over the weir is

$$Q = C_d \times \text{area of section} \times \text{velocity}$$

$$= C_d L H_2 \sqrt{2gH}$$

$$= C_d L(H_1 - H) \sqrt{2gH} \qquad \text{.. (6.12)}$$

It will be reasonable to assume that water which passes over the weir will adjust the flow in such a manner that the depth and velocity across the weir will give maximum discharge for a constant head H_1 on the upstream side of the weir. For maximum value of discharge, we differentiate Q w.r.t. H (eqn. 6–12) and equate to zero.

$$i.e. \frac{dQ}{dH} = C_d L \sqrt{2g} \left[\frac{H_1}{2\sqrt{H}} - \frac{3}{2}\sqrt{H}\right] = 0$$

$$\frac{H_1}{2\sqrt{H}} = \frac{3}{2}\sqrt{H}$$

$$\therefore \quad H_1 = 3H$$

$$\text{or } H = \frac{H_1}{3}$$

$$\therefore \quad H_2 = \frac{2}{3} H_1$$

$$\text{Hence, } Q_{max} = C_d L \times \sqrt{2g} \times \frac{2}{3} H_1 \sqrt{\frac{H_1}{3}}$$

$$= 1{\cdot}71 \ C_d L H_1^{\frac{3}{2}} \qquad \text{.. (6·12a)}$$

Problem-24 : *Find the maximum discharge over 20 m long broad-crested weir when the head is 1·5 m. Take $C_d = 0{\cdot}61$.*

$Q_{max} = 1{\cdot}71 \ C_d L H_1^{\frac{3}{2}}$

$= 1{\cdot}71 \times 0{\cdot}61 \times 20 \times 1{\cdot}5^{\frac{3}{2}} =$ 38·2 m³/sec. or 38,200 litres/sec.

Problem-25 : *A flat topped and broad-crested weir has length of 22 m. The head causing the flow is 0·2 m. Calculate the discharge passing over the weir when it is a suppressed weir and $C_d = 0{\cdot}8$.*

Velocity head, $H = H_1 - H_2 = H_1 - \frac{2}{3} H_1 = \frac{H_1}{3}$

$\therefore$ Head, $H_1 = 3 \times$ velocity head $= 3 \times 0{\cdot}2 = 0{\cdot}6$ m

Discharge, $Q = 1{\cdot}71 \ C_d \ L{\cdot}H_1^{\frac{3}{2}}$

$= 1{\cdot}71 \times 0{\cdot}8 \times 22 \ (0{\cdot}6)^{1{\cdot}5} =$ 14 m³/sec.

Submerged Weir

If the tail water level is above the sill or crest of the weir as shown in fig. 6-17, it is said to be the drowned or submerged

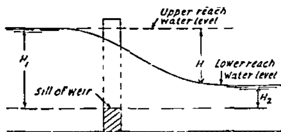


Fig. 6-17 Submerged Weir

weir. The discharge over this weir is found by dividing it into two

horizontal sections such as free portion of the weir and the submerged portion of the weir.

The discharge through the frer portion of the weir is (ref. eqn. 6·2)

$$Q_1 = \frac{2}{3} C_{d1} L \sqrt{2g}\, H^{\frac{3}{2}}$$

The discharge through the submerged portion is

$$Q_2 = C_{d2} L H_2 \sqrt{2gH}$$

Total discharge, $Q = Q_1 + Q_2$

$$\therefore \quad Q = \frac{2}{3} C_{d1} L \sqrt{2g} \cdot H^{\frac{3}{2}} + C_{d2} L H_2 \sqrt{2gH} \qquad .. (6·13)$$

When velocity of approach is taken in account the formula becomes,

$$Q_1 = \frac{2}{3} C_{d1} L \sqrt{2g} \left\{ (H + H_a)^{\frac{3}{2}} - H_a^{\frac{3}{2}} \right\}$$

and $Q_2 = C_{d2} L H_2 \sqrt{2g(H + H_a)}$

Then, the total discharge, $Q = Q_1 + Q_2$

$$= \frac{2}{3} C_{d1} L \sqrt{2g} \left\{ (H + H_a)^{\frac{3}{2}} - H_a^{\frac{3}{2}} \right\} + C_{d2} L H_2 \sqrt{2g(H + H_a)}$$

Problem – 26 : *A weir at the edge of a pond is 3 m wide. The sill of weir is 0·5 m below the water level in the pond and 0·25 m below the tail water level. Compute the discharge over the weir if $C_d = 0·6$.*

Here, $L = 3$ m, $H = 0·5 - 0·25 = 0·25$ m, $H_2 = 0·25$ m

$C_{d1} = C_{d2} = 0·6$

$$Q_1 = \frac{2}{3} C_{d1} L \sqrt{2g}\, H^{\frac{3}{2}}$$

$$= \frac{2}{3} \times 0·6 \times 3 \times 4·43 \times (0·25)^{\frac{3}{2}} = 0·665 \text{ m}^3/\text{sec.}$$

and $Q_2 = C_{d2} L H_2 \times \sqrt{2gH}$

$$= 0·6 \times 3 \times 0·25 \times 4·43 \sqrt{0·25} = 0·995 \text{ m}^3/\text{sec.}$$

Hence, $Q = Q_1 + Q_2 = 0·665 + 0·995 =$ 1·66 m³/sec.

Problem – 27 : *The upper and lower reach water surfaces are 2·5 m and 1 m respectively above their respective bed levels. A submerged weir 30 m long has its crest 60 cm and 30 cm above the beds respectively on the upper and lower reach sides. Calculate the discharge if the mean velocity of approach is 1·5 m/sec and C_{d1} and C_{d2} are 0·6 and 0·65 respectively.*

Here, $L = 30$ m, $H_1 = 2{\cdot}5 - 0{\cdot}6 = 1{\cdot}9$ m, $H_2 = 1 - 0{\cdot}3 = 0{\cdot}7$ m

$$H = H_1 - H_2 = 1{\cdot}9 - 0{\cdot}7 = 1{\cdot}2 \text{ m}, \; H_a = \frac{1{\cdot}5 \times 1{\cdot}5}{19{\cdot}62} = 0{\cdot}115 \text{ m}$$

$$Q_1 = \tfrac{2}{3}\, C_{d1}\, L \sqrt{2g}\, \{ (H + H_a)^{\frac{3}{2}} - H_a^{\frac{3}{2}} \}$$

$$= \tfrac{2}{3} \times 0{\cdot}6 \times 30 \times 4{\cdot}43 \; \{ 1{\cdot}315)^{\frac{3}{2}} - (0{\cdot}115)^{\frac{3}{2}} \}$$

$$= 79{\cdot}6 \text{ m}^3/\text{sec}.$$

$$Q_2 = C_{d2}\, L{\cdot}H_2 \sqrt{2g\,(H + H_a)}$$

$$= 0{\cdot}65 \times 30 \times 0{\cdot}7 \times 4{\cdot}43 \sqrt{1{\cdot}315} = 46{\cdot}4 \text{ m}^3/\text{sec}.$$

Hence $Q = Q_1 + Q_2 = 79{\cdot}6 + 46{\cdot}4 = \underline{126 \text{ m}^3/\text{sec.}}$

Problem-28 : *Show that a correction factor to allow for the velocity of approach for a rectangular notch can be expressed in the form* $\left[\left(1 + \frac{\alpha h_1}{H} \right)^{3/2} - \left(\frac{\alpha h_1}{H} \right)^{3/2} \right]$ *where* H *is the measured head,* h_1 *is equal to* $\frac{V_a^2}{2g}$ *(where* V_a *is the velocity of approach) and* α *is the factor to allow for the velocity not being uniform over the section of the channel.*

Let h be the depth at which a horizontal strip of water of thickness dh is considered as shown in fig 6-4. The theoretical velocity of water flowing through the strip is $\sqrt{2gh}$. Discharge through the strip, $dQ = C_d \times$ *area of strip* $\times$ *velocity*

$$= C_d \times L \times dh \sqrt{2gh}$$

$$\text{Velocity head} = \alpha \frac{V_a^2}{2g} = \alpha h_1$$

$$\therefore \quad \text{total effective head} = H + \alpha h_1$$

$\therefore$ total discharge through the rectangular weir considering effects of velocity approach,

$$Q_1 = \int_{\alpha h_1}^{H+\alpha h_1} C_d \cdot L \cdot dh \sqrt{2gh}$$

$$= \tfrac{2}{3} C_d L \sqrt{2g} \left[h^{\frac{3}{2}} \right]_{\alpha h_1}^{H+\alpha h_1}$$

$$= \tfrac{2}{3} C_d \cdot L \sqrt{2g} \left[(H + \alpha h_1)^{\frac{3}{2}} - (\alpha h_1)^{\frac{3}{2}} \right]$$

If the velocity of approach is neglected, then discharge through rectangular notch, $Q = \frac{2}{3} C_d L \sqrt{2g}\, H^{\frac{3}{2}}$.

Now correction factor which is the ratio of discharge with effect of velocity of approach to the dischare neglecting the effect of velocity of approch is

$$\text{correction factor} = \frac{Q_1}{Q} = \frac{\frac{2}{3} C_d\, L \sqrt{2g} \left[(H + \alpha h_1)^{\frac{3}{2}} - (\alpha h_1)^{\frac{3}{2}} \right]}{\frac{2}{3} C_d\, L \sqrt{2g}\, H^{\frac{3}{2}}}$$

$$= \left[\left(1 + \frac{\alpha h_1}{H}\right)^{\frac{3}{2}} - \left(\frac{\alpha h_1}{H}\right)^{\frac{3}{2}} \right]$$

Syphon

Now that use of weir *viz.* keeping the water level in reservoirs at constant height, is well known to us, a better method of achieving

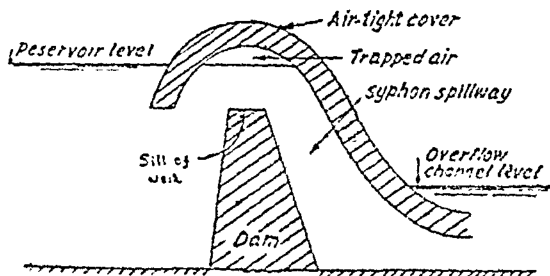


Fig. 6-18 Syphon spillway

the said goal can be thought of. It happens at times that due to heavy rains a great bulk of fresh water comes into the reservoir and

this should be discharged quickly. Since the velocity of water is dependent upon the head of water over the sill of the weir and as the fresh water is spread over a large area of the reservoir, it takes a very long time to reject all this water because the head is too small. A greater head can be made use of by constructing the syphon spillway in the dam

2. A rectangular notch 4 m wide has constant head of H m. If the discharge through the notch is 1,000 litres/sec, determine the value of H. Take the coefficient of discharge of 0·64.

[26 cm]

3. A rectangular notch is to be made to discharge 300 litres of water per sec. with a head over the sill equal to half the width of the notch. Neglecting velocity of approach and end contractions, determine the width of the notch and the head of water above the sill. Take $C_d = 0{\cdot}6$.

[74·5 cm ; 37·25 cm]

4. A weir 8 m long is to be built across a rectangular channel to discharge 10,000 litres/sec. If maximum depth of water on the upstream side of the weir is to be 2 m what should be the height of weir ? Neglect end contractions. Take $C_d = 0{\cdot}62$.

[1·008 m]

5. A weir is to be built across a rectangular channel to discharge 12,000 litres/sec. of water. The height of the weir above the bed of the channel is 1·5 m. The maximum depth of water on the upstream side is 3·5 m. Neglecting end contractions, determine the width of the weir. Take discharge coefficient as 0·62.

[2·31 m]

6. A rectangular channel 2·5 m wide carries a maximum discharge of 500 litres/sec. and a minimum of 300 litres/sec. It discharges over a rectangular weir 2 m wide. At what level from the channel bed, should the notch be placed to give the maximum depth of 1·5 m for the channel ? What is then the minimum depth of water in the channel ? Assume value of 0·6 for discharge coefficient. Neglect velocity of appoach and end contraction.

[1·231 m; 1·426 m]

7. A weir 8 m long is to be built across a rectangular channel to discharge 9 m^3/sec. If the maximum depth of water on the upstream side of the weir is to be 2·5 m, what should be the height of the weir ? Neglect end contraction. Coefficient of discharge be taken as 0·62.

[5·276 m]

V-notch

8. Water flows through a V-notch having an angle of 120° to a depth of 1 metre, and afterwards passes over a rectangular notch 2·5 m wide. Taking coefficient of discharge as 0·62 for both notches, find the head of water over the sill of the rectangular notch.
[0·67 m]

9. During the test in a laboratory, water which has passed through a venturimeter flows over a right angled V-notch, the head at the notch being registered. The larger diameter of the venturimeter is 30 cm and the diameter at the throat is 15 cm. The coefficient of the meter is unity while that for the V-notch is 0·52. If the difference of pressure heads between the entrance and throat registered at the venturimeter is three times the steady head over the sill of the V-notch, determine the discharge and the head over the V-notch.
[78 litres/sec., 30·9 cm]

10. Water is fed into the channel 2 m wide by an orifice 20 cm in diameter and discharging under a constant head of 4 m. A right angle V-notch is built across the channel such that its apex is 2 m above the bed of the channel. Determine the head of flow over the weir and depth of water in channel just upstream of the weir. Coefficient of discharge for the orifice = 0·62; coefficient of discharge for the V-notch = 0·64. Neglect velocity of approach.
[0·422 m; 2·422 m]

11. Find the depth and top width of a triangular notch capable of discharging maximum quantity of 4·5 m^3/sec and such that the head shall be 7·5 cm when the discharge is 0·0054 m^3/sec. For a right angled notch take C = 1·47.
[1 m; 6·12 m]

Rectangular and V-notch

12. Water flows over a rectangular weir 1·5 m wide to a depth of 20 cm and afterwards passes over a right angled V-notch. Taking coefficients of discharge for the notches as 0·62 and 0·6 respectively, find the depth of water over the V-notch.
[49·34 cm]

13. Water flows over a rectangular weir 3·5 m wide and afterwards passes over a V-notch having an angle of the notch 120°. The coefficient of discharge for the rectangular notch is 0·62 while that for the V-notch is 0·6. If the head of water above the sill of the rectangular notch is $\frac{1}{20}$th of its width, determine the head of water above the sill of the V-notch.

[51·46 cm]

14. A trapezoidal weir is 150 cm wide at the top and 60 cm wide at the bottom, the perpendicular distance between the top and bottom being 45 cm. If the discharge over the weir is 300 litres/sec, determine the head of water above the sill of weir. Take coefficient of discharge for the weir as 0·62.

[35 cm]

15. A trapezoidal weir is 1 m wide at the top and 30 cm wide at the bottom, the perpendicular distance between the top and bottom being 50 cm. If the head of water above the sill of the weir is 40 cm and the coefficient of discharge for the weir is 0·62, determine the discharge over this weir in litres per second.

[948 litres/sec]

16. A stepped notch is 40 cm wide from the bottom upto 10 cm, then changes suddenly to 60 cm for 20 cm and then changes suddenly to 1 m for 40 cm. Taking the coefficient of

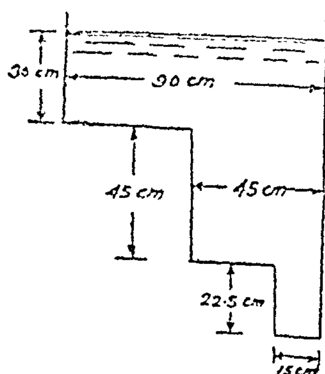


Fig. 6-19

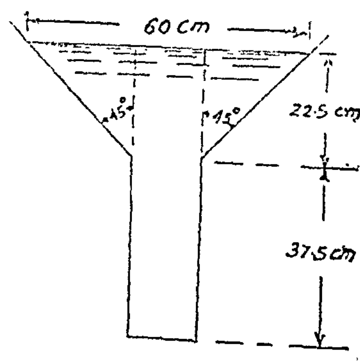


Fig. 6-20

discharge as 0·62 for all portions of the notch, find the discharges through the notch. The surface level of water is at the top of the notch.

[785 litres/sec.]

17. Find the quantity of water flowing through the stepped notch shown in fig. 6-19 assuming the coefficient of discharge 0·64 for all portions of the notch. The level of water coincides with the top of the notch.

[712 litres/sec.]

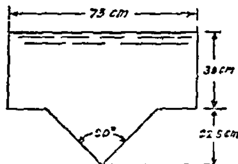


Fig 6-21

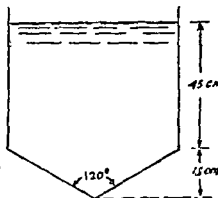


Fig. 6-22

18 A stepped rectangular notch is to be designed to discharge 500 litres/sec of water such that the maximum depth of water is to be 90 cm The height of successive steps from top to bottom is to be increased by 6 cm and the width is to be decreased by 20 cm. The level of water coincides with the top of the notch. Take coefficient of discharge as 0 62 for all portions of the notch.

[*Top width - 59·5 cm, top height - 24 cm,* middle width - 39 5 cm, middle height - 30 cm, bottom width - 19 5 cm, bottom height - 36 cm]

19. Find the discharge through the notch shown in fig. 6-20. Coefficient of discharge is 0·6 for all portions.

[156·7 litres/sec.]

20. Find the discharge through the notch shown in fig. 6–21. Take discharge coefficient as 0·6 for all portions.

[301·4 litres/sec.]

21. Find the discharge through a triangular notch shown in fig. 6–22. Take coefficient of discharge as 0·62 for all portions.

[362 litres/sec.]

Francis' and Bazin's Formulae

22. A weir is 300 m long. Water passes over the weir under a head of 1·15 m. Determine the discharge by using (a) Bazin's formula, and (b) Francis' formula.

[(a) 677 m^3/sec. (b) 680 m^3/sec]

23. A weir 10 m long is divided into four parts by vertical wooden posts, each 30 cm wide. Find the discharge when the head is 1 m.

[15·1 m^3/sec]

24. Find the discharge, using Bazin's formula, for a suppressed rectangular weir when the head is 30 cm and the length is 2 m.

[650 litres/sec]

25. A rectangular weir is designed to discharge 18,000 litres of water per minute with a head over the sill of $\frac{3}{4}$th width of the notch. Neglecting the velocity of approach and allowing for the end contractions, determine the width of the notch and the head of water above the sill.

[0·616 m; 0·462 m]

26 A sharp-crested rectangular contracted weir 1·25 m long is provided in the channel. The channel has a flow of 950 litres/sec. Determine the head of water above the sill of the weir. Use Francis' formula.

[0·6 m]

27 A reservoir has a catchment area of 25×10^6 m^2. The rainfall over the catchment area is 7·5 cm in 24 hours; and 45%

of the rain water reaches the reservoir. If the length of the weir is 6 m, determine the head of water above the sill of the weir.

[97 cm]

28 A reservoir has catchment area of 75×10^6 m². What length of weir will it require to carry away a rainfall of 10 cm in 24 hours, 60% of which reaches the tank with a head of 1·5 m over the weir?

[16·1 m]

29 The catchment area of a tank is 32×10^6 m². The maximum rainfall in the catchment area is 5 cm per hour. Find the length of the waste weir necessary if 40% of rainfall reaches the tank and the head of water over the sill of the weir is not to exceed 1 m. The waste weir has to carry piers 0·5 m wide, 2 m apart for supporting a footway.

[8 m]

30 The maximum flood discharge of 850 m³/sec. passes over a weir built across a river. A road bridge is provided with 0·8 m wide pillars over the weir which is divided into a number of openings each of 4·5 m span. Find the number of openings required; if the head over the crest is not to exceed 1·5 m. Neglect the velocity of approach and use Francis' formula. What is the total length of the bridge?

[58; 308·2 m]

Velocity of approach

31 Find the discharge over a weir 3 m long under a measured head of 1 m if the channel approaching the weir is 4·5 m wide and 1·5 m deep. Account for the velocity of approach.

[520 5 litres/sec]

32 A stream approaches a waterfall having a fall of 60 m. Just before the fall the flow of the stream is measured by a weir built across the stream. The measured head over the sill of the weir is found to be 0·3 m and the length of the weir is 3·5 m. The

mean velocity of approach is 1·5 m/sec. Find the volume of water available in the stream and the horse-power that could be developed from the fall if 50% of the energy could be utilised.

[1,368 litres/sec. 547 H. P.]

33 The discharge of a rivulet is being measured by a sharp edged right angled V-notch. The depth of water above the apex of the notch is 35 cm. The sill of the notch is 45 cm above the bottom of the approach channel which is 2 m wide. The water level 1·5 m behind the weir is found to be 2·5 cm above water level on the weir. If discharge coefficient is 0·62, determine the discharge of the stream in litres/sec.

[131·3 litres/sec.]

34 Derive an expression for discharge over a rectangular weir taking into account the velocity of approach. What will be the effect of lateral contractions ?

Find the discharge which will flow over a narrow crested weir 4 m long when the measured head over the sill of the weir is found to be 1 m. The approach channel is 4·25 m wide and the sill of the weir is 15 cm above the bed of the channel.

[7,780 litres/sec.]

35 Show that a correction factor to allow for the velocity of approach for a rectangular notch can be expressed in the form $\left[\left(1+\frac{\alpha h_1}{H}\right)^{\frac{3}{2}}-\left(\frac{\alpha h_1}{H}\right)^{\frac{3}{2}}\right]$ where H is the measured head, h_1 is equal to $\frac{V_a^2}{2g}$ (where V_a is the velocity of approach) and α is a factor to allow for the velocity not being uniform over the section of the channel.

A rectangular channel 2 m wide has at the end a rectangular sharp-edged notch 1·22 m wide, with the sill 15 cm from the bottom. Assuming $\alpha = 1{\cdot}1$, calculate the flow when the head is 30 cm using Francis' formula.

[366 litres/sec]

Time of emptying vessels fitted with notches

36. A reservoir has an area of 100,000 m^2 and is provided with a weir 6 m long. Find how long it will take for the level of water above the sill to fall from 2 m to 1 m. Deduce the formula you use and state the assumptions made. Take $C_d = 0{\cdot}62$.
[9 min. 25 sec]

37. A small pond has an area of 300 m^2. Supposing no water enters the pond, find the time taken to lower the water level above the vertex of the right-angled V-notch from 1 m to 0·5 m. Take $C_d = 0{\cdot}62$.
[4 min. 8 sec]

38. A pond having a water spread of $1{\cdot}67 \times 10^6$ m^2 is provided with a narrow crested weir 6 m long which discharges with a head 1·22 m on its crest If head is to be lowered to 90 cm find the time taken Take $C_d = 0{\cdot}62$.
[12 hr. 34 min. 56 sec]

39. A rectangular channel is 100 m long and 3 m wide. The depth of water in the channel is 2·25 m. A right-angled triangular notch is fitted at one end of the channel with apex 0·75 m above the bed of the channel. Find the time taken to change the water level by 1 m. Neglect velocity of approach and take $C_d = 0{\cdot}6$.
[5 min. 23 sec.]

Broad-crested weirs

40. Deduce an expression for the rate of discharge over a broad-crested weir.

Find the maximum discharge over 25 m broad crested weir when the head is 80 cm. Take $C_d = 0{\cdot}61$.
[18,650 litres/sec]

41. Find the maximum discharge over 20 m long broad-crested weir when the head is 95 cm. $C_d = 0{\cdot}62$.
[20,600 litres/sec]

42. A flat topped and broad-crested weir has length of 10 m. The head causing the flow is 0·5 m. Calculate the discharge passing over the weir when it is suppressed weir and $C_d = 0{\cdot}8$.
[25·18 m^3/sec]

Submerged weir

43. A weir at the edge of a reservoir has its sill 2 m below the water level in the reservoir and 60 cm below the tail water

level. If water flows over the weir at the rate of 42·5 m³/sec, determine the length of the weir to be constructed. Take $C_{d1} = 0{\cdot}6$ and $C_{d2} = 0{\cdot}64$.

[8·63 m]

44. The upper and lower reach water surfaces of a reservoir are 3 m and 0·6 m respectively above their bed levels. A submerged weir 12 m long has its crest 1 m and 15 cm above the beds respectively on the upper and lower reach sides. Calculate the discharge if the mean velocity of approach is 1·25 m/sec. and C_{d1} and C_{d2} are 0·62 and 0·64 respectively.

[64,580 litres/sec]

45. A weir at the edge of a pond is 4 m wide. The sill of the weir is 1 m below the water level in the pond and 0·5 m below the tail water level. Compute the discharge over the weir if $C_d = 0{\cdot}62$.

[6·48 m³/sec]

Miscellaneous

46. What length of Cippoletti weir is required for flow of 0·58 m³/sec., if the maximum head is limited to 0·25 m. The constant of the weir is 1·822.

[2·55 m]

47. What length of Cippoletti weir is required for flow of 850 litres/sec. if the maximum head is limited to $\frac{1}{12}$th of its length.

[3·73 m]

48. A rectangular notch 15 cm wide discharges 25 litres/sec. The coefficient of discharge for the notch is 0·6. Using Thomson's principle of geometric similarity, work out the head and length of the notch necessary to make the discharge three times.

[32 cm, 23·3 cm]

49. In measuring a discharge of 70 litres/sec. of water a rectangular notch 50 cm wide and then a right angled V–notch are used. If in measuring the head of water over the weir an error of $\frac{1}{20}$ cm is made in each case, determine the percentage error in finding out the discharge. Take discharge coefficient as 0·62 for each notch.

[0·23%; 0·292%; ratio of error = 0·785]

7
FLOW THROUGH PIPES

Introduction

In the previous chapters we have studied the different methods for measurement of fluid flow. When fluid flows, it flows through some media. The fluid flows from one place to another in an open channel or in a closed conduit which is known as pipe. In the latter case the flow may be under pressure while in the former case it will be at atmospheric pressure as it is open to atmosphere.

In practice as we think about the flow in pipes we find that water flows from the reservoir to the town in every water works project. Oil and gas are also carried through pipes to their places of use. In this chapter we shall discuss the flow of water through pipe lines.

Fluid Friction

When a body is moved through the fluid or the fluid moves over the surface of a body, the resistance to motion betewen the body and fluid comes into play which is known as *fluid friction.*

We have seen that the flow may be steady streamline or turbulent. If the velocity of the fluid changes from lower to higher value, then the flow changes from steady *streamline flow* to *unsteady turbulent flow.* Hence in between these two stages there will be one stage at which the flow changes from one type to another one and the velocity of flow at this stage is known as *critical velocity.*

When the fluid flows through a pipe, resistance is set up which opposes its motion. In overcoming this resistance to the motion,

some portion of energy is lost to the system and this loss of energy is known as frictional loss.

In case of horizontal pipe lines the loss of energy takes place at the expense of pressure energy, while in case of inclined pipe lines it takes place at the expense of both pressure energy and potential energy. Thus the source of energy loss depends on the layout of the pipe lines. The loss will take place according to certain causes which finally depend on the type of flow of water in the pipe line. The laws were determined by a series of experiments carried out by eminent persons in the field.

Laws of fluid friction : These laws depend upon the type of flow of water, *i. e.* whether steady streamline flow or unsteady turbulent flow is present. For steady streamline flow the frictional laws are as under :

(1) Friction is proportional to the velocity of fluid in the media.

(2) It is independant of pressure under which the fluid flows.

(3) It is proportional to the wetted surface.

(4) It varies greatly with the temperature.

(5) It does not depend upon the nature of the surface in contact.

For the steady streamline flow as the velocity of the fluid is below the critical velocity a film of stationary liquid will be formed over the wetted surface and hence there will be resistance due to viscosity. It therefore can be stated as *viscous flow.*

If the flow velocity is beyond the critical velocity the flow is turbulent and the flow is subjected to the following frictional laws.

(1) Friction is proportional to the square of the velocity.

(2) It is proportional to the density of the fluid.

(3) It is proportional to the area of surface in contact.

(4) It is independent of pressure under which the flow takes place.

(5) It is dependent on the wetted surface.

(6) It varies only slightly with the temperature.

Froude's experiment : William Froude carried out a series of experiments to determine the resistance due to motion of a body in some media. He used a 90 m long basin containing water in which thin wooden boards were towed endwise. The planks were connected to a carriage running on rails outside. Boards of lengths varying from 25 cm to 15 m were used and their surfaces being covered with varnish, sand, tinfoil etc in turn. The force required to tow the boards at various speeds was measured by means of dynamometer fitted on it. He concluded that:

(a) Resistance to the motion of body is proportional to V^n where V is velocity of the body and n, the index power;

(b) Resistance varies with the surface roughness Rougher the surface greater is the resistance. For smooth surfaces he found the value of n to be about 1.85; while for the very rough surfaces the power reached the value of 2.

(c) Total resistance increases with the length but the mean resistance per square metre decreases as the length increases.

Total resistance, $R = f A V^n$ for any surface ..(7·1)

and $R = f A V^2$ for very rough surface ..(71a)

where f is the mean resistance in kg/m² of surface, A is surface area and V is velocity of the body.

Reynolds' experiments Osborne Reynolds demonstrated that the flow of a real fluid may be either of the two types One may be laminar flow while the other one turbulent flow. He carried out experiments on different parameters to determine the type of flow. The apparatus used by him was similar to the one shown in fig. 7-1.

The apparatus consists of a supply tank which stores water, It is fitted with an arrangement to fit different diameters of bell mouthed glass tubes, one being shown in fig. 7-1(a). There is also a *regulating valve on the glass tube to regulate the flow of water.*

A small tank is provided to carry the dye stuff which is injecting fine stream of dye into the glass tube at its centre. There is also a measuring tank.

Water is allowed to pass through the tube and knowing the discharge its velocity can be calculated. He allowed to increase the flow and hence the velocity. So long as the velocity was below the critical velocity the dye injected was appearing to be a fine straight line stream. Such type of flow was called laminar flow. When the velocity was more than the critical velocity, the dye filament became wavered and got diffused with water as shown in fig. 7-1 (b).

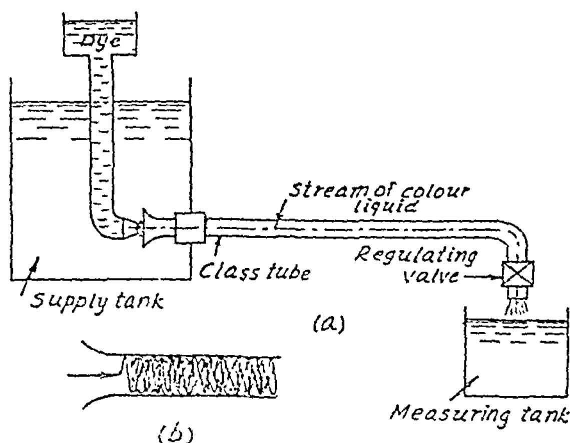


Fig. 7-1 Reynolds' apparatus

This type of flow is known as the turbulent flow. He carried out an experiment further by reducing the velocity instead of increasing and he got different truth. The velocity at which the flow changed changed from laminar to turbulent was different than that when the flow from turbulent to laminar. Thus there are two critical velocities, *viz.* (i) upper critical velocity, (ii) lower critical velocity. Upper critical velocity is that velocity at which the flow changes from laminar to turbulent, while the lower critical velocity is that velocity at which the flow changes from turbulent to laminar.

Reynolds' number: Reynolds from his experiments discovered that the type of flow depends not only on velocity but on other

factors such as diameter of the tube, viscosity of the fluid and specific mass. He found out a dimensionless term which is known as Reynolds' number in his honour. This is as under :

$$R_e = \frac{Vd}{\nu} \quad ..(7.2)$$

where, R_e – abstract number known as Reynolds' number,

V – mean velocity of flow in a pipe, m/sec.,

d – diameter of circular pipe, m,

ν – kinematic viscosity of liquid,

$= \frac{\mu}{\rho}$ in which μ is the absolute viscosity of liquid and ρ is the mass density of liquid.

Reynolds found that if R_e is less than 2,000 the flow in the pipe is laminar or viscous and if R_e is greater than 4,000 the flow is turbulent. For the value of R_e from 2,000 to 4,000 the flow is unpredictable. The mean velocity, for the value of $R_e = 2{,}000$ is called lower critical velocity, while for the value of $R_e = 4{,}000$ the velocity is called higher critical velocity.

Hydraulic Gradient of Pipe

Water flows from a tank A to the tank B through a pipe line shown in fig. 7-2. The tank is at a level Z_A from datum level and contains water under head H_A above the centre of the pipe line. Consider two sections of the pipe line 1-1 and 2-2 such that

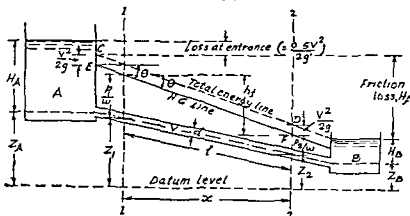


Fig. 7-2 Hydraulic gradient

the length of the pipe line between the sections is l, and the loss of head between the sections h_f. As water enters the pipe line there will be a loss of head at entrance equal to $\frac{0.5\,V^2}{2g}$. Hence the total energy line, CD will be a dotted line joining the two points C & D as shown in fig. 7-2. Now if we draw another line EF at a distance equal to $\frac{V^2}{2g}$, we get another line EF known as hydraulic gradient line.

Let, Z_1 – centre of the pipe line above datum at section 1-1,

Z_2 – centre of the pipe line above datum at section 2-2,

$\frac{p_1}{w}$ – pressure head at section 1-1,

$\frac{p_2}{w}$ – pressure head at section 2-2.

If we mark ordinates $\frac{p_1}{w}$ at section 1-1 and $\frac{p_2}{w}$ at section 2-2 above the centre of the pipe line, and join the tops of these ordinates, we get a line called the pressure head line or hydraulic gradient line, it will coincide with the hydraulic gradient. Thus the ordinate between the hydraulic gradient line and the pipe centre line shows the pressure head at that point.

The vertical component of hydraulic gradient denotes the loss of head h_f due to friction in the pipe line of length l. Let θ be the inclination of hydraulic gradient to the horizontal.

$$\text{Then,}\quad \tan\theta = \frac{h_f}{x}$$

$$\text{Let}\ \tan\theta = i$$

$$\therefore\quad i = \frac{h_f}{x}$$

When the pipe is horizontal or if it is sloping very slowly $x = l$,

$$i = \frac{h_f}{l} \qquad \text{.. (7·3)}$$

i is termed as the slope or gradient.

The vertical distance from datum level upto the line of total energy represents the total energy at any point along the pipe line.

The slope of total energy line decides the direction of flow of water along the pipe line.

Loss of Head due to Friction in Pipe

Consider *water flowing through a horizontal pipe of uniform* cross-sectional area as shown in fig. 7-3.

Let, A – area of cross-section of the pipe,

K – wetted perimeter of the pipe,

V – *mean velocity of water which is constant along the pipe line,*

l – length of the pipe line considered between sections 1-1 and 2-2,

p_1 – intensity of pressure of water at section 1-1,

p_2 – intensity of pressure of water at section 2-2, and

f – frictional resistance per unit area at unit velocity.

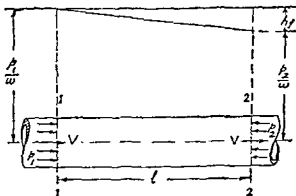


Fig 7-3

Let P_1 be the total force acting at section 1-1 in the forward direction on water column and P_2 be the total force acting at section 2-2 in the backward direction Hence the net force acting on the water column between sections 1-1 and 2-2 is (P_1-P_2). Now, water flows through the pipe line with uniform velocity. This force must be equal to the frictional resistance R, as seen earlier in this chapter.

$$R = f' \times \text{wetted area} \times V^n$$

$$\therefore \quad R = f' \times K \times l \times V^n \text{ as wetted area} = Kl$$

$$\therefore \quad P_1 - P_2 = f' \cdot K \cdot l \cdot V^n$$

Now, $P_1 = p_1 A$ and $P_2 = p_2 A$

$$\therefore \quad (p_1 - p_2)A = f' \, K \cdot l \cdot V^n$$

Dividing both sides by the density of water w, we have,

$$\frac{p_1}{w} - \frac{p_2}{w} = f' \cdot \frac{K}{A} \frac{l \cdot V^n}{w}$$

Let h_f be the head lost due to friction in the pipe line. It will be equal to $\frac{p_1}{w} - \frac{p_2}{w}$.

$$\text{i.e.} \quad h_f = \frac{p_1}{w} - \frac{p_2}{w} = f' \frac{K}{A} \cdot \frac{l \cdot V^n}{w}$$

The *hydraulic mean depth* is the ratio of the cross sectional area of flow A to the wetted perimeter K, and it is denoted by m.

$$\text{Hence,} \quad m = \frac{A}{K}$$

$$\therefore \quad h_1 = f' \cdot \frac{l \cdot V^n}{mw}$$

The value of n found from experiments is 2 and let $\frac{h_f}{l} = i$ be the slope of the hydraulic gradient.

$$\text{Then,} \quad i = \frac{f' \, V^2}{mw}$$

$$\therefore \quad V = C\sqrt{mi} \qquad \text{.. (7·4)}$$

$$\text{where } C = \sqrt{\frac{w}{f'}}$$

This form is known as the Chezy formula where C is the constant which is determined experimentally. However, the most common form for the loss of head due to friction will be derived as under.

$$h_f = \frac{f' l V^2}{mw}$$

Now, $m = \frac{A}{K} = \frac{\frac{\pi}{4} d^2}{\pi d} = \frac{d}{4}$

$$\therefore \quad h_f = \frac{f' \cdot l \cdot V^2}{\frac{d}{4} w}$$

Let, $f' = \frac{fw}{2g}$ .. (7·4a)

$$\therefore \quad h_f = \frac{fw}{2g} \cdot \frac{4l}{d} \cdot \frac{V^2}{w}$$

$$= \frac{4fl}{d} \cdot \frac{V^2}{2g} \quad .. (7·5)$$

Also since discharge, $Q = A \times V$

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} d^2}$$

$$\therefore \quad h_f = \frac{4fl}{d} \cdot \frac{Q^2}{\frac{\pi^2}{16} d^4} \times \frac{1}{2g}$$

$$= \frac{64}{2g} \times \frac{flQ^2}{\pi^2 d^5} = \frac{flQ^2}{3{\cdot}026 d^5} \simeq \frac{flQ^2}{3d^5} \quad ..(7·5a)$$

This form for head lost on friction will be most suitable for determining the discharge through pipes when loss of head due to friction is known. f is termed as coefficient of friction and it is a dimensionless number.

Expression, $h_f = \frac{4fl}{d} \cdot \frac{V^2}{2g}$ is known as Darcy's formula. Its advantages are : (i) It involves no fractional indices, (ii) The frictional loss is expressed in terms of velocity head. To find the value of f, Darcy gives the following :

$$\left.\begin{aligned} \text{For new pipes, } f &= 0{\cdot}005\left(1+\frac{1}{40d}\right) \\ \text{For old pipes, } f &= 0{\cdot}01\left(1+\frac{1}{40d}\right) \end{aligned}\right\} \qquad \ldots\ldots (7{\cdot}6)$$

where d is the diameter of the pipe in metres.

Problem-1 : *Assuming frictional resistance to be 4 kg/m² at 4 m/sec, find the resistance in kg, of a steamer speeding at 37 km/hr and having an immersed surface of 1,100 m². Find also the horse power required to propell the steamer.*

Resistance to flow, $R = KAV^2$ (eqn. 7·1)

$$\therefore \quad K = \frac{R}{AV^2} = \frac{4}{1 \times 4 \times 4} = 0{\cdot}25 \text{ kg/m}^2 \text{ per unit velocity}$$

$$\text{Velocity, } V = \frac{37{,}000}{3{,}600} = 10{\cdot}25 \text{ m/sec}$$

$$\text{Resistance, } R = 0{\cdot}25\, A{\cdot}V^2 = 0{\cdot}25 \times 1{,}100 \times 10{\cdot}25^2 = 29{,}000 \text{ kg}$$

$$\text{H.P. required} = \frac{RV}{75} = \frac{29.000 \times 10{\cdot}25}{75} = \underline{3{,}960 \text{ H.P.}}$$

Problem-2 : *Oil of specific gravity 0·9 and viscosity 0.0132 kg sec/m². is flowing in a pipe of 15 cm diameter. If the discharge is 1,700 litres per min, state whether the flow is laminar or turbulent.*

Since 1 litre of pure water weighs 1 kg,

1,000 litres = 1 m³.

Now, flow, $Q = AV \times 60 \times 1{,}000$ litres/min

$$\therefore \quad V = \frac{1{,}700}{\frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times 60 \times 1{,}000} = 1{\cdot}6 \text{ m/sec.}$$

Now, kinematic viscosity, $\nu = \dfrac{\mu}{\rho} = \dfrac{\mu}{sw/g}$

where μ – absolute viscosity,✓
s – specific gravity, and✓
w – density of water.

$$\therefore \quad \nu = \frac{0.0132}{0.9 \times 1000/9.81} = 0.000144$$

$$\text{Reynolds' number, } R_e = \frac{VD}{\nu} = \frac{1.6 \times \frac{15}{100}}{0.000144} = 1{,}667$$

Since $R_e < 2{,}000$ the flow is laminar or viscous.

Problem-3 : *Water flows through an old pipe 30 cm in diameter and 500 m long, at the rate of 0·2 m³/sec. Find the head lost in friction, (a) using Chezy's formula, (b) using Darcy's formula.*

(a) From eqn. (7·6) value of f is determined.

$$f = 0.01\left(1 + \frac{1}{40d}\right) = 0.01\left(1 + \frac{1}{40 \times 0.3}\right) = 0.0108$$

For Chezy formula, with help of eqn. (7·4a)

$$C = \sqrt{\frac{w}{f'}} = \sqrt{\frac{2g}{f}}$$

$$= \sqrt{\frac{2 \times 9.81}{0.0108}} = 42.5$$

$$\text{and hydraulic mean depth, } m = \frac{\text{area}}{\text{perimeter}} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4} = \frac{\frac{30}{100}}{4} = 0.075$$

$$\text{Also velocity, } V = \frac{Q}{\text{area}} = \frac{0.2}{\frac{\pi}{4}\left(\frac{30}{100}\right)^2} = 2.83 \text{ m/sec.}$$

Hence using Chezy formula,

$$V = C\sqrt{mi}$$

$$\text{or } 2.83 = 42.5\sqrt{0.075 \times i}$$

$$\therefore \quad i = 0.0589$$

$$\text{Now, since } i = \frac{h_f}{l}$$

head lost, $h_f = i \times l = 0.0589 \times 500 = \underline{29.45 \text{ m of water}}$

$$\therefore \quad 0{\cdot}6 + 2 = 0 + \frac{V_2^2}{2g}\left(1 + 0{\cdot}5 + \frac{4 \times 0{\cdot}01 \times 60}{\frac{10}{100}}\right)$$

$\therefore \quad V^2 = 2{\cdot}91$

$\therefore \quad V = 1{\cdot}71$ m/sec.

Discharge, $Q = AV = \frac{\pi}{4}\left(\frac{10}{100}\right)^2 \times 1{\cdot}71$

$= 0{\cdot}032$ m³/sec. or 32 litres/sec.

Problem - 6 : *Two reservoirs A and B are connected by a pipe 25 cm diameter. Water in the tank A is 9 m above the centre of the pipe while that in the tank B is 3 m above the centre of the pipe. The pipe slopes at 1 in 50 for the first one km then at 1 in 100 for the next 1·5 km, the rest of the length of it is horizontal for another 1·5 km. Find the discharge in litres per min. by using Darcy's formula and neglecting all the minor losses except loss due to friction.*

Friction constant, $f = 0{\cdot}01\left(1 + \frac{1}{40d}\right) = 0{\cdot}01\left(1 + \frac{1}{40 \times 0{\cdot}25}\right)$

$= 0{\cdot}0112$

The drop of level for one km from entrance is

$1{,}000 \times \frac{1}{50} = 20$ m

The drop of level for 1·5 km at a slope of 1 in 100 is

$1{,}500 \times \frac{1}{100} = 15$ m

Hence difference of water levels between two reservoirs is

$H = 9 + 20 + 15 - 3 = 41$ m

The total length of the pipe between two reservoirs is

$l = 1{,}000 + 1{,}500 + 1{,}500 = 4{,}000$ m

$H = h_f$ as minor losses are neglected.

i.e. $41 = \frac{4flV^2}{d\,2g}$

or $41 = \frac{4 \times 0{\cdot}0112 \times 4{,}000}{25/100} \cdot \frac{V^2}{19{\cdot}62}$

$\therefore \quad V^2 = 1{\cdot}142$

$\therefore \quad V = 1{\cdot}07$ m/sec.

Now, discharge, $Q = AV = \frac{\pi}{4}\left(\frac{25}{100}\right)^2 \times 1{\cdot}07$

$= 0{\cdot}0515$ m³/sec. or 51·5 litres/sec.

Problem-7 : *A tank supplies water to a short horizontal pipe 75 m long and open to atmosphere at the discharge end. The water level in the tank is 6 m above the centre of the pipe line. The pipe is 23 cm diameter for first 15 m length and then it suddenly reduces to 15 cm diameter for 30 m length and then it suddenly enlarges to 30 cm diameter for another 30 m length. Taking all the losses into account, find the discharge in m³/sec. Take $C_c = 0{\cdot}62$ and $f = 0{\cdot}006$.*

Since discharge is to be calculated, all losses are expressed in terms of velocity in the last leg of the pipe.

$$\text{Discharge} = A_1V_1 = A_2V_2 = A_3V_3$$

$$\therefore \quad V_1 = \frac{A_3}{A_1}V_3 = \left(\frac{d_3}{d_1}\right)^2 V_3 = \left(\frac{30}{23}\right)^2 V_3 = 1{\cdot}71V_3;$$

and $V_1^2 = 3{\cdot}16V_3^2$;

$$V_2 = \frac{A_3}{A_2}V_3 = \left(\frac{d_3}{d_2}\right)^2 V_3 = \left(\frac{30}{15}\right)^2 V_3 = 4V_3$$

and $V_2^2 = 16V_3^2$

$$\text{Head lost at entrance} = 0{\cdot}5\frac{V_1^2}{2g} = \frac{0{\cdot}5 \times 3{\cdot}16V_3^2}{2g} = \frac{1{\cdot}58V_3^2}{2g}$$

$$\left.\begin{array}{l}\text{Head lost due}\\ \text{to friction}\end{array}\right\} = \frac{4fl_1V_1^2}{d_1 2g} = \frac{4 \times 0{\cdot}006 \times 15}{0{\cdot}23} \times \frac{3{\cdot}16V_3^2}{2g} = \frac{4{\cdot}94V_3^2}{2g}$$

$$\left.\begin{array}{l}\text{Head lost due to}\\ \text{sudden contraction}\end{array}\right\} = \left(\frac{1}{C_c}-1\right)^2 \frac{V_2^2}{2g} = \left(\frac{1}{0{\cdot}62}-1\right)^2 \times \frac{16V_3^2}{2g} = \frac{5{\cdot}98V_3^2}{2g}$$

$$\left.\begin{array}{l}\text{Head lost due}\\ \text{to friction}\end{array}\right\} = \frac{4fl_2}{d_2}\cdot\frac{V_2^2}{2g} = \frac{4 \times 0{\cdot}006 \times 30}{0{\cdot}15} \times \frac{16V_3^2}{2g} = \frac{76{\cdot}8V_3^2}{2g}$$

$$\left.\begin{array}{l}\text{Head lost due to}\\ \text{sudden enlargement}\end{array}\right\} = \frac{(V_2 - V_3)^2}{2g} = \frac{(4V_3 - V_3)^2}{2g} = \frac{9V_3^2}{2g}$$

$$\left.\begin{array}{l}\text{Head lost due}\\ \text{to friction}\end{array}\right\} = \frac{4fl_3V_3^2}{d_3 2g} = \frac{4 \times 0{\cdot}006 \times 30}{0{\cdot}3} \times \frac{V_3^2}{2g} = \frac{2{\cdot}4V_3^2}{2g}$$

Velocity head at exit $= \frac{V_3^2}{2g}$

The total available head, *viz.* 6 m is lost on all this items.

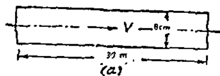
$$\therefore \quad 6 = (1{\cdot}58 + 4{\cdot}94 + 5{\cdot}98 + 76{\cdot}8 + 9 + 2{\cdot}4 + 1)\frac{V_3^2}{2g} = 101{\cdot}7\frac{V_3^2}{2g}$$

$$\therefore \quad V_3 = \sqrt{\frac{6 \times 19{\cdot}62}{101{\cdot}7}} = 1{\cdot}08 \text{ m/sec.}$$

Hence, $Q = A_3V_3 = \frac{\pi}{4}\, 0{\cdot}3^2 \times 1{\cdot}08 = 0{\cdot}0763 \text{ m}^3\text{/sec.}$

Problem-8 : *(a) A pipe of 8 cm diameter is 30 m long and carries a discharge of 0·15 m³/sec. Find the energy lost per second due to friction.*

(b) Now the central 18 m length of pipe is replaced by a pipe of 23 cm diameter the changes of sections being sudden. What will be the saving in H.P. due to the adoption of this alternative ? Take C_c = 0·64 and f = 0·01.



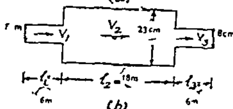


Fig. 7-4

(a) Discharge, $Q = A \times V$

$$\therefore \quad V = \frac{Q}{A} = \frac{0{\cdot}15}{\frac{\pi}{4}\left(\frac{8}{100}\right)^2} = 29{\cdot}8 \text{ m/sec.}$$

$$\text{Friction head, } h_f = \frac{4fl}{d}\frac{V^2}{2g} = \frac{4 \times 0{\cdot}01 \times 30}{8/100} \times \frac{29{\cdot}8^2}{2 \times 9{\cdot}81}$$

$$= 680 \text{ m of water}$$

$$\text{Energy lost per second} = wQh_f = 1{,}000 \times 0{\cdot}15 \times 680$$

$$= 10{,}02{,}000 \text{ kg-m}$$

(b) The new picture of flow is as in fig. 7-4 (b).

$$Q = A_1V_1 = A_2V_2$$

$$\therefore \quad V_2 = \frac{A_1}{A_2} \times V_1 = \left(\frac{d_1}{d_2}\right)^2 \times V_1 = \left(\frac{8}{23}\right)^2 \times V_1 = \frac{V_1}{9}$$

$$\therefore \quad V_2^2 = \frac{V_1^2}{81}$$

And $A_1V_1 = A_3V_3 \quad \therefore \quad V_1 = V_3$

Loss of head due to sudden enlargement is

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{\left(V_1 - \frac{V_1}{9}\right)^2}{2g} = 0.97\,\frac{V_1^2}{2g}$$

Loss of head due to sudden contraction is

$$h_c = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_3^2}{2g}$$

$$= \left(\frac{1}{0.64} - 1\right)^2 \frac{V_1^2}{2g} = 0.314\,\frac{V_1^2}{2g}$$

$$h_{f_1} = h_{f_3} = \frac{4fl_1}{d_1} \cdot \frac{V_1^2}{2g} = \frac{4 \times 0.01 \times 6}{8/100} \times \frac{V_1^2}{2g} = \frac{3.0\,V_1^2}{2g}$$

$$h_{f_2} = \frac{4fl_2}{d_2} \cdot \frac{V_2^2}{2g} = \frac{4 \times 0.01 \times 18}{23/100} \cdot \frac{V_1^2}{81 \times 2g} = \frac{0.0386V_1^2}{2g}$$

$\therefore$ total loss of head, $H_t = h_{f_1} + h_e + h_{f_2} + h_c + h_{f_3}$

$$= \frac{3.0V_1^2}{2g} + \frac{0.79V_1^2}{2g} + \frac{0.0386\,V_1^2}{2g} + \frac{0.314V_1^2}{2g} + \frac{3.0\,V^2}{2g}$$

$$= \frac{7.1426V_1^2}{2g} = \frac{7.1426 \times (29.8)^2}{19.62}$$

$= 323$ m of water

$\therefore$ saving in H. P.

$$= \frac{wQ\,(h_t - H_1)}{75} = \frac{1{,}000 \times 0.15 \times (680 - 323)}{75} = \underline{714 \text{ H. P.}}$$

Problem – 9 : *A town having population of 1,50,000 is to be supplied with water from a reservoir 6·5 km distant and it is stipulated that one half of the daily supply of 136 litres per head should be delivered in 8 hours. What must be the size of the pipe to carry this supply if the head available is 15 m ? C = 80 for Chezy formula.*

Total supply = 1,50,000 × 136 = 2,04,00,000 litres

Since half of this quantity is required in 8 hours, maximum flow in the pipe per second is

$$Q = \frac{1,02,00,000}{1,000 \times 8 \times 3,600} = 0{\cdot}36 \text{ m}^3/\text{sec.}$$

i, the slope of the gradient $= \dfrac{h_f}{l} = \dfrac{15}{6,500} = 0{\cdot}00231$

Chezy formula is

$$V = C\sqrt{mi}$$

$$= 80\sqrt{\frac{d}{4} \times 0{\cdot}00231}$$

Discharge, $Q = \dfrac{\pi}{4} d^2 \times 80\sqrt{\dfrac{d}{4} \times 0{\cdot}00231}$

Squaring both sides,

$$(0{\cdot}36)^2 = \frac{\pi^2}{16} d^5 \times 6,400 \times \tfrac{1}{4} \times 0{\cdot}00231 = 2{\cdot}27 d^5$$

$$\therefore \quad d = \left(\frac{0{\cdot}36}{2{\cdot}27}\right)^{\frac{1}{5}} = 0\;692 \text{ m} = \underline{69{\cdot}2 \text{ cm}}$$

Problem – 10 : *Power is to be transmitted hydraulically over a distance of 10 km by means of a number of 15 cm pipes laid at a slope of 1 in 1,000. The pressure at the inlet end is maintained constant at 42 kg/cm². Determine the minimum number of pipes required to ensure transmission efficiency of 90%, when 350 H P. is delivered. Friction coefficient for the pipes may be taken as 0·0075.*

Pressure head, $H_1 = \dfrac{10^4 p}{w} = \dfrac{10^4 \times 42}{1,000} = 420 \text{ m}$

Static head, $H_2 = l \sin \alpha = \dfrac{10 \times 1,000}{1,000} = 10 \text{ m}$

$\therefore$ total available head, $H = H_1 + H_2 = 420 + 10 = 430 \text{ m}$

Efficiency of transmission $= \dfrac{H - h_f}{H}$

$$\text{or } 0\;9 = \frac{430 - h_f}{430}$$

$$\therefore \quad h_f = 430\,(1 - 0{\cdot}9) = 43 \text{ m of water}$$

Loss of head due to friction is $h_f = \frac{4fl}{d} \cdot \frac{V^2}{2g}$

or $43 = \frac{4 \times 0{\cdot}0075 \times 10 \times 1{,}000}{0{\cdot}15} \cdot \frac{V^2}{19{\cdot}62}$

$\therefore \; V = 0{\cdot}656$ m/sec

Now, $Q = AV = \frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times 0{\cdot}656 = 0{\cdot}0116$ m³/sec.

Water horse power $= \frac{wQ \times \text{available head}}{75}$

Available head $= 430 - 43 = 387$ m

Water horse power delivered per pipe $= \frac{1{,}000 \times 0{\cdot}0116 \times 387}{75} = 60$ H.P.

$\therefore$ number of pipes required $= \frac{\text{total H.P. delivered}}{\text{H.P. delivered per pipe}}$

$= \frac{350}{60} = 5{\cdot}83$

Hence, number of pipes required is 6.

Problem – 11 : *A part of a straight length of pipe conveys water at the rate of 0·0425 m³/sec. and it slopes 1 in 200 in the upward direction between the points A and D. The first 45 m of pipe (AB) is 23 cm in diameter and the last 30 m of pipe (CD) is 15 cm diameter, and both are connected by a tapering pipe BC 3 m long as shown in fig. 7–5. The pressure at end A of 23 cm pipe is 4 kg/cm². If the flow in the pipe is in the upward direction, find the pressures at points B, C and D, assuming there is a loss of head in the tapering pipe equivalent to 2% of the mean velocity head. Take f = 0·005 for both pipes.*

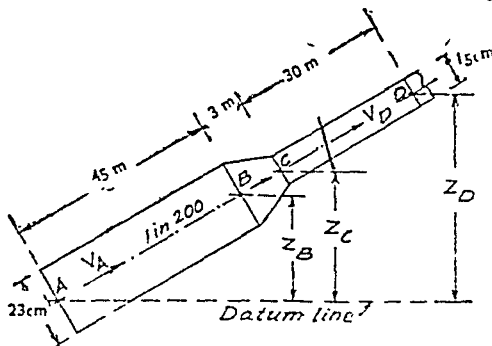


Fig. 7–5

Here, we have $Z_A = 0$, $Z_B = \frac{45 \times 1}{200} = 0.225$ m, $p_A = 4$ kg/cm^2

$Z_C = \frac{48 \times 1}{200} = 0.24$ m; $Z_D = \frac{78 \times 1}{200} = 0.39$ m

$$V_A = V_B = \frac{Q}{\frac{\pi}{4}\left(\frac{23}{100}\right)^2} = \frac{0.0425}{\frac{\pi}{4} \times (0.23)^2} = 1.025 \text{ m/sec}$$

$$V_C = V_D = \frac{Q}{\frac{\pi}{4}\left(\frac{15}{100}\right)^2} = \frac{0.0425 \times 4}{\pi \times 0.0225} = 2.4 \text{ m/sec}$$

$$\text{Mean velocity} = \frac{V_B + V_C}{2} = \frac{1.025 + 2.4}{2} = 1.713 \text{ m/sec}$$

$$\text{Mean velocity head} = \frac{1.713^2}{2 \times 9.81} = 0.15 \text{ m of water}$$

Applying Bernoulli's theorem between A and B, we have,

$$Z_A + \frac{p_A}{w} + \frac{V_A^2}{2g} = Z_B + \frac{p_B}{w} + \frac{V_B^2}{2g} + \text{loss due to friction}$$

$$\therefore \quad \frac{p_B}{w} = Z_A - Z_B + \frac{p_A}{w} - \text{loss due to friction (as } V_A = V_B)$$

$$= 0 - 0.225 + \frac{10,000 \times 4}{1,000} - \frac{4 \times 0.005 \times 4.5}{0.23} \times \frac{1.025^2}{19.62} = 39.56 \text{ m}$$

$$\therefore \quad p_B = \frac{1,000 \times 39.56}{10,000} = 3.956 \text{ kg/cm}^2$$

Applying Bernoulli's theorem between B and C, we have,

$$Z_B + \frac{p_B}{w} + \frac{V_B^2}{2g} = Z_C + \frac{p_C}{w} + \frac{V_C^2}{2g} + \text{loss of head in the tapering pipe}$$

$$\therefore \quad \frac{p_C}{w} = (Z_B - Z_C) + \frac{p_B}{w} + \frac{V_B^2 - V_C^2}{2g} - \frac{2}{100} \times 0.15$$

$$= 0.225 - 0.24 + 39.56 + \frac{(1.025)^2 - (2.4)^2}{19.62} - 0.003$$

$$= 39.79 \text{ m of water}$$

$$\therefore \quad p_C = \frac{39.79 \times 1,000}{10,000} = \underline{3.979 \text{ kg/cm}^2}$$

Applying Bernoulli's theorem between C and D, we have,

$$Z_C + \frac{p_C}{w} + \frac{V_C^2}{2g} = Z_D + \frac{p_D}{w} + \frac{V_D^2}{2g} + \text{loss of head due to friction}$$

$$\therefore \quad \frac{p_D}{w} = (Z_C - Z_D) + \frac{p_C}{w} - \frac{4fl}{d} \cdot \frac{V_D^2}{2g} \quad (\text{as } V_C = V_D)$$

$$= 0{\cdot}24 - 0{\cdot}39 + 39{\cdot}79 - \frac{4 \times 0{\cdot}005 \times 30}{0{\cdot}15} \times \frac{(2{\cdot}4)^2}{2 \times 9{\cdot}81}$$

$$= 38{\cdot}46 \text{ m of water}$$

$$\therefore \quad p_D = \frac{38{\cdot}46 \times 1{,}000}{10{,}000} = 3{\cdot}84 \text{ kg/cm}^2$$

Problem-12 : *A pipe PQRS of uniform diameter is made up of three straight lengths (fig. 7-6). PQ measures 120 m, QR 150 m and RS 60 m. The survey levels of the points P, Q, R & S are 60 m, 45 m, 75 m and 90 m repectively above the datum. A gauge at P shows pressure of 3 kg/cm² and at S 0·7 kg/cm². Neglecting all losses except frictional, find in which direction water will flow and state the pressures at Q and R.*

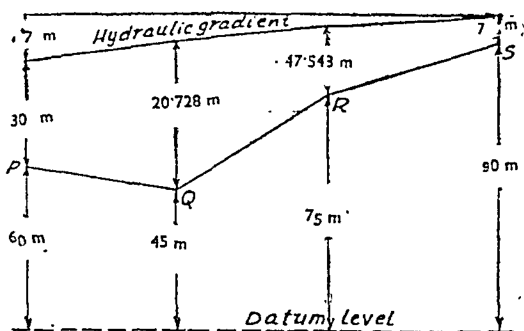


Fig. 7-6

$$\text{Pressure at P} = \frac{10{,}000 \times 3}{1{,}000} = 30 \text{ m of water}$$

$$\text{Pressure at S} = \frac{10{,}000 \times 0{\cdot}7}{1{,}000} = 7 \text{ m of water}$$

Total head at P except velocity head = 60 + 30 = 90 m

Total head at S except velocity head = 90 + 7 = 97 m

As the diameter of pipe line is uniform, the velocity head will be the same, and hence the difference in the total energy at S and P is 97 – 90 = 7 m.

Therefore water is flowing from S towards P and loses 7 m head on account of friction.

Total length of pipe = 120 + 150 + 60 = 330 m

i, the slope or the gradient $= \frac{h_f}{l} = \frac{7}{330} = 0.0212$

Loss of head from S to R = 60 × 0.0212 = 1.272 m
Loss of head from R to Q = 150 × 0.0212 = 3.18 m
Loss of head from Q to P = 120 × 0.0212 = 2.544 m

To find the pressure head at Q and R, either plot to scale the pipe axis and gradient and measure the vertical distances between them, as shown in fig 7-6, or calculate as follows :

Pressure head at R = total head at S – potential head of R – loss of head between S and Q

= 97 – 75 – 1.272 = 20.728 m

Pressure head at Q = total head at S – potential head of Q – loss of head between S and Q

= 97 – 45 – 1.272 – 3.18 = 47.5 m

Pressure at R $= \frac{1{,}000 \times 20.728}{10{,}000} = 2.0728 \text{ kg/cm}^2$

Pressure at Q $= \frac{1{,}000 \times 47.5}{10{,}000} = 4.75 \text{ kg/cm}^2$

Syphon Pipe

Sometimes, while laying out a pipe line to connect two reservoirs, it so happens that an obstacle in the form of a high ridge or high ground comes up such that a straight pipe line cannot be laid. If we want a straight pipe line, we have to cut the ridge and lay the pipe line, but it will be difficult for repairs after laying and the cost of laying will be more. Hence it is usually laid on the surface of the ridge. Thus the summit of the pipe will be above the water level in the supply reservoir. This type of pipe is known

s the syphon pipe. If the pressure at the summit is more than apour pressure of water the flow will take place otherwise he flow will stop. This will be better understood by illustrating ne problem shown below.

Problem – 13 : *A reservoir P supplies water to a lower level eservoir Q through a 45 cm diameter pipe line which rises above the evel of water in the higher reservoir by 2 m. The pipe length etween reservoir P and summit R is 450 m. The difference between evels of water in two reservoirs is 45 m and the total length of the ipe line is 2,700 m. What will be the discharge if the minimum ressure is not to fall below 2·5 m of water absolute at the summit ? 'ake f = 0·005.*

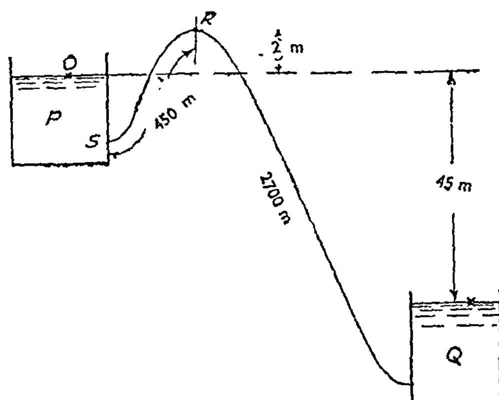


Fig. 7-7

Applying Bernoulli's equation to points O and R we have,

$$10{\cdot}4 = 2 + 2{\cdot}5 + \left(0{\cdot}5 + \frac{4fl}{d} + 1\right)\frac{V^2}{2g} \quad \left\{\begin{array}{l}\text{where 10·4 m is}\\ \text{atmospheric pressure.}\end{array}\right\}$$

$$= 4{\cdot}5 + \left(0{\cdot}5 + \frac{4 \times 0{\cdot}005 \times 450}{0{\cdot}45}\right)\frac{V^2}{2g}$$

$\therefore V = 2{\cdot}36$ m/sec.

Discharge, $Q = \frac{\pi}{4}\left(\frac{45}{100}\right)^2 \times 2{\cdot}36 = \underline{0{\cdot}375 \text{ m}^3/\text{sec.}}$

Problem 14 : *A pipe line, 8 cm diameter, 15 m long has its ends connected to two different tanks and it is used as a model of a syphon. The difference between water levels in the two tanks is 2·5 m. The length of the pipe line from inlet to summit is 6 m at a slope 1 in 5. The length of a portion of this pipe line (on L.H.S. of summit) below water level in the supply tank is 2 m. Take $f = 0{\cdot}008$. Determine the discharge in litres/min. and the pressure at summit. If the vapour pressure is 0·245 kg/cm² absolute, state whether the syphon will operate or not. Atmospheric pressure = 10 m of water.*

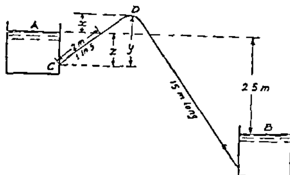


Fig. 7-8

Taking water level in tank B as datum,

total energy at A = total energy at B + losses

$$i.e.\ 2{\cdot}5 + 10 = 10 + \frac{0{\cdot}5V^2}{2g} + \frac{4fl}{d} \cdot \frac{V^2}{2g} + \frac{V^2}{2g}$$

$$i.e.\ 2{\cdot}5 = \frac{V^2}{2g}\left(0{\cdot}5 \times \frac{4 \times 0{\cdot}008 \times 15}{8/100} + 1\right)$$

$$\therefore \quad V^2 = \frac{2{\cdot}5 \times 2 \times 9{\cdot}81}{7{\cdot}5} = 6{\cdot}55$$

$$\therefore \quad V = 2{\cdot}56 \text{ m/sec.}$$

Now, discharge, $Q = \frac{\pi}{4} d^2V = \frac{\pi}{4} \left(\frac{8}{100}\right)^2 \times 2{\cdot}56 = 0\,01286$ m³/sec.

or 12·86 litres/sec. or 772 litres/min.

Referring to fig. 7-8,

$y = 6 \times \frac{1}{5} = 1{\cdot}2$ m ; $z = 2 \times \frac{1}{5} = 0{\cdot}4$ m,

and $x = y - z = 1{\cdot}2 - 0\,4 = 0{\cdot}8$ m

Now, taking water at A as datum, we have,

total energy at A = total energy at D

$$i.e.\ \ 10 = 0{\cdot}8 + \frac{p_d}{w} + 0{\cdot}5\,\frac{V^2}{2g} + \frac{4fl_1}{d}\cdot\frac{V^2}{2g} + \frac{V^2}{2g}$$

$$\text{or}\ \ 9{\cdot}2 = \frac{p_d}{w} + \frac{V^2}{2g}\left(0{\cdot}5 + \frac{4\times 0{\cdot}008\times 6}{\frac{8}{100}} + 1\right)$$

$$\therefore\ \ \frac{p_d}{w} = 9{\cdot}2 - \frac{3{\cdot}9\ V^2}{2g} = 9{\cdot}2 - \frac{3{\cdot}9\times 2{\cdot}56^2}{2\times 9{\cdot}81} = 7{\cdot}9 \text{ m of water}$$

$$\therefore\ \ p_d = \frac{7{\cdot}9\times 1{,}000}{10{,}000} = 0{\cdot}79 \text{ kg/cm}^2$$

Since pressure at summit is more than the vapour pressure of 0·245 kg/cm² absolute, the syphon will operate.

Equivalent Size of a Pipe

Sometimes it is necessary to change a pipe line which is already laid and which is of different diameters at different sections. If it is replaced by a single pipe of uniform diameter, such that the loss of head as well as the discharge is the same in both cases, then the new size found out is known as an equivalent size of a pipe. The diameter of the equivalent pipe will be different from that of the original pipe. The conception of equivalent pipe is useful to calculate easily the loss of head due to friction.

Let a pipe line of lengths l_1, l_2, l_3 etc. have diameters d_1, d_2, d_3 etc. and the velocity of water in the respective pipe lines be V_1, V_2, V_3 etc. Neglecting all the minor losses, the total head causing the flow must be equal to the sum of all the frictional losses.

$$\therefore\ \ H = \frac{4fl_1}{d_1}\cdot\frac{V_1^2}{2g} + \frac{4fl_2}{d_2}\cdot\frac{V_2^2}{2g} + \frac{4fl_3}{d_3}\cdot\frac{V_3^2}{2g} + \dots\ \dots$$

Now let l be length of new pipe, d be equivalent diameter of the new pipe, and V be velocity of water in the new pipe,

$$\text{Then,}\ \ \frac{4fl}{d}\cdot\frac{V^2}{2g} = \frac{4fl_1}{d_1}\cdot\frac{V_1^2}{2g} + \frac{4fl_2}{d_2}\,\frac{V_2^2}{2g} + \frac{4fl_3}{d_3}\cdot\frac{V_3^2}{2g} + \dots\ \dots$$

$$\text{Discharge,}\ \ Q = \frac{\pi}{4}\,d^2V = \frac{\pi}{4}\,d_1^2V_1 = \frac{\pi}{4}\,d_2^2V_2 = \frac{\pi}{4}\,d_3^2V_3$$

$$\therefore \quad V = \frac{4Q}{\pi d^2}\ ;\ V_1 = \frac{4Q}{\pi d_1^2}\ ;\ V_2 = \frac{4Q}{\pi d_2^2}\ ;\ V_3 = \frac{4Q}{\pi d_3^2}$$

$$\therefore \quad \frac{4fl}{2gd}\left(\frac{4Q}{\pi d^2}\right)^2 = \frac{4fl_1}{2gd_1}\left(\frac{4Q}{\pi d_1^2}\right)^2 + \frac{4fl_2}{2g \cdot d_2}\left(\frac{4Q}{\pi d_2^2}\right)^2 + \frac{4fl_3}{2gd_3}\left(\frac{4Q}{\pi d_3^2}\right)^2 \quad ..$$

$$\therefore \quad \frac{l}{d^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5} + \quad .. \quad .. \quad ..$$

$$\therefore \quad d = \left\{\frac{l}{\Sigma \frac{l}{d^5}}\right\}^{\frac{1}{5}} \qquad .. (7.7)$$

Problem – 15 : *Find the diameter of a uniform pipe necessary to replace a compound pipe line having the following sections : 45 cm diameter pipe for 900 m length; 30 cm diameter for 300 m length and 15 cm diameter for 600 m length. The coefficient of friction for the pipe line is 0·01. The pressure head at inlet end is 45 m of water while that at the delivery end is 6 m of water. Determine the the discharge in L. P. M.*

The total length, $l = 900 + 300 + 600 = 1800$ m

$$d_{equi} = \left\{\frac{l}{\Sigma \frac{l}{d^5}}\right\}^{\frac{1}{5}}$$

$$= \left\{\frac{1,800}{\frac{900}{\left(\frac{45}{100}\right)^5} + \frac{300}{\left(\frac{30}{100}\right)^5} + \frac{600}{\left(\frac{15}{100}\right)^5}}\right\}^{\frac{1}{5}} = \left\{\frac{1,800}{48,700 + 1,23,600 + 79,00,000}\right\}^{\frac{1}{5}}$$

$= 0{\cdot}186$ m or $18{\cdot}6$ cm

Head causing the flow $= 45 - 6 = 39$ m

$$H = \frac{4fl}{d}\frac{V^2}{2g}$$

$$i.e.\ 39 = \frac{4 \times 0{\cdot}01 \times 1,800}{0{\cdot}186} \times \frac{V^2}{19{\cdot}62}$$

$$\therefore \quad V = 1{\cdot}408 \text{ m/sec.}$$

Discharge, $Q = \frac{\pi}{4} d^2 \times V = \frac{\pi}{4} (0{\cdot}186)^2 \times 1{\cdot}408 \times 1{,}000 \times 60$

$= 2{,}300$ litres per min.

Parallel Flow through Pipes

This may be divided in two types. (i) Water is led from the supply tank through a pipe which is connected after some distance to two parallel pipes of different diameters and finally supplying to the receiving tank as shown in fig. 7–9a. (ii) A pipe conveying from one tank to another is looped for certain length as shown in fig. 7–9 (b).

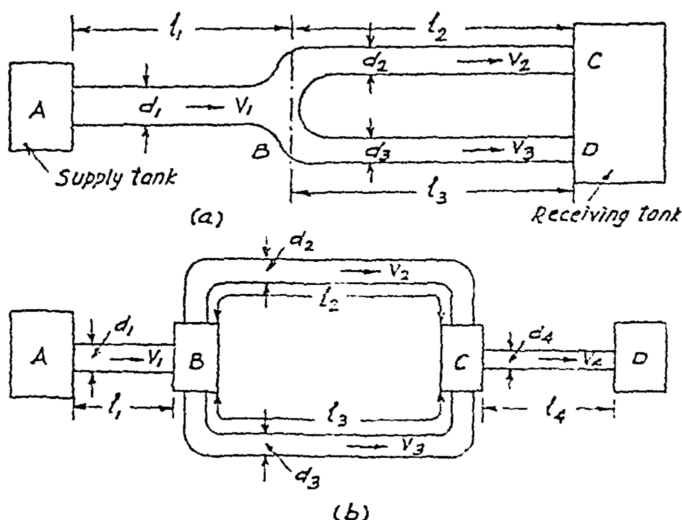


Fig. 7–9 Parallel flow in pipes

(i) The total discharge Q_1 in the pipe AB is divided in two parts at junction B. Let Q_2 and Q_3 be the discharge of pipes BC and BD respectively.

$$\therefore \quad Q_1 = Q_2 + Q_3 \qquad \text{.. (i)}$$

The total loss of head will be equal to the difference of water levels between two tanks for any path of flow.

$$h_f = h_{f_1} + h_{f_2} \quad ..(ii)$$

$$\text{Also } h_f = h_{f_1} + h_{f_3} \quad ..(iii)$$

From eqns. (ii) & (iii), we have, $h_{f_1} + h_{f_2} = h_{f_1} + h_{f_3}$

$$\therefore \quad h_{f_2} = h_{f_3} \quad ..(iv)$$

Equations (i), (ii) and (iv) will be useful in finding the unknown quantities.

(ii) In this case the sum of discharges through loop pipes be equal to the flow before looping and also equal to flow after looping. For any path of flow the total loss will be equal to water levels between the two tanks.

$$\therefore \quad Q_1 = Q_2 + Q_3 = Q_4 \quad ..(v)$$

$$\text{Head lost, } h_f = h_{f_1} + h_{f_2} + h_{f_4} \quad ..(vi)$$

$$\text{Also } h_f = h_{f_1} + h_{f_3} + h_{f_4} \quad ..(vii)$$

From eqns (vi) & (vii), we have,

$$h_{f_2} = h_{f_3} \quad ..(viii)$$

Equations v), (vi), (vii) and (viii) will be useful in the solution of problems.

Problem 16 *Two pipes P and Q 15 cm and 23 cm in diameter respectively, branch from a point R to a point S, which is 9 m below R. Pipe P is 300 m long and pipe Q is 600 m long. Water is supplied at R under a head of 45 m. A short pipe 10 cm in diameter is fitted at S. Find the delivery when this pipe is fully open to atmosphere. Take $V = C\sqrt{mi}$ for pipes P and Q and C = 90.*

Let V_1, V_2 and V be the velocities of water in the pipes P, Q and 10 cm pipe line respectively.

Total head = 45 + 9 = 54 m

Consider pipe P.

$$m = \frac{d}{4} = \frac{0.15}{4} = 0.0375$$

$$V_1 = 90\sqrt{0.0375 \times h_{f_1}/300}$$

$$\therefore \quad h_{f_1} = V_1^2$$

Time of Emptying Tank through Pipe

Let, A – area of cross-section of tank,

d – diameter of the pipe line,

H_1 – initial head of water above the outlet end of the pipe,

H_2 – final head of water above the outlet end of the pipe,

V – velocity of water in the pipe line,

h – head of water for elementary strip above the outlet end of the pipe,

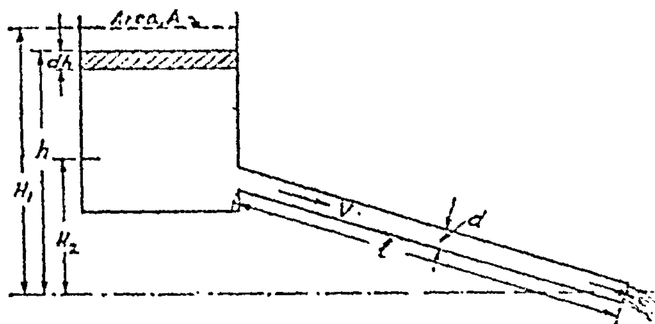


Fig. 7–10

Consider an elementary strip of thickness dh at head h. This head h is used up to overcome various losses.

$$\therefore \quad h = 0{\cdot}5\frac{V^2}{2g} + \frac{4fl}{d}\frac{V^2}{2g} + \frac{V^2}{2g} = \left(1{\cdot}5 + \frac{4fl}{d}\right)\frac{V^2}{2g}$$

$$\therefore \quad V = \frac{\sqrt{2gh}}{\sqrt{1{\cdot}5 + \dfrac{4fl}{d}}}$$

Volume of the strip is, $dQ = -Adh$

This volume leaves the tank in time dT through the pipe.

$$\therefore \quad dQ = \text{area} \times \text{velocity} \times dT$$

$$= \frac{\pi}{4} d^2 \times V{\cdot}dT$$

$$\therefore \quad \frac{\pi}{4} d^2 V{\cdot}dT = -Adh$$

$$\therefore \quad dT = \frac{-A dh}{\frac{\pi}{4} d^2 V}$$

$$= \frac{-4A\left(\sqrt{1.5 + \frac{4fl}{d}}\right) h^{-\frac{1}{2}} dh}{\pi d^2 \sqrt{2g}}$$

Integrating both sides, we get

$$T = -\frac{4A\sqrt{1.5 + \frac{4fl}{d}}}{\pi d^2 \sqrt{2g}} \int_{H_1}^{H_2} h^{-\frac{1}{2}} dh$$

$$= \frac{8A\sqrt{1.5 + \frac{4fl}{d}}}{\pi d^2 \sqrt{2g}} \left[\sqrt{H_1} - \sqrt{H_2}\right] \qquad ..(7\cdot8)$$

If the tank is to be emptied completely, $H_2 = 0$.

$$\therefore T = \frac{8A\sqrt{1.5 + \frac{4fl}{d}} \sqrt{H_1}}{\pi d^2 \sqrt{2g}} \qquad ..(7\cdot8a)$$

Problem – 18 : *A tank 2 m in diameter is fitted with a vertical pipe line, 8 cm in diameter, 9 m long, at its base whose lower end discharges water in the atmosphere. The tank contains water to a depth of 3 m. Find the time taken to empty the tank. Consider all losses and take value of f = 0·01.*

Area of tank, $A = \frac{\pi}{4} 2^2 = 3.14 \text{ m}^2$

Diameter of pipe $d = \frac{8}{100} = 0.08$

Time of emptying, $T = \frac{8A\sqrt{1.5 + \frac{4fl}{d}} \sqrt{H}}{\pi d^2 \sqrt{2g}}$

$$= \frac{8 \times 3.14\sqrt{1.5 + \frac{4 \times 0.01 \times 9}{0.08}} \sqrt{3}}{\pi \times (0.08)^2 \times \sqrt{2 \times 9.81}}$$

$= 1{,}200$ sec or 20 min

Time of Flow from One Reservoir to Another through Long Pipe

Let, A_1 – cross sectional area of supplying tank,

A_2 – cross-sectional area of receiving tank,

a – cross sectional area of connecting pipe,

d – diameter of the pipe,

l – length of the pipe,

h – difference of water levels at a particular instant which changes to h' in a small interval of time dT, and

dH – change of water level in the supply tank in time dT.

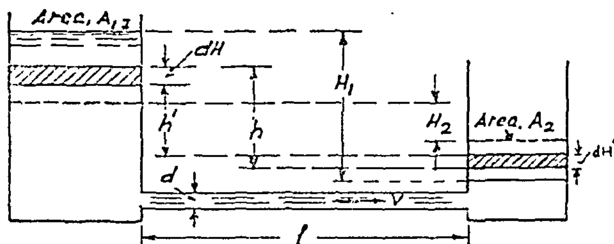


Fig. 7-11

$$dh = h - h' = (dH + h' + dH') - h' = dH + dH'$$

$$= dH + \frac{A_1}{A_2}\,dH = \left(1 + \frac{A_1}{A_2}\right) dH$$

$$\therefore \quad dH = \frac{dh}{1 + \frac{A_1}{A_2}}$$

Also, $h = 0.5\,\frac{V^2}{2g} + \frac{4fl}{d}\frac{V^2}{2g} + \frac{V^2}{2g} = \left(1.5 + \frac{4fl}{d}\right)\frac{V^2}{2g}$

Velocity of water in pipe at given instant, $V = \dfrac{\sqrt{2gh}}{\sqrt{1.5 + \frac{4fl}{d}}}$

The volume of water displaced from one tank to another is

$$dQ = -A_1 dH$$

This quantity flows out through the connecting pipe.

$$\therefore \quad dQ = \text{area} \times \text{velocity} \times \text{time}$$
$$= a \times V \times dT$$

$$\therefore \quad -A_1 dH = a \cdot V \cdot dT$$

$$\therefore \quad dT = -\frac{A_1 dH}{a \times V} = -\frac{A_1 dH \sqrt{1 \cdot 5 + \frac{4fl}{d}}}{a\sqrt{2gh}}$$

$$\text{But,} dH = -\frac{dh}{1 + \frac{A_1}{A_2}}$$

$$\therefore \quad dT = -\frac{A_1 \sqrt{1 \cdot 5 + \frac{4fl}{d}}}{\left(1 + \frac{A_1}{A_2}\right) a\sqrt{2g}} h^{-\frac{1}{2}} dh$$

Let H_1 be the initial difference and H_2 the final difference. Then integrating between these limits, we have,

$$\int_0^T dT = -\frac{A_1 \sqrt{1 \cdot 5 + \frac{4fl}{d}}}{\left(1 + \frac{A_1}{A_2}\right) a\sqrt{2g}} \int_{H_1}^{H_2} h^{-\frac{1}{2}} dh$$

$$\therefore \quad T_1 = \frac{2A_1 \sqrt{1 \cdot 5 + \frac{4fl}{d}} \left(H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}}\right)}{\left(1 + \frac{A_1}{A_2}\right) \times a\sqrt{2g}} \quad \text{.. (7·9)}$$

If we neglect the other minor losses except friction for very long pipes, we have,

$$T = \frac{2A_1 \sqrt{\frac{4fl}{d}} \left(\sqrt{H_1} - \sqrt{H_2}\right)}{\left(1 + \frac{A_1}{A_2}\right) \times a \times \sqrt{2g}}$$

Time of Flow from One Reservoir to Another through Long Pipe

Let, A_1 – cross sectional area of supplying tank,

A_2 – cross-sectional area of receiving tank,

a – cross sectional area of connecting pipe,

d – diameter of the pipe,

l – length of the pipe,

h – difference of water levels at a particular instant which changes to h' in a small interval of time dT, and

dH – change of water level in the supply tank in time dT.

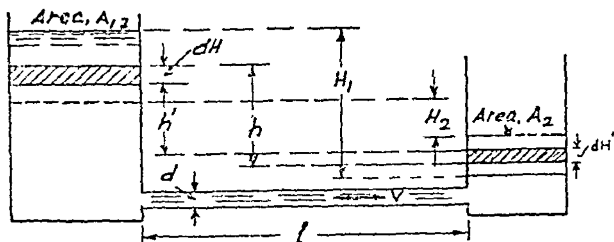


Fig. 7–11

$$dh = h - h' = (dH + h' + dH') - h' = dH + dH'$$

$$= dH + \frac{A_1}{A_2}\,dH = \left(1 + \frac{A_1}{A_2}\right) dH$$

$$\therefore \quad dH = \frac{dh}{1 + \frac{A_1}{A_2}}$$

Also, $h = 0{\cdot}5\,\frac{V^2}{2g} + \frac{4fl}{d}\frac{V^2}{2g} + \frac{V^2}{2g} = \left(1{\cdot}5 + \frac{4fl}{d}\right)\frac{V^2}{2g}$

Velocity of water in pipe at given instant, $V = \dfrac{\sqrt{2gh}}{\sqrt{1{\cdot}5 + \frac{4fl}{d}}}$

The volume of water displaced from one tank to another is

$$dQ = -A_1 dH$$

This quantity flows out through the connecting pipe.

$$\therefore \quad dQ = \text{area} \times \text{velocity} \times \text{time}$$
$$= a \times V \times dT$$

$$\therefore \quad -A_1 dH = a \cdot V \cdot dT$$

$$\therefore \quad dT = -\frac{A_1 dH}{a \times V} = -\frac{A_1 dH \sqrt{1 \cdot 5 + \frac{4fl}{d}}}{a\sqrt{2gh}}$$

$$\text{But,} dH = -\frac{dh}{1 + \frac{A_1}{A_2}}$$

$$\therefore \quad dT = -\frac{A_1 \sqrt{1 \cdot 5 + \frac{4fl}{d}}}{\left(1 + \frac{A_1}{A_2}\right) a\sqrt{2g}} h^{-\frac{1}{2}} dh$$

Let H_1 be the initial difference and H_2 the final difference. Then integrating between these limits, we have,

$$\int_0^T dT = -\frac{A_1 \sqrt{1 \cdot 5 + \frac{4fl}{d}}}{\left(1 + \frac{A_1}{A_2}\right) a\sqrt{2g}} \int_{H_1}^{H_2} h^{-\frac{1}{2}} dh$$

$$\therefore \quad T_1 = \frac{2A_1 \sqrt{1 \cdot 5 + \frac{4fl}{d}} \left(H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}}\right)}{\left(1 + \frac{A_1}{A_2}\right) \times a\sqrt{2g}} \quad \text{.. (7·9)}$$

If we neglect the other minor losses except friction for very long pipes, we have,

$$T = \frac{2A_1 \sqrt{\frac{4fl}{d}} \left(\sqrt{H_1} - \sqrt{H_2}\right)}{\left(1 + \frac{A_1}{A_2}\right) \times a \times \sqrt{2g}}$$

$$\therefore \quad T = \frac{2\sqrt{\frac{4fl}{d}}\left(\sqrt{H_1} - \sqrt{H_2}\right)}{a \times \sqrt{2g}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)} \quad \ldots (7{\cdot}10)$$

If the supply tanks is a large reservoir or tank in which the water level practically remains constant, the time for filling the tank may be obtained by putting $A_1 = \infty$ and solving for T.

Problem-19 : *Two tanks A and B, 8 m and 3 m in diameter, are connected by a 25 cm pipe 300 m long. The level of the oil in tank A is 10 m higher than that in tank B when the flow was started. Determine the time taken to transfer 18,000 litres of oil from tank A to tank B. Take f = 0·0075 and sp. gr. of oil as 0·9.*

Area, $A_1 = \frac{\pi}{4}\, 8^2 = 50{\cdot}3 \text{ m}^2, \qquad A_2 = \frac{\pi}{4}\, 3^2 = 7{\cdot}09 \text{ m}^2$

Let y be the height of fall in the tank A and z be the height of rise in the tank B. Now, one litre $= \frac{1}{1{,}000}\ 0{\cdot}001 \text{ m}^3$.

Flow, $Q = 18{,}000 \times 0{\cdot}001 = 18 \text{ m}^3$

$$\therefore \quad y = \frac{Q}{A_1} = \frac{18}{50{\cdot}3}\ 0{\cdot}358 \text{ m}$$

$$z = \frac{Q}{A_2} = \frac{18}{7{\cdot}09} = 2{\cdot}54 \text{ m}$$

$H_2 = 10 - y - z = 10 - 0{\cdot}358 - 2{\cdot}54 = 7{\cdot}102 \text{ m}, \ H_1 = 10 \text{ m}$

Neglecting all losses except friction, we have,

$$T = \frac{2\sqrt{\frac{4fl}{d}}\left(\sqrt{H_1} - \sqrt{H_2}\right)}{a\sqrt{2g}\left(\frac{1}{A_1} + \frac{1}{A_2}\right)}$$

$$= \frac{2\sqrt{\frac{4 \times 0{\cdot}0075 \times 300}{0{\cdot}25}}\left(\sqrt{10} - \sqrt{7{\cdot}102}\right)}{\frac{\pi}{4}(\cdot 25)^2\sqrt{2 \times 9{\cdot}81}\left(\frac{1}{50{\cdot}3} + \frac{1}{7{\cdot}09}\right)}$$

$= \underline{750 \text{ sec.}}$

Branched Mains

This is the case in which the flow is divided into number of pipes. In this there are two cases. In the first case one tank may supply water to the junction and then from the junction it may flow in different pipes, say two pipes In the other case a number of pipes, say two, may supply water to junction from which water may flow through one pipe to the receiving tank.

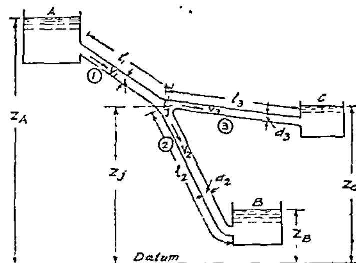


Fig. 7-12 Branching of supply mains

Case I : Water flows from tank A to the junction J and then from it to tanks B and C. The flow depends upon the slope of the hydraulic gradient.

Applying Bernoulli's equation to A and J,

$$Z_A = Z_J + \frac{p_J}{w} + \frac{V_1^2}{2g} + h_{f_1} \quad ..(i)$$

Applying Bernoulli's equation to J and B, we have,

$$Z_J + \frac{p_J}{w} + \frac{V_2^2}{2g} = Z_B + h_{f_2} \quad ..(ii)$$

Applying Bernoulli's equation to J and C, we have,

$$Z_J + \frac{p_J}{w} + \frac{V_2^2}{2g} = Z_C + h_{f_2} \quad \text{..(iii)}$$

Total discharge from pipe 1 will be equal to the sum of discharge from pipes 2 and 3.

$$\therefore \quad Q_1 = Q_2 + Q_3$$

$$\therefore \quad a_1V_1 = a_2V_2 + a_3V_3$$

With the help of the above four equations the desired quantities can be evaluated.

Problem - 20 : *A reservoir P supplies water to two reservoirs Q and R situated at a lower level. The difference of levels between the surfaces of P and Q is 30 m and between P and R is 40 m. A pipe line, 30 cm diameter, supplies water from the tank P to the junction S situated at a distance of 300 m from the tank P. From the junction point S, a 15 cm pipe line 450 m long supplies water to the tank Q while a pipe line, 25 cm diameter and 150 m long, supplies water to tank R. Find the quantity of water discharged per minute into the reservoirs Q and R. Neglect all losses except friction. Take f = 0·01.*

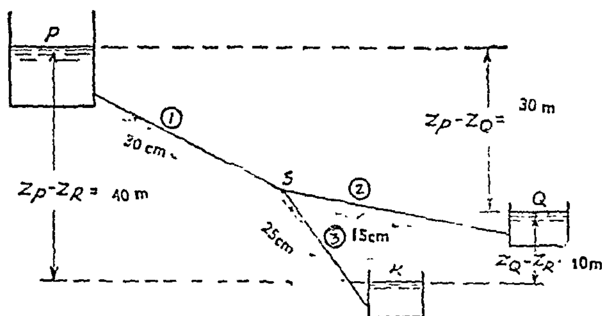


Fig. 7-13

The various frictional heads are

$$h_{f_1} = \frac{fl_1Q_1^2}{3d_1^5} = \frac{0{\cdot}01 \times 300 \times Q_1^2}{3 \times (0{\cdot}3)^5} = 412Q_1^2 \text{ m}$$

$$h_{f_2} = \frac{fl_2Q_2^2}{3d_2^5} = \frac{0{\cdot}01 \times 450 \times Q_2^2}{3 \times (0{\cdot}15)^5} = 19{,}700\ Q_2^2 \text{ m}$$

$$h_{f3} = \frac{f l_3 Q_3^2}{3 d_3^5} = \frac{0.01 \times 150 \times Q_3^2}{10 \times (0.25)^5} = 512 Q_3^2 \text{ m}$$

Further, $$V_1^2 = \left(\frac{Q_1}{A_1}\right)^2 = \left\{\frac{Q_1}{\frac{\pi}{4}(0.3)^2}\right\}^2 = \frac{1}{0.005} Q_1^2$$

$$V_2^2 = \left(\frac{Q_2}{A_2}\right)^2 = \left\{\frac{Q_2}{\frac{\pi}{4}(0.15)^2}\right\}^2 = \frac{1}{0.00312} Q_2^2$$

$$V_3^2 = \left(\frac{Q_3}{A_3}\right)^2 = \left\{\frac{Q_3}{\frac{\pi}{4}(0.25)^2}\right\}^2 = \frac{1}{0.00241} Q_3^2$$

$$\therefore \quad \frac{V_1^2}{2g} = 10.2\, Q_1^2, \frac{V_2^2}{2g} = 163.5 Q_2^2, \frac{V_3^2}{2g} = 21.1 Q_3^2$$

Applying Bernoulli's equation to tank P and junction S, we have

$$Z_P = Z_s + \frac{p_s}{w} + \frac{V_1^2}{2g} + h_{f1} \qquad \text{..(i)}$$

Similarly for points S and Q, we have,

$$Z_s = \frac{p_s}{w} + \frac{V_2^2}{2g} = Z_Q + h_{f2} \qquad \text{..(ii)}$$

and for points S and R, we have,

$$Z_s = \frac{p_s}{w} + \frac{V_3^2}{2g} = Z_R + h_{f3} \qquad \text{..(iii)}$$

Subtracting (ii) from (i), we have.

$$(Z_P - Z_Q) - h_{f2} = h_{f1} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

or $$30 - 19{,}700 Q_2^2 = 412 Q_1^2 + 10.2 Q_1^2 - 163.5 Q_2^2$$

$$\therefore \quad 30 = 422.2 Q_1^2 + 19{,}536.3 Q_2^2$$

$$\therefore \quad Q_2 = \sqrt{0.001535 - 0.0216 Q_1^2} \qquad \text{..(iv)}$$

Subtracting (iii) from (i), we have,

$$(Z_P - Z_R) - h_{f3} = h_{f1} + \frac{V_1^2}{2g} - \frac{V_3^2}{2g}$$

or $40 = 512Q_3^2 + 412Q_1^2 + 10{\cdot}2Q_1^2 - 21{\cdot}1Q_3^2$

$\therefore$ $40 = 422{\cdot}2Q_1^2 + 490{\cdot}4Q_3^2$

or $40 = 422{\cdot}2Q_1^2 + 490{\cdot}9Q_3^2$ ..(v)

$\therefore$ $Q_3 = \sqrt{0{\cdot}0815 - 0{\cdot}86Q_1^2}$

Now $Q_1 = Q_2 + Q_3 = \sqrt{0{\cdot}001535 - 0{\cdot}0216Q_1^2} + \sqrt{0{\cdot}0815 - 0{\cdot}86Q_1^2}$

Try $Q_1 = 0{\cdot}2$. Then, L. H. side = 0·2 and R.H. side = 0·211

The right hand and left hand sides are nearly equal.

Hence, $Q_1 = 0{\cdot}21$ m³/sec or $0{\cdot}21 \times 60 = 12{\cdot}6$ m³/min.

$Q_2 = \sqrt{0{\cdot}001535 - 0{\cdot}0216 \times 0{\cdot}21^2} = 0{\cdot}0241$ m³/sec
or 1·446 m³/min.

$Q_3 = \sqrt{0{\cdot}0815 - 0{\cdot}86 \times 0{\cdot}4^2} = 0{\cdot}209$ m³/sec
or 12·54 cu.m/min

Case 2 : Here, tanks A and B are supplying water to the junction J, from which water flows to the tank C through one pipe.

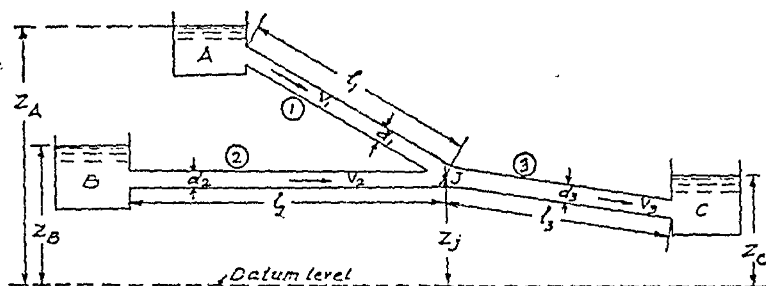


Fig. 7–14 Branching of supply mains

Applying Bernoulli's equation to points A and J, we have

$$Z_A = Z_J + \frac{p_J}{w} + \frac{V_1^2}{2g} + h_{f1} \quad ..(\text{i})$$

Applying Bernoulli's equation to points B and J, we have,

$$Z_B = Z_J + \frac{p_J}{w} + \frac{V_2^2}{2g} + h_{f2} \quad ..(\text{ii})$$

Applying Bernoulli's equation to points J and C, we have,

$$Z_J + \frac{p_J}{w} + \frac{V_3^2}{2g} = Z_C + h_{f_3} \qquad .. \text{(iii)}$$

Also, $Q_1 + Q_2 = Q_3$

or $A_1V_1 + A_2V_2 = A_3V_3 \qquad .. \text{(iv)}$

The desired quantities can be evaluated with the help of these four equations.

Problem – 21 : *Two reservoirs A and B having water levels at 60 m and 30 m above datum, feed through 30 cm and 25 cm mains respectively into a common main of 45 cm diameter which ultimately supplies water to a reservoir C situated in such a way that water level in it is 15 m above the datum. If the lengths of 30 cm, 25 cm, and 45 cm diameter pipes are respectively 3, 2·5 and 4 km, determine the rate of flow of water into the tank C. If the height of the junction point J is 20 m above datum, determine the pressure at the junction. Assume f = 0·005 and neglect all losses except friction.*

Loss of head in the pipe AJ *i. e.* pipe 1 is

$$h_{f_1} = \frac{4 \times 0{\cdot}005 \times 3 \times 1{,}000}{30/100} \times \frac{V_1^2}{2g} = 200\,\frac{V_1^2}{2g}$$

Loss of head in the pipe BJ *i. e.* pipe 2 is

$$h_{f_2} = \frac{4 \times 0{\cdot}005 \times 2{\cdot}5 \times 1{,}000}{25/100} \times \frac{V_2^2}{2g} = 200\,\frac{V_2^2}{2g}$$

Loss of head in the pipe JC *i. e.* pipe 3 is

$$h_{f_3} = \frac{4 \times 0{\cdot}005 \times 4 \times 1{,}000}{0{\cdot}45} \times \frac{V_3^2}{2g} = 178\,\frac{V_3^2}{2g}$$

Applying Bernoulli's equation to the tank A and the junction point J, we have,

$$Z_A = Z_J + \frac{p_J}{w} + \frac{V_1^2}{2g} + 200\,\frac{V_1^2}{2g} \qquad .. \text{(i)}$$

Applying Bernoulli's equation to tank B and the junction J,

$$Z_B = Z_J + \frac{p_J}{w} + \frac{V_2^2}{2g} + 200\,\frac{V_2^2}{2g} \qquad .. \text{(ii)}$$

Applying Bernoulli's equation to the junction J and the tank C,

$$Z_J + \frac{p_J}{w} + \frac{V_3^2}{2g} = Z_C + 178\frac{V_3^2}{2g} \quad .. \text{(iii)}$$

Subtracting (iii) from (i), we have,

$$(Z_A - Z_C) - 178\frac{V_3^2}{2g} = 201\frac{V_1^2}{2g} - \frac{V_3^2}{2g}$$

$$\text{or } (60-15) - 177\frac{V_3^2}{2g} = 201\frac{V_1^2}{2g}$$

$$\therefore\ V_1 = \sqrt{\frac{45\times 19\cdot 62}{201} - \frac{177}{201}V_3^2} = \sqrt{4\cdot 4 - 0\cdot 88\,V_3^2} \quad ..\text{(iv)}$$

Similarly, subtracting (iii) from (ii), we have,

$$(Z_B - Z_C) - 178\frac{V_3^2}{2g} = 201\frac{V_2^2}{2g} - \frac{V_3^2}{2g}$$

$$\text{or } (30-15) - 177\frac{V_3^2}{2g} = 201\frac{V_2^2}{2g}$$

$$\therefore\ V_2 = \sqrt{\frac{15\times 19\cdot 62}{201} - \frac{177}{201}V_3^2} = \sqrt{1\cdot 465 - 0\cdot 88\,V_3^2} \quad ..\text{(v)}$$

Now, $Q = A_1V_1$; $Q_2 = A_2V_2$ and $Q_3 = A_3V_3$

and $Q_1 + Q_2 = Q_3$

$\therefore\ A_1V_1 + A_2V_2 = A_3V_3$

i.e. $\frac{\pi}{4}d_3^2 \times V_3 = \frac{\pi}{4}d_1^2V_1 + \frac{\pi}{4}d_2^2V_2$

$$\text{or } \left(\frac{45}{100}\right)^2 V_3 = \left(\frac{30}{100}\right)^2 V_1 + \left(\frac{25}{100}\right)^2 V_2$$

$$\text{i.e. } 2\cdot 03V_3 = 0\cdot 9V_1 + 0\cdot 625V_2 \quad .. \text{(iv)}$$

From eqn. (iv), (v) & (vi), we have,

$$2\cdot 03V_3 = 0\cdot 9\sqrt{4\cdot 4 - 0\cdot 88V_3^2} + 0\cdot 625\sqrt{1\cdot 465 - 0\cdot 88\,V_3^2}$$

To find out approximate value of V_3 by trial, first neglect $0\cdot 88V_3^2$ in the R. H. S. and find V_3. Then real value of V_3 will be less than one found out and which is 1·3 m/sec. Let it be 1 m/sec.

The value of $V_3 = 1$ gives L.H.S. = 2·167 and R.H.S = 2·03. This value is lower, and higher value should be taken, say $V_3 = 1{\cdot}05$ m/sec. L. H. S. = 2·1 and R. H. S. = 2·13. Thus, L. H. S. ≃ R. H. S. when $V_3 = 1{\cdot}05$ m/sec.

$$\therefore \quad V_1 = \sqrt{4{\cdot}4 - 0{\cdot}88\,V_3^2} = 1{\cdot}85 \text{ m/sec.}$$

$$V_2 = \sqrt{1{\cdot}465 - 0{\cdot}88\,V_3^2} = 0{\cdot}705 \text{ m/sec.}$$

$$Q = A_3 V_3 = \frac{\pi}{4}\left(\frac{45}{100}\right)^2 \times 1{\cdot}05 = 0{\cdot}167 \text{ m}^3\text{/sec.}$$

$$\text{Again, } Z_A = Z_J + \frac{p_J}{w} + 200\,\frac{V_1^2}{2g} + \frac{V_1^2}{2g} \;\{\text{eqn. (i)}\}$$

$$\therefore \quad \frac{p_J}{w} = (Z_A - Z_J) - 201\,\frac{V_1^2}{2g} = (60 - 20) - 201 \times \frac{3{\cdot}44}{19{\cdot}62} = 4{\cdot}7 \text{ m of water}$$

$$\therefore \quad p_J = \frac{4{\cdot}7 \times 1{,}000}{100 \times 100} = 0{\cdot}47 \text{ kg/cm}^2$$

Transmission of Power through Pipes

We know that when water is to be transmitted from the reservoir to the power-generating station, it is transmitted over a long distance under high pressure. The power generated finally depends on pressure-head at the generating station and the quantity of water supplied to it. The pressure-head depends on the amount of loss of head in the pipe (penstock) due to friction as the supply head is constant which depends on the level of water in the reservoir above the discharge end of pipe at the generating station. Thus more power will be available if the loss is minimum or in other words more power is transmitted through pipe. The efficiency of transmission will be maximum if the loss of power in the pipe line is minimum.

Let, H – pressure-head at entrance to the pipe,
h_f – head loss due to friction in the pipe,
l – length of the pipe, and
V – velocity of water in the pipe.

Total head available at outlet of pipe is

$$H - h_f = H - \frac{4fl}{d}\cdot\frac{V^2}{2g}$$

$$\text{Available horse-power} = W\left(H - \frac{4fl}{d}\frac{V^2}{2g}\right)$$

$$\text{But weight, } W = wQ = wAV = w\,\frac{\pi}{4}\,d^2V$$

$$\therefore \quad \text{available H. P.} = \frac{w\,\frac{\pi}{4}\,d^2\left(HV - \frac{4fl}{d}\frac{V^3}{2g}\right)}{75}$$

This will be maximum when the expression $HV - \frac{4fl}{d}\frac{V^3}{2g}$ has least value.

Differentiating this expression w. r. t. V, we have,

$$H - \frac{3 \times 4fl}{d}\frac{V^2}{2g} = 0$$

$$\text{or } H - 3h_f = 0 \qquad \text{as } \frac{4fl}{d}\frac{V^2}{2g} = h_f$$

$$\therefore \quad h_f = \frac{H}{3}$$

$$\text{Efficiency, } \eta \text{ of transmission} = \frac{W(H - h_f)}{WH} = \frac{H - h_f}{H}$$

$$\text{Maximum } \eta \text{ of transmission} = \frac{H - H/3}{H} \times 100 = 66{\cdot}67\%$$

Problem – 22 : *A hydraulic machine is supplied with water through a horizontal pipe line 1·5 km long and having an efficiency of transmission 80%. A gauge fitted at the supply end of the pipe line shows the pressure as 56 kg/cm². Mechanical efficiency of the machine is 90% and its output is 60 H.P. If coefficient of friction for the pipe is 0·007, determine (a) pressure guage reading at the machine, (b) velocity of water in the pipe, and (c) the diameter of the pipe.*

$$\text{Efficiency of transmission, } \eta = \frac{H - h_f}{H}$$

$$\text{But, } H = \frac{10{,}000 \times 56}{1{,}000} = 560 \text{ m of water}$$

$$\therefore \quad 0{\cdot}8 = \frac{560 - h_f}{560}$$

$$\therefore \quad h_f = 560\,(1 - 0{\cdot}8) = 112 \text{ m of water}$$

$$\text{Again, } h_f = \frac{4fl}{d} \cdot \frac{V^2}{2g}$$

$$\text{or } 112 = \frac{4 \times 0{\cdot}007 \times 1{,}500 \times V^2}{d \times 19{\cdot}62}$$

$$\therefore \quad \frac{V^2}{d} = 52{\cdot}4$$

$$\therefore \quad d = \frac{V^2}{52{\cdot}4} \qquad \ldots (\text{i})$$

Available head $= 560 - 112 = 448$ m

$$\therefore \quad p = \frac{wH}{10^4} = \frac{448 \times 1{,}000}{10{,}000} = 44{\cdot}8 \text{ kg/cm}^2$$

$$\text{Available H.P.} = \frac{448 \times W}{75}$$

Output H. P. = available H. P. × Mech. η

$$\therefore \quad 60 = \frac{448 \times W}{75} \times 0{\cdot}9$$

$$\therefore \quad W = \frac{60 \times 75}{448 \times 0{\cdot}9} = 11{\cdot}12 \text{ kg/sec}$$

Also $W = wAV$

$$\text{or } 11{\cdot}12 = 1{,}000 \times \frac{\pi}{4} d^2 V$$

$$\therefore \quad d^2 V = \frac{4 \times 11{\cdot}12}{\pi \times 1{,}000} = 0{\cdot}0142 \qquad \ldots (\text{ii})$$

From (i) and (ii), we have,

$$\frac{V^4}{52{\cdot}4^2} \times V = 0{\cdot}0142$$

$$\therefore \quad V = (52{\cdot}4^2 \times 0{\cdot}0142)^{\frac{1}{5}} = \underline{2{\cdot}08 \text{ m/sec}}$$

$$\text{and } \quad d = \frac{V^2}{52{\cdot}4} \text{ m} \quad \therefore \quad d = \frac{(2{\cdot}08)^2}{52{\cdot}4} \times 100 = 8{\cdot}25 \text{ cm}$$

Flow through Nozzles

A nozzle is fitted at the end of a pipe which is used for transmission of power as in the case of hydraulic-power station. Nozzle is a tapering mouth-piece which converts total head available at the nozzle into equivalent velocity head. The pressure

at outlet end of the nozzle is atmospheric hence the energy is entirely in kinetic form.

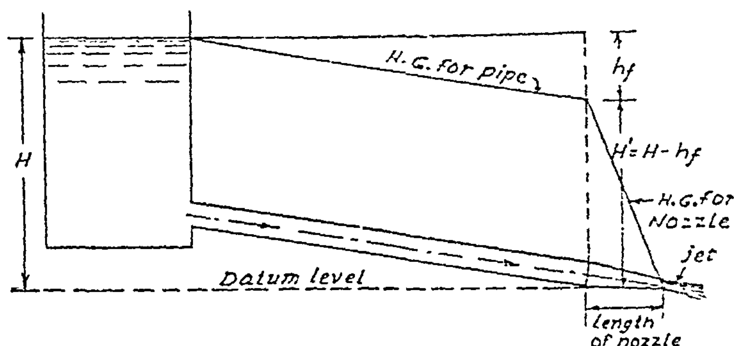


Fig. 7-15 Flow through nozzle

The head available to the nozzle is

$$H' = H - h_f$$

For maximum transmission of power, $h_f = \dfrac{H}{3}$

$$\therefore \quad H' = H - \frac{H}{3} = \tfrac{2}{3}H$$

But, $3h_f = H$

$$\therefore \quad H' = \tfrac{2}{3} \times 3h_f = 2h_f$$

or $h_f = \frac{1}{2}H'$

Now the whole available head is converted into velocity head.

$$\therefore \quad H' = \frac{V_1^2}{2g}$$ where V_1 is velocity of the jet.

$$\therefore \quad h_f = \frac{1}{2}\frac{V_1^2}{2g}$$

and $h_f = \dfrac{4fl}{d} \cdot \dfrac{V^2}{2g}$ where V is velocity of water in the pipe line.

$$\therefore \quad \frac{4fl}{d} \cdot \frac{V^2}{2g} = \frac{1}{2}\frac{V_1^2}{2g}$$

$$\therefore \quad \frac{8fl}{d} = \frac{V_1^2}{V^2} \qquad ..(i)$$

Also discharge, $Q = \frac{\pi}{4} d^2 V = \frac{\pi}{4} d_1^2 V_1$

where d = diameter of the pipe line, and
d_1 = diameter of the nozzle.

$$\therefore \quad \frac{V_1}{V} = \left(\frac{d}{d_1}\right)^2 \qquad ..(ii)$$

From (i) and (ii), we have,

$$\frac{8fl}{d} = \left\{\left(\frac{d}{d_1}\right)^2\right\}^2 = \left(\frac{d}{d_1}\right)^4$$

$$\therefore \quad \frac{d}{d_1} = \left(\frac{8fl}{d}\right)^{\frac{1}{4}}$$

This holds good only for maximum H.P. condition.

$$\text{H.P. of the jet} = \frac{wA_1V_1}{75} \times \frac{V_1^2}{2g} = \frac{wA_1V_1^3}{2g \times 75} \qquad ..(7\cdot21)$$

In case the maximum H.P. condition is not given, then neglecting minor losses, we have,

$$H' = H - h_f = H - \frac{4fl}{d} \cdot \frac{V^2}{2g}$$

Also, $H' = \frac{V_1^2}{2g}$

$$\therefore \quad H - \frac{4fl}{d} \cdot \frac{V^2}{2g} = \frac{V_1^2}{2g}$$

But, $AV = A_1 V_1 \quad \therefore \quad V = \frac{A_1}{A} V_1$

$$\therefore \quad H - \frac{4fl}{d}\left(\frac{A_1}{A}\right)^2 \frac{V_1^2}{2g} = \frac{V_1^2}{2g}$$

$$\text{or} \quad H = \left\{1 + \frac{4fl}{d}\left(\frac{A_1}{A}\right)^2\right\}\frac{V_1^2}{2g}$$

$$\therefore \quad V_1 = \sqrt{\frac{2gH}{1 + \dfrac{4fl \cdot A_1^2}{d \cdot A^2}}}$$

$$\text{and H.P. of the jet} = \frac{wA_1V_1^3}{2g \times 75} \qquad ..(7\cdot12)$$

Problem–23 : *A 15 cm diameter pipe 2,500 m long is supplied with water from a reservoir under pressure of 7 kg/cm². Determine the H.P. delivered by 4 cm diameter nozzle fitted at the end of pipe line if coefficient of friction is 0·01. Neglect all minor losses except friction.*

The velocity of water from the nozzle is

$$V_1 = \sqrt{\frac{2gH}{1 + \dfrac{4fl}{d}\left(\dfrac{A_1^2}{A^2}\right)}}$$

$$\text{and } H = \frac{10{,}000 \times 7}{1{,}000} = 70 \text{ m of water}$$

$$\text{and } \frac{A_1}{A} = \left(\frac{d_1}{d}\right)^2 = \left(\frac{4}{15}\right)^2 = \frac{1}{14\cdot05} \text{ and } \left(\frac{A_1}{A}\right)^2 = \frac{1}{197\cdot5}$$

$$\therefore \quad V_1 = \sqrt{\frac{19\cdot62 \times 70}{1 + \dfrac{4 \times 0\cdot01 \times 2{,}500}{0\cdot15} \times \dfrac{1}{197\cdot5}}} = 17\cdot8 \text{ m/sec.}$$

$$\text{H. P.} = \frac{wA_1V_1^3}{2g \times 75} = \frac{1{,}000 \times \dfrac{\pi}{4}\left(\dfrac{15}{100}\right)^2 \times 17\cdot8^3}{19\cdot62 \times 75} = 67 \text{ H.P}$$

Problem–24 : *A 5 cm diameter, 60 m long pipe is fed with water under a head of 6 m at one end, while a nozzle 6 mm diameter is fitted at the other end. Find the height to which a jet for a fountain will reach. Take f = 0·01.*

The ratio of areas of nozzle and pipe is

$$\frac{A_1}{A} = \left(\frac{d_1}{d}\right)^2 = \left(\frac{0\cdot6}{5}\right)^2 = 0\cdot0144 \text{ and } \left(\frac{A_1}{A}\right)^2 = 0\cdot000208$$

The velocity of the jet is

$$V_1 = \sqrt{\frac{2gH}{1 + \frac{4fl}{d}\left(\frac{A_1}{A}\right)^2}} = \sqrt{\frac{2 \times 9{\cdot}81 \times 6}{1 + \frac{4 \times 0{\cdot}01 \times 60 \times 0{\cdot}000208}{5/100}}}$$

$$= 10{\cdot}7 \text{ m/sec.}$$

Equivalent head for this velocity $= \frac{10{\cdot}7^2}{2g} = \frac{10{\cdot}7^2}{2 \times 9{\cdot}81} = 5{\cdot}93$ m

The jet will rise to 5·93 m height

Water Hammer

A pipe line is conveying water and is fitted with a regulating valve. If this valve is suddenly closed when water is fully flowing through it; water near the valve end will immediately come to rest, the velocity head getting converted into pressure, while water behind it is moving with high velocity and will cause sudden rise of pressure because of the momentum being destroyed. This rise of pressure will be transmitted along the pipe line in the form of pressure waves which will cause knocking The magnitude of pressure is dependent on the length of the line and the speed of closing the valve.

Let, l – length of the pipe line,

A – area of cross-section of the pipe,

V – velocity of water in the pipe, and

t – time taken by water to come to rest on closing a valve.

Retardation of water is, $a = \frac{V}{t}$

Mass of moving water $= \frac{W}{g} = \frac{wAl}{g}$

Since, force = mass × retardation

Force on the valve, $P = \frac{wAl}{g} \times a = \frac{wAl}{g} \times \frac{V}{t}$

Intensity of pressure on valve is

$$p = \frac{P}{A} = \frac{wAl}{A \times g} \times \frac{V}{t} = \frac{wlV}{gt} \qquad \text{.. (7.12)}$$

The sudden rise in pressure in a pipe due to stoppage of flow is known as hammer blow or water hammer.

Problem-25 : *Water is supplied to a hydraulic turbine through a pipe line 800 m long. Water flows in the pipe line at the rate of 1·2 m/sec. A valve near the turbine end is closed in 30 sec. Find the rise in pressure due to this if the elasticity of pipe wall and the compressibility of liquid are neglected.*

According to eqn. (7·12), pressure rise is, $p = \frac{wlV}{gt}$

$$\therefore \quad p = \frac{1{,}000 \times 800 \times 1{\cdot}2}{9{\cdot}81 \times 30} = 3{,}260 \text{ kg/m}^2$$

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TUTORIAL-7

Fluid friction

1 Assuming frictional resistance to be 5 kg/m² at 4·6 m/sec find the propelling force in kg for a steamer having a speed of 24 km/hr and immersed surface of 1,200 m². Find also the horse-power required to propell the steamer.

[12,550 kg; 1,120 H.P.]

2 A steamer has an immersed surface area of 2,000 m². The steamer requires 2,200 H.P. when steaming at a speed of 20 km/hr. Determine the increase in the propelling force when the speed of the steamer is increased to 24 km/hr.

[13,050 kg]

3 Oil of sp. gr. 0·8 and viscosity 0·0125 kg/sec/m² is flowing in a pipe of 22·5 cm diameter. If the discharge is 1,810 litres/min state whether the flow is laminar or turbulent.

[Laminar]

Hydraulic gradient

4 Water flows through a pipe, 45 cm diameter and 3·2 km long at the rate of 150 litres/sec. Find the head lost in friction, (a) using Chezy's formula, (b) using Darcy's formula.
[(a) 13·6 m of water, (b) 13·4 m of water]

5 A cast iron pipe, 22·5 cm diameter and 600 m long connects two reservoirs. If the difference of water levels in the two reservoirs is 30 m find the discharge through the pipe. Take $f=0.01$. Ignore all losses other than friction.
[93·2 litres/sec.]

6 A cast iron pipe, 25 cm diameter conveys water at the rate of 80 litres/sec from one reservoir to another reservoir. If the difference in water level between the two reservoirs is 50 m, find the length of the pipe line between the two reservoirs. Friction coefficient for the pipe is 0·01. Ignore all losses other than friction and velocity head.
[2,280 m]

7 A tank containing water upto a depth of 2 m above the entrance of 15 cm diameter pipe line discharges into the atmosphere. The pipe is laid at a slope of *1 in 100* and is 60 m long. Determine the discharge in litres/sec. Take $f=0.01$.
[32]

8 A pipe 100 m long discharges water into the atmosphere from a tank containing water upto a height of 4 m above the entrance of a pipe. The pipe is laid at a slope of 1 in 150. If the discharge is 60 litres/sec., determine the diameter required for the pipe. Take friction coefficient as 0·008, and take all the losses into account.
[18·5 cm]

9 Two reservoirs *A* and *B* are connected by a pipe *30 cm* diameter. Water in the tank *A* is 10 m above the centre of the pipe at inlet end while that in the tank *B* is 3 m above the centre of the pipe at outlet end. The pipe line is placed at a certain gradient for the first 2 km and then it is laid horizontally for 3 km and then slopes up at 1 in 200 for 1 km. If

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[18·5 cm]

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[93·2 litres/sec.]

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[2,280 m]

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[18·5 cm]

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the pipe discharges water at rate of 60 litres/sec into the tank B; determine the gradient of the first 2 km pipe line and the pressure at the end of 2 km pipe line. Neglect all losses except friction. Take $f = 0{\cdot}01$.

[1·62 in 1,000; 11·5 m of water]

Loss due to friction

10 A tank supplies water to a short horizontal pipe 120 m long and open to atmosphere at the discharge end. The water level in the tank is 8 m above the centre of the pipe line. The pipe is 20 cm in diameter for first 20 m length and then it suddenly enlarges to 30 cm diameter for 50 m length and then it suddenly reduces to 25 cm diameter for the rest of its length. Taking all losses into account, find the discharge in litres/sec. Take $C_c = 0{\cdot}6$ and $f = 0{\cdot}01$.

[124·5 litres/sec]

11 (a) A pipe of 20 cm diameter is 60 m long and carries discharge of 100 litres/sec. Find the energy lost per second due to friction.

(b) Now the central 40 m length of pipe is replaced by a pipe of 30 cm diameter, the changes of sections being sudden. What will be the saving in H.P. due to adoption of the alternative ? Take $C_c = 0{\cdot}6$ and $f = 0{\cdot}01$

[(a) 618 kg-m, (b) 4·26 H.P.]

12 A jet condenser of a marine engine is 2·44 m below the water line. If the vaccum in the condenser is 102 mm of mercury, calculate the quantity of cooling water passing through 38 mm diameter pipe 18·3 m long. Take $f = 0{\cdot}01$.

[2·23 litres/sec]

13 A town having population of 2,20,000 is to be supplied with water from a reservoir at a distance of 10 km through a pipe line 60 cm in diameter. It is stipulated that one half of the daily supply of 120 litres per head should be delievered in 6 hours. What must be the head at which water should be supplied from the tank if $C = 90$ for Chezy formula ?

[38·2 m]

14 Power is to be transmitted hydraulically over a distance of 15 km by means of 4 pipe lines each 20 cm diameter and laid at a slope of 1 in 1,500. The supply end has constant pressure of 390 m of water. The efficiency of transmission for the pipe line is 80% and the friction coefficient is 0·008. Determine the total H.P. delivered by the pipe lines.

[432 H.P.]

15 Power is to be transmitted hydraulically over a distance of 8 km by means of a number of 25 cm pipes laid at a slope of 1 in 800. The pressure at the inlet end is maintained constant at 80 kg/cm^2. Determine the minimum number of pipes required to ensure transmission efficiency of 85% when 975 H.P. is delivered. Friction coefficient for the pipes may be taken as 0·01.

[2]

16 In the distribution system of water supply for a small town, water is supplied through a 30 cm main and then through a 1·5 cm lead service pipe to each house. The highest level of one of these service pipes is 10 m above the main and its total length is 60 m. If the pressure in the main at the point of take-off of the service pipe is 1·5 kg/cm^2, what quantity of water should be expected from the highest point ?

[39 litres/min.]

17 In the distribution system of water supply for a small town, water is supplied through a 20 cm diameter pipe line and then through a 1·5 cm lead service pipe to each house. The highest level of one of these service pipes is 16 m above the main and its total length is 100 m. If the pressure in the main at the point of take off of the service pipe is 4 kg/cm^2, what quantity of water should be expected from the highest point ? Take the Chezy constant as 50.

[15·96 litres/min.]

18 A part of a straight length of a pipe conveys water at the rate of 80 litres/sec., and it slopes at 1 in 150 in the upward direction between the points A and D. The first 60 m of pipe

(AB) is 30 cm in diameter, and the last 50 m of pipe (CD) is 20 cm in diameter, and both are connected by a tapering pipe BC 10 m long tapering from 30 cm diameter at B to 20 cm diameter at C. The pressure at end A of 30 cm diameter pipe is 10 kg/cm². If the flow in the pipe is in the upward direction, find the pressures at points B, C and D assuming there is loss of head in the tapering pipe equivalent to 4% of the mean velocity head. Take $f = 0.008$ for both pipes.

[$p_b = 0.918$ kg/cm², $p_c = 0.7$ kg/cm², $p_d = 0.625$ kg/cm²]

19 A compound pipe line laid up at a slope of 1 in 100 consists of portion AB of uniform diameter '*d*' for 100 m length; then tapering portion BC which tapers from diameter '*d*' to diameter '2*d*' in 10 m length; and portion CD of uniform diameter '2*d*' for 300 m length. When water flows through the pipe the difference of pressures between the entrance end A and the delivery end D is 14.43 kg/cm² and the pressure difference between the points A and B is 13.2 kg/cm². The loss of head in tapering portion BC is 5% of the mean velocity head. Determine the discharge in litres/sec and the diameter of the pipe AB as well as BC.

[15.9 litres/sec.; 10.2 cm; 20.4 cm]

20 A pipe PQRS of uniform diameter is made up of three straight lengths such that PQ measures 200 m, QR 300 m, and RS 100 m. The survey levels of the points P,Q,R & S are 150 m, 60 m, 200 m and 250 m respectively above the datum. A gauge at Q shows pressure of 15 kg/cm² and at S 2 kg/cm². Neglecting all losses except frictional, find in which direction water will flow and state the pressures at P and R.

[from S towards Q; 4 kg/cm²; 6 kg/cm²]

Syphon

21 A reservoir P supplies water to a lower level reservoir Q through a 30 cm diameter pipe line, 4 km long, which rises above the level of water in the reservoir P by 4 m. The pipe

length between the reservoir P and highest point R is 600 m. Determine the pressure at the point R if discharge is 50 litres/sec. What will be the maximum percentage increase in discharge if separation takes place when pressure inside the pipe falls below 0·3 kg/cm² absolute ? Take f = 0·008. Barometer reads 10 m of water.

[0·432 kg/cm²; 33·4%]

22 A pipe line, 8 cm in diameter 15 m long, has its ends connected to two different tanks and it is used as a model of a syphon. The difference between water levels in two tanks is 2·5 m. The length of the pipe line from inlet to summit is 6 m at a slope of 1 in 5. The length of this pipe line below water level in the supply tank is 1·5 m The coefficient of friction is 0·008. Determine the discharge in litres/min. and the pressure at summit. If the vapour pressur is 0·25 kg/cm² absolute, state whether the syphon will operate or not.

[772 litres/min ; 0·78 kg/cm²; operates]

23 A pipe line 25 m long has its end connected to two different tanks and it is used as a model of a syphon discharging 3,600 litres/min. The difference between water levels in two tanks is 4 m. The length of the pipe line from inlet summit is 8 m at a slope of 1 in 8. The length of a portion of this pipe line (on L.H.S. of summit) below water level in the supply tank is 3 m. Take f = 0·008. Determine the diameter of the pipe and the pressure at summit If the vapour pressure is 0·32 kg/cm² absolute; state whether the syphon will operate or not. Atmospheric pressure = 10 m of water.

[13 m]

Equivalent pipe

24 Find the diameter of a uniform pipe necessary to replace a compound pipe line having the following sections : 4·5 cm. diameter pipe for 600 m length, 45 cm diameter pipe for 350 m length, and 30 cm diameter pipe for 750 m length. The coefficient of friction for the pipe line is 0·01. The pressure at inltet end is 15 kg/cm² Determine the discharge in litres./min.

[28 cm; 1,280 litres/min,]

(AB) is 30 cm in diameter, and the last 50 m of pipe (CD) is 20 cm in diameter, and both are connected by a tapering pipe BC 10 m long tapering from 30 cm diameter at B to 20 cm diameter at C. The pressure at end A of 30 cm diameter pipe is 10 kg/cm². If the flow in the pipe is in the upward direction, find the pressures at points B, C and D assuming there is loss of head in the tapering pipe equivalent to 4% of the mean velocity head. Take $f = 0.008$ for both pipes.

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[15·9 litres/sec.; 10·2 cm; 20·4 cm]

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[13 m]

Equivalent pipe

24 Find the diameter of a uniform pipe necessary to replace a compound pipe line having the following sections : 4·5 cm. diameter pipe for 600 m length, 45 cm diameter pipe for 350 m length, and 30 cm diameter pipe for 750 m length. The coefficient of friction for the pipe line is 0·01. The pressure at inltet end is 15 kg/cm². Determine the discharge in litres./min.

[28 cm; 1,280 litres/min.]

25 Find the diameter of a uniform pipe necessary to replace a compound pipe line having the following sections :– 10 cm diameter pipe for 1 km length; 20 cm diameter pipe for 2 km length, 25 cm diameter pipe for 1·5 km length and 30 cm diameter pipe for 3·5 km length. The coefficient of friction for the pipe line is 0·01. The pressure at inlet is 20 kg/cm^2 while that at the discharge end is 4 kg/cm^2. Determine the discharge in litres/min.

[15 cm; 1,284]

Parallel flow

26 Two pipes A and B 30 cm and 45 cm diameter respectively, branch from a point C to a point D, which is 6 m below C. Pipe A is 600 m long and pipe B is 900 m long. Water is supplied at C at a pressure of 1·4 kg/cm^2. A short pipe 22 cm diameter is fitted at D. Find the delivery when this pipe is open to atmosphere. Take $V = K\sqrt{mi}$ for pipes A and B and $K = 100$.

[640 litres/sec.]

27 Two pipes P and Q 5 cm and 15 cm diameter respectively, branch from a point R to a point S, which is 5 m below R. Pipe P is 500 m long and pipe Q is 1,500 m long. Water is supplied at R under a pressure of 3 kg/cm^2. A short pipe M, 10 cm diameter, is fitted at S. Find the delivery when this pipe is fully open to atmosphere. Take $V = C\sqrt{mi}$ for pipes P and Q and $C = 120$.

[66 litres/sec.]

28 Two pipes, one of 30 cm diameter, 450 m long and the other of 15 cm diameter, 900 m long are connected in parallel to a pipe line D cm diameter, 300 m long. Head lost in the main pipe line of diameter D is half the head lost in 30 cm pipe line. Water passes through the main pipe line at the rate of 100 litres/sec. Find the discharge through each parallel pipe line and the value of D. Take $f = 0{\cdot}005$ for all pipes.

[88·9 litres/sec; 11·1 litres/sec; D = 33·3 cm]

29 Two pipes, one of 20 cm diameter and 1 km long and other of 30 cm diameter and 2 km long are connected in parallel

to a pipe line 'd' cm in diameter and 0·5 km long. Head lost in the main pipe ('d' cm diameter) is double the head lost in 20 cm diameter pipe. Water passes through the main pipe at the rate of 80 litres/sec. Find the discharge through each of the parallel pipes and determine d. Take $f = 0{\cdot}01$ for all pipes.

[27 litres/sec.; 53 litres/sec; $d = 45{\cdot}2$ cm]

Time of emptying

30 A tank 1·25 m square is fitted with a vertical pipe, 15 cm diameter 15 m long at its base, and it discharges water into the atmosphere. The tank contains water to a depth of 4 m. Find the time taken to empty the tank. Consider all losses and take friction coefficient as 0·01.

[3 min. 3 sec.]

31 A cylindrical tank is fitted with a vertical pipe line, 10 cm in diameter, 20 m long at its base and its lower end discharges water into the atmosphere. The depth of water level changes from 5 m to 2 m in 5 min. If friction coefficient is 0·01, determine the diameter of the tank.

[1·61 m]

Flow between reservoirs

32 Two tanks P and Q, 10 m and 6 m in diameter are connected by a 40 cm diameter pipe, 800 m long. The level of water in the tank P is 16 m higher than that in tank Q when the flow was started. Determine the time taken to change water level in two tanks by 7 m; and find the amount of water that flows from the tank P to the tank Q in the above time. Take friction coefficient as 0·01.

[35 min. 20 sec.; 145 m^3]

33 A reservoir discharges to a lower reservoir through 4 km long, 30 cm diameter pipe, the difference of levels being 20 m. It is required to supply a third reservoir, the surface of which is 10 m below that of the lower reservoir, by a branch pipe taken from the 30 cm main, 2 km from the entrance end at the upper reservoir and the length of this pipe between the junction and the lowest reservoir is 3 km. Calculate the diameter of the new branch pipe in order that the flow into both lower reservoirs may be the same. Take friction coefficient as 0·01.

[25·2 cm]

34 Two reservoirs, A and B having water levels at 60 m and 45 m above datum feed through 30 cm and 20 cm mains respectively into a common main of 40 cm diameter and it supplies water to a third reservoir C having surface level 20 m above datum. If the lengths of the pipes A, B and C are respectively 6 km, 3 km and 4 km; determine the flow in each of the feeding mains, neglecting all losses of head except that due to friction. If the height of junction point, F is 25 m above datum, determine the pressure at the junction. Assume $f = 0{\cdot}008$.

[$Q_A = 68{\cdot}9$ litres/sec., $Q_B = 25{\cdot}4$ litres/sec. $Q_c = 94{\cdot}3$ litres/sec., $p_J = 0{\cdot}42$ kg/cm^2]

35 A tank A supplies water to the tank B through two parallel pipe lines each 40 cm diameter over 5 km length at a downward slope of 1 in 200 and then through 60 cm diameter single pipe line for 4 km length. The water level in tank A is 5 m above the entrance ends. The difference of water levels between the two tanks is 103 m. Determine the discharge in litres/min. and the pressure at the junction point where two pipe lines meet. Take friction coefficient as 0·008.

Power transmission

36 A hydraulic machine is supplied with water through a horizontal pipe line 6,000 m long and having an efficiency of transmission 70%. A gauge fitted at the supply end of the pipe line shows the pressure as 85 kg/cm^2. Mechanical efficiency of the machine is 80% and its output is 80 H.P. If friction coefficient is 0·008, determine, (a) gauge reading at the machine, (b) the the velocity of water in the pipe, (c) the diameter of the pipe.

[(a) 59·5 kg/min (b) 1·612 m/sec. (c) 10 cm]

37 A hydraulic machine is supplied with water through a horizontal pipe line 5 km long and having an efficiency of transmission 60%. A gauge fitted at the supply end of the pipe line shows the pressure as 60 kg/cm^2. Mechanical efficiency of the machine is 70% and its output is 60 H.P. If friction coefficient

is 0 008, determine (a) gauge reading at the machine, (b) the velocity of water in the pipe, and (c) the diameter of the pipe.

[(a) 17 9 kg/cm², (b) 1·815 m/sec , (c) 11·25 cm]

38 A hydraulic machine is supplied with water through a horizontal 30 cm diameter pipe line 6 km long. A gauge fitted at supply end reads the pressure 1·5 times the pressure read by the gauge at outlet. The output of the machine is 36 H. P. and its efficiency is 90%. If the friction coefficient is 0.005, determine, (a) the efficiency of transmission, (b) pressure at the supply end, and (c) the velocity of water in the pipe.

[(a) 66·6%, (b) 5·95 kg/cm², (c) 0·9865 m/sec.]

39 At a central power station, the pressure intensity at entrance is 70 kg/cm² and at exit end of 20 cm diameter pipe line is 40 kg/cm². Water flows through the pipe line with a velocity of 2 m/sec ; determine, (a) the efficiency of transmission, (b) the output H P. of the machine if mech. efficiency is 75%, (c) the length of the supply pipe line, if frictional coefficient is 0 01.

[(a) 57%; (b) 334 H.P. (c) 7·36 km]

40 A 25 cm diameter pipe line is fed with water under a pressure of 50 kg/cm². If the length of the pipe line is 5·5 km, determine the maximum H.P. delivered at the end of the pipe. Take friction coefficient as 0·008.

[615 H.P.]

41 At central power station the pressure intensity at entrance is 85 kg/cm² and at exit end of pipe line 25 cm diameter is 65 kg/cm². Water flows through the pipe line with velocity of 1·25 m/sec. Determine, (a) the efficiency of transmission, (b) the output H. P. of the machine if mech. efficiency is 80%, (c) the length of the supply pipe line if frictional coefficient is 0 008.

[(a) 76·5%, (b) 422 H.P (c) 1,97,000 m]

42 A 30 cm diameter pipeline is fed with water under a pressure of 70 kg/cm² If the length of the pipe is 6,000 m determine the maximum H P , delivered at the end of the pipe Take f - 0·01.

[1,045 H. P.]

Nozzle

43 A 20 cm diameter pipe 360 m long is supplied with water from a reservoir under pressure of 20 kg/cm^2. Determine the H. P. delivered by 8 cm diameter nozzle fitted at the end of the pipe line if friction coefficient is 0·008. Neglect all minor losses except friction.

[214 H. P.]

44 A 30 cm diameter pipe line 1·5 km long is used to supply water in a hydraulic power station. Water is supplied under a head of 400 m of water at one end. The pipe discharges water through a nozzle fitted to its other end. If the friction coefficient is 0·008, determine the diameter of the nozzle to deliver maximum power and what will be the H.P. delivered in that case ?

[7·1 cm; 1,030 H.P.]

45 A 25 cm diameter pipe line, 600 m long is used to supply water in a hydraulic power station. Water is supplied under a pressure of 30 kg/cm^2 at the end. The pipe discharges water through a nozzle fitted to its other end. If friction coefficient is 0·005, determine the diameter of the nozzle to deliver maximum power and what will be the H.P. delivered in the case ?

[8 cm; 840 H.P.]

46 A 5 cm diameter 600 m long pipe is fed with water under a head of 6 m at one end while a nozzle 6 mm diameter is fitted at the other end. Find the height to which a jet for a fountain will reach. Take $f = 0{\cdot}01$.

[3·54 m]

Water hammer

47 Water is supplied to a hydraulic turbine through a pipe line, 2 km long. Water flows in the pipe line at the rate of 3 m/sec. A valve near the turbine end is closed in 40 sec. Find the rise in pressure due to this if the elasticity of pipe wall and the compressibility of liquid is neglected.

[0·765 kg/cm^2]



HYDRO-ELECTRIC POWER GENERATION

Utilisation of Energy of Water

The river bed along its length represents an inclined surface along which water flows under the action of gravitational force. Situated at some height, water stores in itself energy which is dispensed while flowing along its course, thus doing work. If this energy of the river water is not utilised gainfully, the total reserve of energy is spent on friction of water with the bed, on shocks while taking turns, on washing off the banks, on carrying the

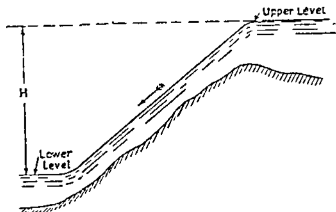


Fig. 8-1 Typical cross-section of a river along its course

soil etc. Fig. 8-1 illustrates the typical cross-section of a river along its course. The energy lost in overcoming friction and other losses is represented in the form of head H here

The energy loss on any portion of the river depends upon the veloctiy of flow of water and the nature of resistance to its flow

in that portion. To create the possibilities for utilisation of water energy of the river, hydraulic structures are erected across the river with the idea of reducing the losses accompanying the flow of water to minimum, so that the remaining energy can be utilised. This is achieved either by reducing various losses or by lowering the flow velocity on separate portions of the river. For example, at some place across the river, when a dam is built, (fig. 8-2) level of water in front of it rises upto its height and a head H_a is created. In front of the dam, the cross-section of the river greatly increases in width and dept. This results in the decrease of velocity which in turn reduces the loss of head in the portion of the river upto the dam. If before the construction of the dam, on the given

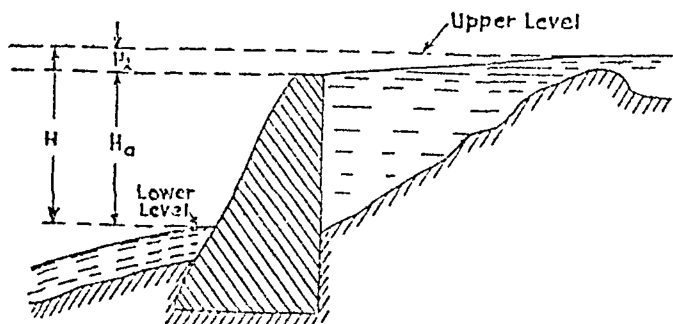


Fig. 8-2 Construction of dam across the river

portion of the river, the head, H was lost, now due to the presence of the dam only head H_l is lost and head H_a is available for converting into work with the help of hydraulic turbines. By allowing the water to flow with head H_a through the turbine, we utilise that energy which is saved because of the reduction in velocity of water in the portion of river ahead of the dam.

Let us now calculate the capacity of the hydro-electric station. Imagine that through the turbines of the station, under the head H_a, Q m³/sec. of water flows. Since it may be possible to allow all the water of the river to flow through the turbine, the quantity, Q becomes the *discharge of the river.* Weight

of water passing through the turbine per second is

$$W = w \times Q$$

where w is the weight of one m^3 of water. Falling from height H_a, the discharge, Q of water can theoretically turn out work equal to the energy of the water current,

$$\begin{aligned} P_a &= WH_a \\ &= wQH_a \text{ kg-m/sec. or} \\ &= kwQH_a \text{ } kW \qquad \text{.. (8·1)} \end{aligned}$$

where, $k = \frac{1}{102}$

The energy utilised by the turbines of the station will be somewhat less, as part of the energy of water is lost in various ways. This is accounted by the efficiency of the station, *e.g.*

$$\eta = \frac{\text{power output}}{\text{water power}} \qquad \text{.. (8·2)}$$

$$= \frac{P_o}{P_a}$$

where P_o is the power output of the station. From this,

$$\text{power output} = \eta P_a = \eta kwQH_a$$

$$= \frac{1}{102} \eta wQH_a \text{ kW} \qquad \text{.. (8·3)}$$

Development of Hydraulic Power Stations

Hydro-electric projects may be used entirely for power generation. Sometimes, they are undertaken for flood control and irrigation purposes in which case they are known as multi-purpose projects *e.g.* Bhakra-Nangal and Damodar Valley Corporation (D.V.C.) projects. The former is mainly for power generation and irrigation, whereas the latter is mainly concerned with flood control.

Hydro-electric power stations are usually situated at a great distance from the industrial area which is the consumer of electric energy to be developed by the power station. The emergence of the successful technique of high voltage transmission of electricity

possible regimes of discharge of the river, as also the maximum discharge. While selecting the size of the turbine to be installed in the station this consideration has to be carefnlly weighed : which discharge should be taken for purposes of calculation ? If the maximum possible discharge is adopted, for greater part of time the turbines would not be ensured the adequate supply of water and the turbines would not work. If the minimum discharge is taken into account, a major part of water carried by the river is required to be rejected over the dam without its utilisation and the station installation would seem to be less economical. While adopting the design discharge, this is considered : can the plant use all the water which flows to the reservoir ? Additionally these things also are required to be considered : (i) The expected electric load of the region which is to be served by the station; (ii) convenience for making the reservoir suitable for navigation, (iii) inclusion of a scheme of water supply of the place, etc. The question of selection of the hydraulic head also is not less complicated. From viewpoint of power engineering, the highest head is most advantageous. But, the greater the height of the dam, the greater is the area which gets filled with water ahead of the dam. In doing, so, the cultivable land, huge forests and a number of villages get inundated under water and the villages have to be rehabilitated at other places. In a number of cases it is necessary to construct special walls which safeguard the nearby cities from inundation. In other words, increase in the design head of the hydro-station involves greater expenditure on the equipment of the station and that way hampers the economy of a vast region. Hence the head and the discharge are selected on the basis of results of financial analysis of a great number of alternatives.

Types of Hydro Electric Stations

There are a few varieties of these stations, chief amongst them are : (a) Power stations located at dam site, and (b) Canal power stations.

Power stations located at dam site. These hydro-electric stations are characterised by the fact that all elements of the equip-

ment of the station are located in one portion of the course of the river. In this, the turbines are installed in the house of the power

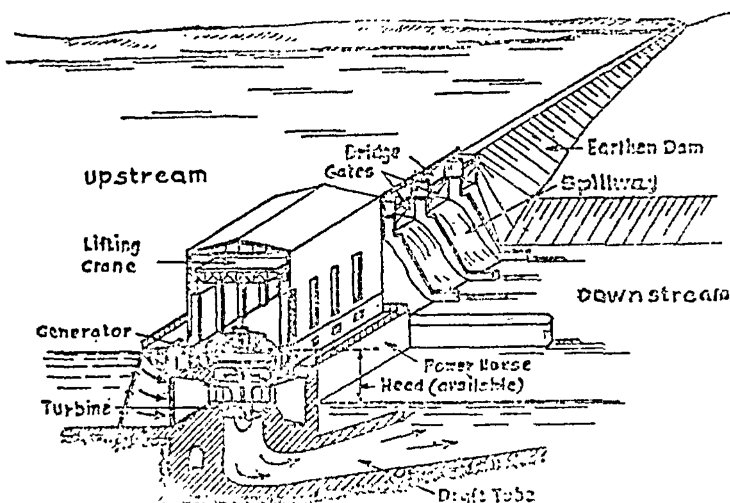


Fig. 8-4 Power station in the dam

station which is, as a rule, an extension of the dam. This is shown in fig. 8-4. In this arrangement, the course of the river is

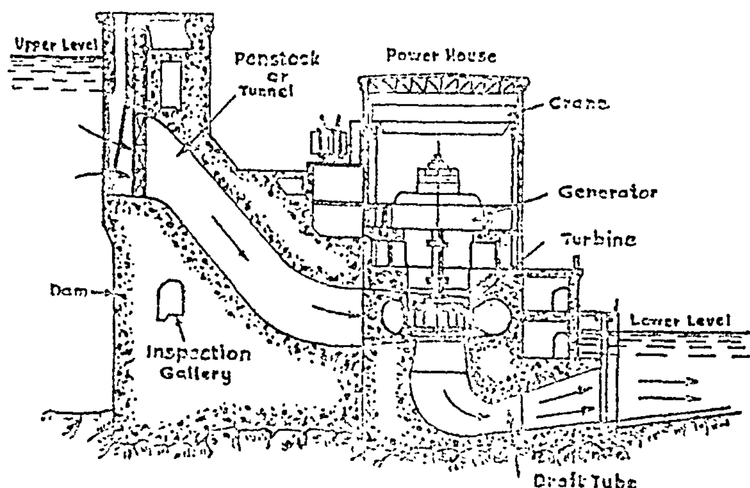


Fig. 8-5 Power station behind the dam

obstructed by the dam. By doing so, the reservoir is formep and the total head of the station is created. When the head is relatively greater, the building of the station is often located immediately behind the dam on the side of tail race, as shown in fig. 8-5. In

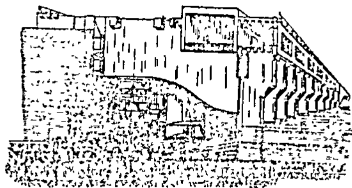


Fig. 8-6

that case, the pressure of water is not transferred to the building of the station, but the whole pressure is received by the dam. Water to the turbines is led through passages in the dam which connect the dam to the power house. In recent times another varient has got prominence; in that case, the turbines are located in the dam itself as shown in fig. 8-6.

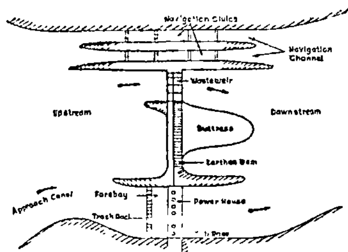


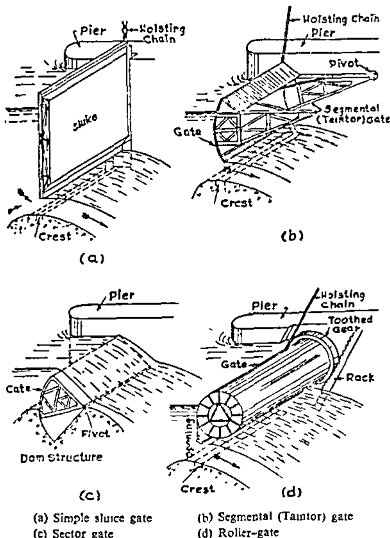
Fig. 8-7 Plan of a hydrostation illustrating basic equipment

In fig. 8-7 is presented the plan of a hydrostation of the above type, wherein the basic elements of the equipment are seen. The dam usually consists of two parts: water-washed and earthen. The water washed part is always made of concrete (cement concrete) and is envisaged for throwing away the extra water from the reservoir at the time of floods and overflowing. The earthen part cannot serve this purpose as the fast flowing water would dig away the dam. In the concrete dam, gates are provided which when opened permit the water to be rejected to the downstream side. There exist different constructions of the gates. The majority of them being found in use are described below.

The *simple sluice gates* (fig. 8-8a) are simple in construction. They are opened for permitting exit area upto 300 sq. metres. However, they require greater effort to lift them which is supplied by special lifting devices. The *segmental lock gates* (or *Taintor gates*, fig. 8-8b) can open to the extent of 200 sq. metres and do not demand great effort for their lifting, however, their construction is more complicated. The *sector gates* (fig. 8-8c) are installed for low opening (upto 5 metres) but, for greater lengths (upto 60 metres) they are lifted by allowing water to come under the gate into the pressure chamber. By this, the gate is floated and water flow over the gate is reduced. When water from the pressure chamber is pumped out, the gate lowers down. The *roller gates* (fig. 8-8d) are employed for still heavy duty (length upto 50 metres, height upto 2-10 metres). They consist of hollow cylinders coupled with steel ropes, which operate on the edge of the dam. Their lift is effected along the inclined rack, with the help of the toothed gear. Inside the cylinder, passage is provided to communicate between the walls. These gates work very well under conditions of severe winter and do not require a big effort for their lifting.

The *concrete dam* shown in fig. 8-9 has a triangular prismatic form, which is very convenient for the water to flow over it. The dam is safe against sliding due to its own weight. For this purpose, the dam is made quite massive. Water, overflowing the

dam has a high velocity and contains a big amount of energy. In order that the base of the dam is not damaged by the fast moving overflowing water, water resisting arrangement is provided which



(a) Simple sluice gate (b) Segmental (Taintor) gate
(c) Sector gate (d) Roller-gate

Fig. 8-8 Various types of lock gates

faces the pounding of water and dissipates its energy. The material used for this purpose is the hard edged stones, capable of resisting destroying forces of water.

Earthen dams are constructed for hydraulic heads upto 100 metres. The materials for their construction are usually the varieties of locally available earth (clinker). The cross-sectional view of an earthen dam is shown in fig. 8–10. The profile (slope)

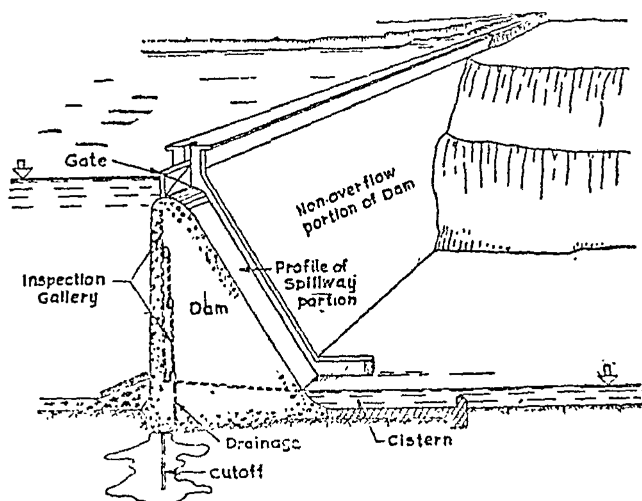


Fig. 8-9 Concrete dam

of the dam is usually made flat in order to avoid the washing away of the earth. To stop the washing of the dam from the lake side a certain portion, from the top, of the dam surface is strengthened by concrete face or by stone lining. A special wall (screen) or a

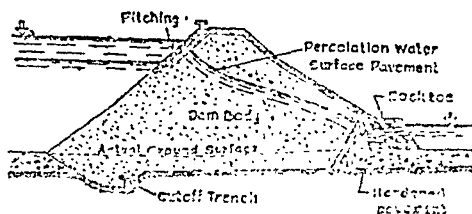


Fig. 8-10 Earthen dam

R.C.C. diaphragm is erected in the centre of the dam (fig. 8–10) to prevent the infiltration of water from lake side through the dam

on the downstream side. The dam slope is covered by grass or stone paving to prevent the washing out of the dam by rains. Often a dam is constructed from stones instead from earth (fig. 8-12). In that case, from the lake side the slope is covered by impenetrable screen.

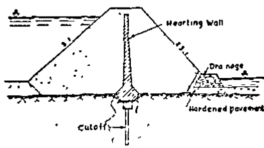


Fig. 8-11

In front of the building of the hydrostation, usually a protection wall is constructed to prevent dirt, wooden pieces and ice cubes from entering the water turbines. On one of the banks of navigable rivers a sluice gate is constructed for the passage of

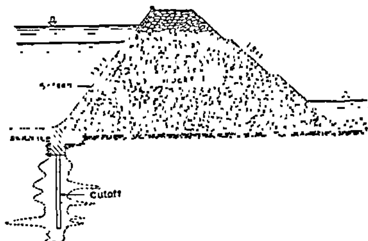


Fig. 8-12 Stone masonry dam

ships. The sluice consists of a chamber closed from both ends by special gates. The ship enters, for example, into the gate from the downstream side, behind it the gate is closed. The chamber is then filled with water from the reservoir, due to which the ship is raised to the level of the upstream of the reservoir. Now the gate on the

upstream side opens and the ship moves out of the chamber and enters the lake. The reverse operation is carried out in case the ship is to be lowered. If the head is considerably great, the sluice usually consists of a few stages.

Canal hydrostation : In contrast to the hydrostations considered above, the dam for the canal hydrostation is constructed not for the creation of head, but for the storage and diversion of water. The *diversion* comprises a water carrier (a canal or a tunnel) with small inclination. Water from the reservoir flowing along the diversion towards the pipes (penstocks) passes through the turbines and then flows again into the natural course of the river (fig. 8-13).

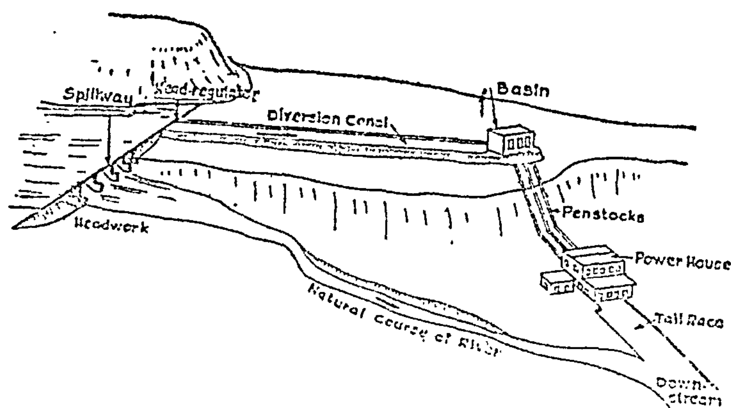


Fig. 8-13 Canal hydrostation

On the portion of the river from dam to the building of the power station, called the "old course" water flows only in that case, when water overflows through the dam. The loss of head in this type of arrangement is much less than that loss which would have taken place if water had flown along its older course and for this only the useful head is created.

Canal hydrostations are usually built on mountainous rivers. For the equipment of such a plant there are 3 basic elements : the head section (headworks); the diversion section and the station section. The head section is required to ensure the flow of water

into the diversion and protection of diversion section from fall of water (other than that coming from the lake) into it and screening of foreign material. The dam which forms part of the headwork is provided with gates for rejecting extra water over the spillway as in the case of dams of previous power stations. In front of the diversion, is arranged a water receiver which prevents rubbish from entering the canal. These canals are provided with sluice gates for regulating quantity of water flowing in the canals At the end of the canal an equalising reservoir is constructed from which water enters into the penstock pipes and flows to the power house. This equalising reservoir commonly known as surge tank reduces the pressure variation (pulsations) in the canal in the event of change in mass flow in it, thus safeguarding the diversion from possible destruction.

An important component in this type of power station is the head regulating device (or surge tank) of sufficient size which helps to compensate for the variation in discharge through the turbine passage through regulating gates and penstocks etc Situated near the surge tank is a valve house, housing electrically operated sluice valves, meant for regulation of flow in the penstock. These valves also help to close the penstocks and thus, facilitate cleaning and repairing.

At times power stations are classified according to the heads under which they work *viz high head, medium head low head* and plants. The following figures give a rough idea of the head under which the various power stations work :

(a) High head power plants–100 metres and above,

(b) Medium head power plants – 30 to 50 metres,

(c) Low head power plants – 3 to 30 metres

It may be noted that the figures given above overlap one another. Therefore, it is difficult to classify the plants rigidly on the basis of head.

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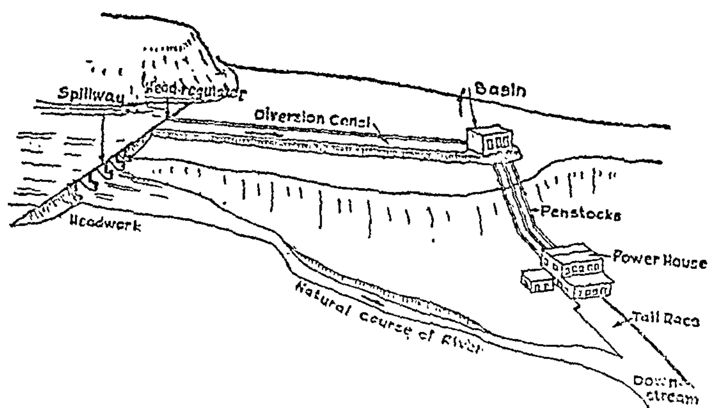


Fig. 8–13 Canal hydrostation

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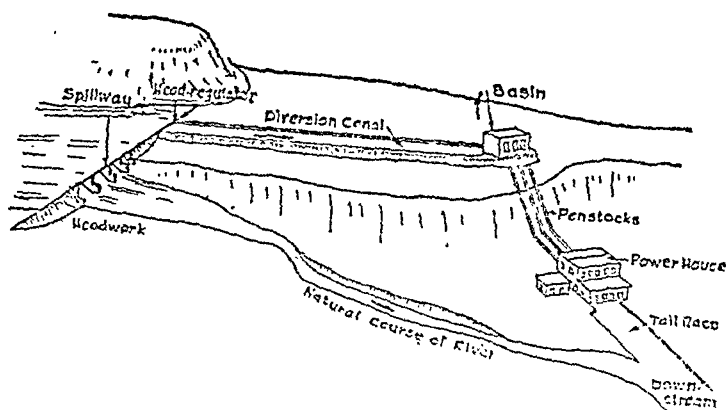


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On the portion of the river from dam to the building of the power station, called the "old course" water flows only in that case, when water overflows through the dam. The loss of head in this type of arrangement is much less than that loss which would have taken place if water had flown along its older course and for this only the useful head is created.

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into the diversion and protection of diversion section from fall of water (other than that coming from the lake) into it and screening of foreign material. The dam which forms part of the headwork is provided with gates for rejecting extra water over the spillway as in the case of dams of previous power stations. In front of the diversion, is arranged a water receiver which prevents rubbish from entering the canal. These canals are provided with sluice gates for regulating quantity of water flowing in the canals. At the end of the canal an equalising reservoir is constructed from which water enters into the penstock pipes and flows to the power house. This equalising reservoir commonly known as surge tank reduces the pressure variation (pulsations) in the canal in the event of change in mass flow in it, thus safeguarding the diversion from possible destruction.

An important component in this type of power station is the head regulating device (or surge tank) of sufficient size which helps to compensate for the variation in discharge through the turbine passage through regulating gates and penstocks etc Situated near the surge tank is a valve house, housing electrically operated sluice valves, meant for regulation of flow in the penstock These valves also help to close the penstocks and thus, facilitate cleaning and repairing.

At times power stations are classified according to the heads under which they work *viz high head, medium head low head* and plants. The following figures give a rough idea of the head under which the various power stations work :

(a) High head power plants–100 metres and above,

(b) Medium head power plants – 30 to 50 metres,

(c) Low head power plants – 3 to 30 metres

It may be noted that the figures given above overlap one another. Therefore, it is difficult to classify the plants rigidly on the basis of head.

upstream side opens and the ship moves out of the chamber and enters the lake. The reverse operation is carried out in case the ship is to be lowered. If the head is considerably great, the sluice usually consists of a few stages.

Canal hydrostation : In contrast to the hydrostations considered above, the dam for the canal hydrostation is constructed not for the creation of head, but for the storage and diversion of water. The *diversion* comprises a water carrier (a canal or a tunnel) with small inclination. Water from the reservoir flowing along the diversion towards the pipes (penstocks) passes through the turbines and then flows again into the natural course of the river (fig. 8-13).

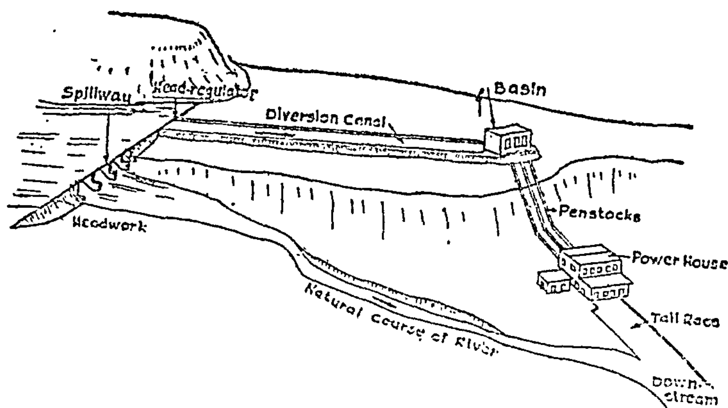


Fig. 8-13 Canal hydrostation

On the portion of the river from dam to the building of the power station, called the " old course " water flows only in that case, when water overflows through the dam. The loss of head in this type of arrangement is much less than that loss which would have taken place if water had flown along its older course and for this only the useful head is created.

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Surge Tank

The rapid variation in local conditions imposes severe hydraulic conditions on turbine pipe lines, *i.e.* rapid changes in velocity are necessary to comply with variations in load. If this is allowed to exist unchecked, it would generate serious water hammers. The greatest stress in the material of the pipe line will normally occur at a point near the turbine inlet. Here, the static pressure is of the order of 35 to 70 kg/cm^2 or more. This necessitates a greater thickness of pipe. It is the usual practice to construct the pipe line in two parts; the upper part being of greater diameter and the lower part of smaller diameter, though this involves a higher frictional loss. This is done in the interest of lower cost.

Besides this, the implication of water Hammers or pressure or inertia surges in the pipe line on the governor of the turbine has to be vividly understood. When the load on the turbine, for instance, decreases, the governor mechanism cuts down the discharge by gate closing. This sudden gate closing leads to a decrease in velocity in the pipe line which, in turn, increases the pressure. This increased pressure travels upward in the form of waves. Further, the effect of this higher pressure or pressure surge is to cause greater discharge through the turbine and consequently to increase its speed. This, in turn, compels the governor to close the gates to a further extent. This phenomena will get repeated. This leads to unsteady operation or hunting of the governor. A reverse phenomena will occur in the pipe line when the turbine load suddenly increases.

A partial rectification of this defect is secured by adopting the deflector and relief valve for Pelton wheel and reaction turbines respectively (ref. chapter-9). These two devices are, however, useful, only in times of decrease of load. A full scale remedy lies, however, in the adoption of what is known as *surge tank*, *surge shaft*, or *surge chamber*.

The principle of working of the surge tank is illustrated in fig. 8-14. At a point, very close to the turbine, the pressure pipe communicates freely with an open tank (surge tank whose height

is kept greater than the highest level in the reservoir. When the load on the turbine is steady, *oaa'* represents the normal pressure gradient. In this case the surge tank water surface will be lower than the reservior surface by an amount equivalent to frictional

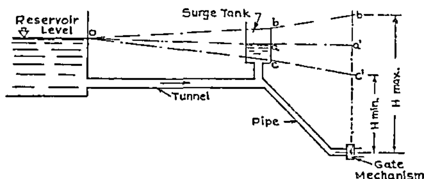


Fig. 8-14 Schematic diagram of surge tank operation

loss in the tunnel. At a time of increase in load, which calls for increased discharge, the level in the surge tank will fall and *occ'* will represent pressure gradient. Similarly, in the event of a falling load, level in the surge tank will rise and the pressure gradient is represented by *obb'*. A still more rapid gate closure might cause water to spill over the walls of the surge tank. Thus, the surge tank not only helps limiting the pressure surges in the pipe line upstream of it, but it also acts as a small supplementary stroage reservoir. During gate closure, surplus water is stored in the space between levels *a* and *b*, while, during gate opening, volume of water between levels *a* and *c* is available for flow through turbine.

Though, the ideal location of the surge tank is at a close vicinity of the turbine inlet, this is not feasible for constructional reasons. Hence, it is usually located at the junction of the pressure canal or tunnel and the steel penstocks.

Elements of Power Station

The basic elements of a hydro-power station are :

(a) **Turbine unit :** It converts the static energy of water into mechanical energy of the turbine rotor.

(b) **Electrical generator** : It consumes mechanical energy produced by the turbine units and in exchange produces electrical energy.

(c) **Transformers** : These are necessary to step up the voltage in order to facilitate transmission of electrical energy.

(d) **Switch gears** : They facilitate switching on and switching off of the generaters from the circuit. These are either oil operated or air operated.

(e) **Transmission lines** : Copper conductors in the group of minimum four wires are suspended over transmission towers through insulators. They carry electrical energy at stepped up voltage to distant industrial centres.

Layout of Power Station

There are many variants of arranging the equipment in the building of the hydro-electric station. One which is most characteristic of them is presented in fig. 8-15, in which the elevation and the plan are seen. In the buliding are installed the turbine (1) and generator (9). In the machine hall, regulating devices of the turbine-control valve (14), and pressure regulator for servo machanism (13) are arranged. Above these elements, along the length of the hall, moves a bridge crane (7), whose lifting capacity is sufficient for the heaviest part of the equiment. Height of the crane is determined by the necessary clerance, above the units, required for the biggest size of a unit. Usually the working wheel of turbine (8) or the rotor of the generator (9) serves as such a unit on the upstream side of the building, where water enters the turbine unit, a special locking gate (4) is put up with special mechanism (6). It quickly closes the passage of water turbine in the event of a major breakdown. This is kept in the constant state of readiness for operation which is completed in about 2 minutes' time. The second sluice (3) serves for fixing the filtering network which prevents the turbine from taking in foreign material. The third sulice (2) is meant for closing the water flow when repairs to turbine are carried out.

1 – Turbine

2 – Sluice gate for repair work

3 – Sluice gate for filtration

4 – Special locking gate

5 – Bridge

6 – Special mechanism for lock gate

7 – Bridge crane

8 – Wheel of turbine

9 – Generator

10 – Control room

11 – Transformer

12 – Closing network



The closing network (2) is arranged at the exit from the draft tube and when it closes, it prevents water from entering the turbine from the downstream side when level on this side is high. Over the draft tube is located the control room (10), laboratories etc. and by the side of it are the transformers (11). From the side of the upper level, on the pillars is constructed a bridge (5) on which is laid the highway (15). At the end of the building is provided an assembly shed where the assembly of various aggregates is carried out.

TUTORIAL - 8

1 How can the energy of water flowing in a river be utilised ?

2 How does the discharge in rivers vary during the years ? What are hydrographs ?

3 How are hydro-electric plants classified ? What are their essential features ?

4 For what are gates installed on dams ? Which are their main types ?

5 Describe with the help of a sketch a canal water power plant.

6 Discuss the principle of working of a surge tank. Give a sketch of its location in a high-head project. Review the types of surge tanks.

7 Which are the essential components of equipment of a hydro-electric station ? Explain the function of each of them concisely.

8 Give a typical layout of modern big capacity power station. What are the precautionary arrangements provided in a station ?



IMPACT OF JET AND JET PROPULSION

Introduction

Hydraulic turbines are the machines to convert the energy of water into the mechanical energy which is employed for rotation of the electric generator. From the basic law of hydromechanics, expressed by Bernaulli's equation,

$$\frac{V^2}{2g} + \frac{p}{w} + Z = H = \text{Constant.} \quad \ldots (9{\cdot}1)$$

it is seen that the energy of the jet of water can be utilised by causing change in the value of every member of the equation. Hence, with proper apparatus the static head Z as well as the pressure energy $\frac{p}{w}$ are converted into the kinetic energy of the jet of very high velocity. This jet is permitted to impinge (strike) upon the moving vanes or buckets of the rotor of the turbine and hence the energy of the jet is transferred to the turbine shaft. In order to estimate the energy obtained by the rotor shaft it is required to undertake a detailed study of the action of the jet on vanes of different shapes, sizes and conditions.

When a steady jet of water impinges against a solid surface whether plane or curved, there is no rebound as in the case of impact of two solid bodies. The jet transforms, on reaching the surface, into a thin stream which glides along the surface and leaves it tangentially. As the solid mass of the vane is to be

moved, force is required to be applied to it. This force is created by causing a directional change in the velocity of the jet of water. Calculation of the force can be carried out if the initial and final directions and velocities of the jet are known to us. Newton's second law of motion is always at our disposal. By this law, force = mass × acceleration, *i. e.* the force in a given direction equals the product of the mass by the rate of change of velocity in that direction. First a study of the impact on stationary surfaces and then on moving surfaces such as vanes or blades of a turbine wheel, will now be made.

Impact on Stationary Surfaces

Plane surface perpendicular to the jet : Let a jet of water of cross sectional area a (m^2) impinge with a velocity V (m/sec.) on a fixed plate as shown in fig. 9-1. The weight W (kg) of

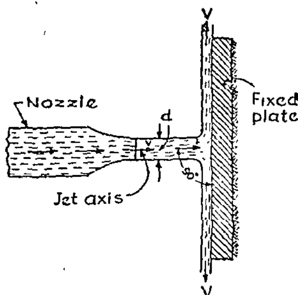


Fig. 9-1 Impact on fixed normal flat plate

water striking against the plane surface per second is waV where w is density of water (kg/m^3). The force, on the surface, is equal to mass times change in jet velocity per second. It is equal to

$\frac{W}{g}(V-0)$, since final velocity in the original direction is zero.

The force on the surface, $F = \frac{waV^2}{g}$ kg .. (9·2)

Plane surface inclined to the jet : If the surface is inclined at an angle θ to the jet, the jet velocity can be resolved into two components, one normal to the surface and other parallel to it. Since water leaves the surface tangentially, there is no component of force in that direction after impinging. Hence, the normal force or thrust on the surface is

$$F_n = \frac{W}{g}(V\sin\theta - 0)$$

$$= \frac{waV^2}{g} \times \sin\theta \text{ kg} \quad .. (9·3)$$

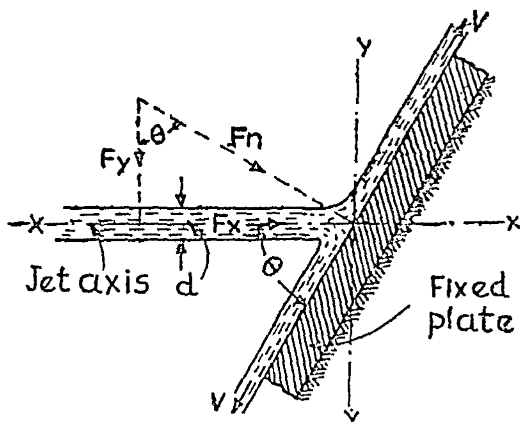


Fig. 9-2 Impact on fixed inclined flat plate

If F_n is resolved in two directions, one parallel to the jet and other perpendicular to the jet, the forces F_x and F_y respectively are given by

$$F_x = F_n \sin\theta = \frac{waV^2}{g}\sin\theta \times \sin\theta$$

$$= \frac{waV^2}{g}\sin^2\theta \text{ kg} \quad .. (9·4)$$

and $F_y = F_n \cos\theta = \frac{waV^2}{g} \sin\theta \times \cos\theta = \frac{waV^2}{2g} \sin 2\theta$ kg (9·5)

If, now the jet impinges on the surface normally, $\theta = 90°$, and,

$$F_n = \frac{waV^2}{g}, \text{ as shown above in eqn. (9·2).}$$

$$F_x = F_n = \frac{waV^2}{g}$$

and $F_y = 0$

In practice, in a majority of cases the curved surfaces, and not the flat surfaces, are more prominently employed. On some of these surfaces the jet of water is introduced tangentially, whereas on others it strikes at some angle to the entrance portion of the surface. Hence both such cases deserve individual consideration.

Curved surface on which the jet strikes normally at the centre : In this case the jet strikes horizontally at the centre of the vane on the concave side. After it strikes, it gets divided, glides over the surface and leaves the vane tangentially with the same velocity V.

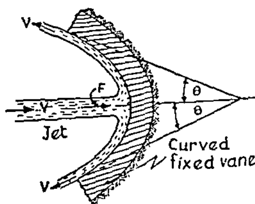


Fig. 9-3 Impact on fixed curved surface

The jet gets deflected through an angle $180 - \theta$, where θ is the inner angle as shown in fig. 9-3.

$$\left.\begin{array}{l}\text{Force } F\text{, normal}\\ \text{to vane}\end{array}\right\} = \frac{W}{g}\left[\left\{\begin{array}{l}\text{jet velocity}\\ \text{normal to}\\ \text{vane when}\\ \text{it strikes it}\end{array}\right\} - \left\{\begin{array}{l}\text{jet velocity}\\ \text{normal to vane}\\ \text{(at the centre)}\\ \text{as it leaves it}\end{array}\right\}\right]$$

$$= \frac{W}{g}\left\{ V - (-V\cos\theta) \right\}$$

$$= \frac{waV}{g}\,(V + V\cos\theta)$$

$$= \frac{waV^2}{g}\,(1 + \cos\theta) \text{ kg} \qquad \text{.. (9·6)}$$

This value of F is greater than waV^2/g, which is the value of F, when θ is made $90°$ ($\cos\theta = 0$) *i. e.* when the jet gets deflected through $90°$ as in the case of a flat plate (fig. 9-1). When $\theta = 0$ ($\cos\theta = 1$) *i. e.* when the vane becomes semicircular and the jets coming off the outlet tips are parallel and reverse in direction to the striking jet,

$$F = \frac{2waV^2}{g} \qquad \text{.. (9·6a)}$$

$$= 2 \times \text{force on flat fixed plate}$$

Thus, it is observed that, as the angle of deflection increases, the value of F increases. When $\theta = 90°$, $F = waV^2/g$ and when $\theta = 0°$, $F = 2waV^2/g$.

Curved surface on which the jet strikes tangentially : Consider a fixed curved surface as shown in fig. 9-4 making an angle θ (at exit) with the issuing jet. It is assumed that ideal conditions exist and jet leaves the surface with the same velocity with which it enters the surface at the lower tip tangentially.

In this case each particle of water in the jet has its direction changed by an angle θ and its velocity in the original direction is reduced from V to $V\cos\theta$.

Therefore, force on the surface in direction of the jet is,

F_x = mass of water striking per second × change of velocity

$$= \frac{W}{g}\,(V - V\cos\theta)$$

$$= \frac{waV^2}{g}\,(1 - \cos\theta) \text{ kg}$$

Similarly, force on the surface in the direction perpendicular to the jet is

$$F_y = \frac{W}{g}(0 - V\sin\theta)$$

$$= -\frac{waV^2}{g}\sin\theta \text{ kg}$$

Negative sign indicates that the force F_y is in the direction opposite to that of the jet at outlet as shown in fig. 9·4.

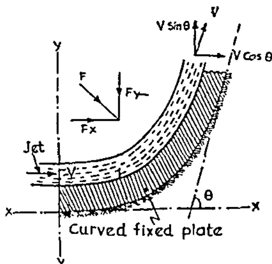


Fig 9-4 Impact on fixed curved surface

The total thrust on the surface *viz* F will be the resultant of F_x and F_y,

i.e. $F = \sqrt{F_x^2 + F_y^2}$

$$= \frac{waV^2}{g}\sqrt{(1-\cos\theta)^2 + \sin^2\theta}$$

$$\frac{waV^2}{g}\sqrt{2(1-\cos\theta)} \text{ kg} \qquad \text{.. (9·7)}$$

when $\theta = 180°$, *i.e.* when the curved surface is in the form of a semicircle, the thrust, $F = \frac{2waV^2}{g}$ kg, which is same as that in eqn. (9·6a). A jet develops double the thrust on a semicircular curved surface than what it exerts on a flat surface.

If now the surface be taken in the form of a curved vane whose exit tips are set at unequal angles to the axis, which is normal to the vane at its centre, then the analysis becomes slightly different. The jet strikes the vane at angle θ to the axis and leaves at an angle ϕ, the vane deflects the jet through an angle of $180 - (\theta + \phi)$. As the vane remains fixed and friction is neglected, velocity of the jet does not change in magnitude as it flows over the vane; the direction of velocity changes. Velocity of the entering jet in the direction of the axis is $V \cos\theta$ and it leaves the vane with a velocity component of $(-V\cos\phi)$ in the same direction.

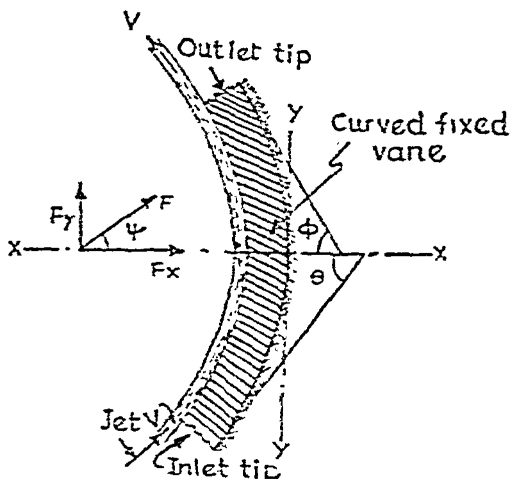


Fig. 9-5 Impact on fixed curved vane

$$\text{Force, } F_x \text{, normal to vane} = \frac{W}{g}\{V\cos\theta - (-V\cos\phi)\}$$

$$= \frac{waV^2}{g}(\cos\theta + \cos\phi) \quad \text{.. (9·8)}$$

If $\phi = \theta$, vane becomes symmetrical about the horizontal axis and

$$\text{force, } F = \frac{2\,waV^2}{g}\cos\theta \quad \text{... (9·9)}$$

If the vane is semicircular, the angles θ and ϕ are each equal to zero, then

$$F_x = \frac{2waV^2}{g}$$

This is seen earlier. This is due to the fact that, with a semicircular vane, use is made of the reaction of leaving water, which exerts the same force on the vane in leaving as in entering. This principle is made use of in the Pelton wheel.

Along with this horizontal force, there is another force which comes into play. The vertical components of the velocities at entrance and exit are $V \sin\theta$ and $V \sin\phi$. These components cause a tangential force on the vane at right angles to the horizontal axis. It is equal to mass of water per sec times change of velocity in y direction.

$$\text{Tangential force, } F_Y = \frac{W}{g}(V \sin\theta - V \sin\phi)$$

$$= \frac{WaV^2}{g}(\sin\theta - \sin\phi)$$

Hence in such a case, there will be a resultant force F, which will make an angle ψ with the horizontal such that $\tan\psi = \frac{F_y}{F_x}$. It may be observed that if the vane is symmetrical, *i.e.* $\theta = \phi$, $F_y = 0$ and the total (resultant) force is same as F_x *i.e* angle $\psi = 0$.

Problem-1: *A nozzle forming a jet of 5 cm diameter is supplied with water under a constant head of 70 m. The jet is directed to strike perpendicularly on a fixed plane plate. Estimate the force exterted by the jet on the plate. Take velocity coefficient, C_v of the nozzle as 0·9.*

Velocity of jet of water, $V = C_v \sqrt{2gH} = 0{\cdot}9\sqrt{2 \times 9{\cdot}81 \times 70}$

$= 37$ m/sec.

Referring to fig. 9-1,

change of velocity in the direction of jet $= V - 0 = 37$ m/sec.

As 1 litre (*i.e.* 1,000 c.c.) of water weighs 1,000 gm or 1 kg, 1 m^3 (*i. e.* 1,000 litres) would weigh 1,000 kg, *i.e.* density of water is 1,000 kg/m^3.

and weight W, of water striking the plate per sec. $\Big\} = waV = 1{,}000 \times \frac{\pi}{4}\left(\frac{5}{100}\right)^2 \times 37$

$$= 72{\cdot}8 \text{ kg/sec.}$$

Hence, force, F on plate $= \frac{W}{g} \times V = \frac{72{\cdot}8}{9{\cdot}81} \times 37 = \underline{274 \text{ kg}}$

Problem-2: *A 3 cm diameter jet exerts a force of 200 kg in the direction of the jet which is normal to the fixed plate. Find the weight of water striking the plate per sec.*

$$F = \frac{waV^2}{g}$$

$$i.e. \quad 200 = \frac{1{,}000 \times \frac{\pi}{4}(\tfrac{3}{100})^2 \times V^2}{9{\cdot}81}$$

$$\therefore \quad V^2 = 2{,}770$$

$$\therefore \quad V = 52{\cdot}7 \text{ m/sec.}$$

Weight of water/sec $= waV = 1{,}000 \times \frac{\pi}{4}\left(\frac{3}{100}\right)^2 \times 52{\cdot}7 = \underline{37 \text{ kg}}$

Problem-3: *A jet of water 7 cm diameter strikes a stationary plate with a certain velocity. The jet is inclinded at an angle of 60° with the plate. If the normal force exerted on the plate is 250 kg determine the velocity of the jet.*

Referring to eqn. (9·3),

normal force, $F_n = \frac{waV^2 \sin\theta}{g}$

where, $a = \frac{\pi}{4}\left(\frac{7}{100}\right)^2 = 0{\cdot}00385 \text{ m}^2.$

$\theta = 60°$ and $\sin 60° = 80{\cdot}66$

$$\therefore \quad 250 = \frac{1{,}000 \times 0{\cdot}00385 \times V^2 \times 0{\cdot}866}{9{\cdot}81}$$

$\therefore \quad V^2 = 740$, hence $V = \underline{27{\cdot}2 \text{ m/sec.}}$

Problem-4: *A jet of water of 12 mm diameter strikes at the centre of a fixed cup (fig. 9-6) which deflects the jet through 165°.*

If the reaction of the cup was found to be 17·5 kg, when the discharge from the jet was 500 kg in 150 sec., estimate the ratio of actual to theoretical force of the jet.

Discharge from jet = $\frac{500}{150}$ = 3·33 kg/sec. = 0·00333 m³/sec.

$\therefore$ velocity of jet = $\dfrac{0{\cdot}00333}{\frac{\pi}{4}\left(\frac{12}{1{,}000}\right)^2}$ = 29·4 m/sec.

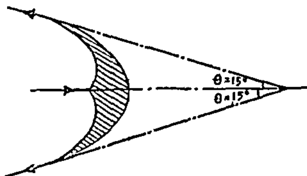


Fig. 9–6

Referring to eqn. (9·6), and assuming frictionless flow over the cup,

$$\text{Theoretical force} = \frac{WV}{g}\ (1 + \cos\theta)$$
$$= \frac{3{\cdot}33}{9{\cdot}81} \times 29{\cdot}4\ (1 + \cos 15°)$$
$$= 19{\cdot}7 \text{ kg}$$

$\therefore$ ratio of actual force to theoretical force = $\dfrac{17{\cdot}5}{19{\cdot}7}$ = <u>0 893</u>

Problem – 5 : *Water under pressure of 3·5 kg/cm² is flowing through a pipe line of 30 cm diameter, at the rate of 280 litres/sec. Find the magnitude and direction of the resultant force on the bend, if the pipe is bent through 30°.*

Total force normal to sections 1–1 and 2–2 of fig. 9–7 due to pressure of water inside the pipe is

$$P_1 = P_2 = \frac{\pi}{4}\,30^2 \times 3{\cdot}5 = 2{,}480 \text{ kg}$$

Component force, F_x on the bend due to P_1 and P_2 is

$$P_1 - P_2 \cos 30° = 2{,}480 - 2{,}480 \times 0{\cdot}866 = 332 \text{ kg}$$

Component force, F_y on the bend due to P_1 and P_2 is

$$- P_2 \sin 30° = - 2{,}480 \times 0{\cdot}5 = - 1{,}240 \text{ kg}$$

$$\text{Velocity of flow} = \frac{Q}{a} = \frac{280/1{,}000}{\frac{\pi}{4}\left(\frac{30}{100}\right)^2} = 3{\cdot}95 \text{ m/sec}$$

Component force, F_x due to change in velocity is

$$\frac{WV}{g}(1 - \cos\theta) = \frac{280 \times 1 \times 3{\cdot}95}{9{\cdot}81}(1 - 3{\cdot}866)$$
$$= 15{\cdot}1 \text{ kg}$$

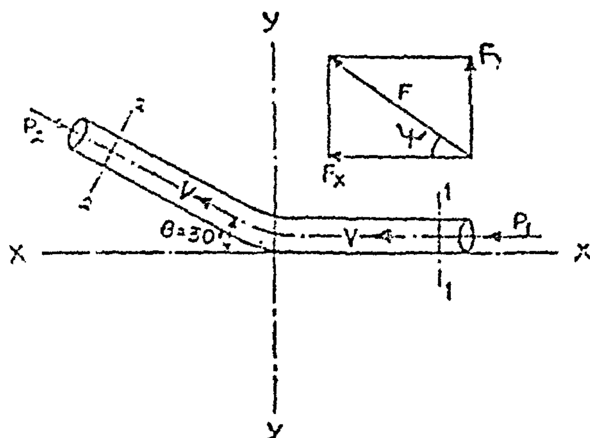


Fig. 9-7

Component force, F_y due to change in velocity is

$$\frac{WV}{g}(0 - \sin 30°) = \frac{280 \times 1 \times 3{\cdot}95}{9{\cdot}81}(0 - 0{\cdot}5)$$
$$= 56{\cdot}5 \text{ kg}$$

Total force in x-direction $= 332 + 15{\cdot}1 = 347{\cdot}1$ kg

Total force in y-direction $= - 1{,}240 - 56{\cdot}5 = - 1{,}296{\cdot}5$ kg

Hence, resultant force, $F = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(347{\cdot}1)^2 + (-1{,}296{\cdot}5)^2}$$
$$= 1{,}340 \text{ kg}$$

and its direction, $\tan\psi = \frac{-1296.5}{347.1} = -3.72$

$$\psi = \underline{-75^\circ}$$

Problem–6 : *A 60°, 25 cm × 15 cm reducing bend in a horizontal pipe line carries 300 litre/sec. Pressure at inlet to the bend is 1·5 kg/cm². Find the force necessary to be applied to hold the bend in position. Neglect friction.*

Area, $a_1 = \frac{\pi}{4}\left(\frac{25}{100}\right)^2 = 0.049 \text{ m}^2$

and $a_2 = \frac{\pi}{4}\left(\frac{15}{100}\right)^2 = 0.0177 \text{ m}^2$

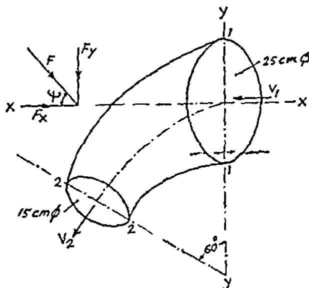


Fig. 9-8

Velocity of flow, $V_1 = \frac{Q}{a_1} = \frac{300}{1{,}000} \times \frac{1}{0.049} = 6.1 \text{ m/sec}$

and $V_2 = \frac{Q}{a_2} = \frac{300}{1{,}000 \times 0.0177} = 17.0 \text{ m/sec}$

Component force, F_X on the bend due to P_1 and P_2 is

$$P_1 - P_2 \cos 30° = 2{,}480 - 2{,}480 \times 0{\cdot}866 = 332 \text{ kg}$$

Component force, F_Y on the bend due to P_1 and P_2 is

$$- P_2 \sin 30° = - 2{,}480 \times 0{\cdot}5 = - 1{,}240 \text{ kg}$$

$$\text{Velocity of flow} = \frac{Q}{a} = \frac{280/1{,}000}{\frac{\pi}{4}\left(\frac{30}{100}\right)^2} = 3{\cdot}95 \text{ m/sec}$$

Component force, F_X due to change in velocity is

$$\frac{WV}{g}(1 - \cos\theta) = \frac{280 \times 1 \times 3{\cdot}95}{9{\cdot}81}(1 - 3{\cdot}866)$$
$$= 15{\cdot}1 \text{ kg}$$

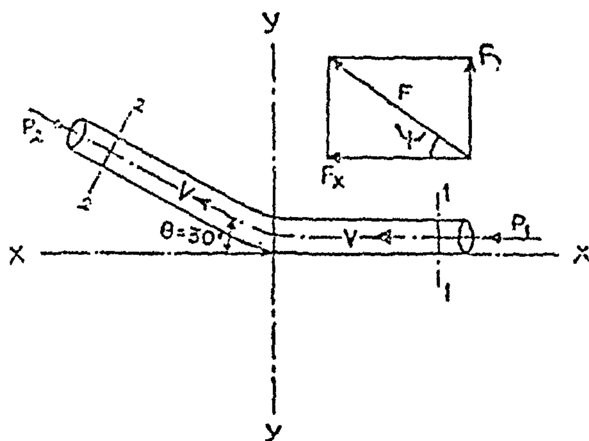


Fig. 9-7

Component force, F_Y due to change in velocity is

$$\frac{WV}{g}(0 - \sin 30°) = \frac{280 \times 1 \times 3{\cdot}95}{9{\cdot}81}(0 - 0{\cdot}5)$$
$$= 56{\cdot}5 \text{ kg}$$

Total force in x-direction $= 332 + 15{\cdot}1 = 347{\cdot}1$ kg

Total force in y-direction $= - 1{,}240 - 56{\cdot}5 = - 1{,}296{\cdot}5$ kg

Hence, resultant force, $F = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(347{\cdot}1)^2 + (-1{,}296{\cdot}5)^2}$$
$$= 1{,}340 \text{ kg}$$

and its direction, $\tan \psi = \frac{-1296.5}{347.1} = -3.72$

$$\psi = \underline{-75^\circ}$$

Problem-6 : *A 60°, 25 cm × 15 cm reducing bend in a horizontal pipe line carries 300 litre/sec. Pressure at inlet to the bend is 1·5 kg/cm². Find the force necessary to be applied to hold the bend in position. Neglect friction.*

Area, $a_1 = \frac{\pi}{4}\left(\frac{25}{100}\right)^2 = 0.049 \text{ m}^2$

and $a_2 = \frac{\pi}{4}\left(\frac{15}{100}\right)^2 = 0.0177 \text{ m}^2$

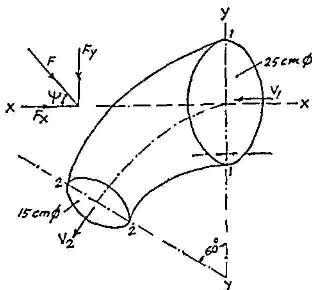


Fig. 9-8

Velocity of flow, $V_1 = \frac{Q}{a_1} = \frac{300}{1{,}000} \times \frac{1}{0.049} = 6.1 \text{ m/sec}$

and $V_2 = \frac{Q}{a_2} = \frac{300}{1{,}000 \times 0.0177} = 17.0 \text{ m/sec}$

By applying Bernoulli's theorem to sections 1–1 and 2–2,

$$\frac{V_1^2}{2g} + \frac{p_1}{w} + Z_1 = \frac{V_2^2}{2g} + \frac{p_2}{w} + Z_2$$

Since, pipe is horizontal, $Z_1 = Z_2$

$$\therefore \quad \frac{p_2}{w} = \frac{p_1}{w} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

$$i.e. \quad \frac{10^4\, p_2}{1{,}000} = \frac{10^4 \times 1{\cdot}5}{1{,}000} + \frac{6{\cdot}1^2}{2 \times 9{\cdot}81} - \frac{17^2}{2 \times 9{\cdot}81}$$

$$= 15 + 1{\cdot}9 - 14{\cdot}8 = 2{\cdot}1 \text{ m}$$

$$\therefore \quad p_2 = \frac{2{\cdot}1 \times 1{,}000}{10^4} = 0{\cdot}21 \text{ kg/cm}^2$$

Total force in x – direction $\Big\}$ $F_x = -p_1 a_1 + p_2 a_2 \cos 60° - \frac{wQ}{g}\,(V_1 - V_2 \cos 60°)$

$$= -1{\cdot}5 \times 10^4 \times 0{\cdot}049 + 0{\cdot}21 \times 10^4 \times 0{\cdot}977 \times 0{\cdot}5 - \frac{1{,}000 \times 300}{9{\cdot}81 \times 1{,}000}\,(6{\cdot}1 - 17 \times 0{\cdot}5)$$

$$= -735 + 18{\cdot}6 + 73{\cdot}5 = -643 \text{ kg}$$

Total force in y – direction $= p_2 a_2 \sin 60° - \frac{wQ}{g}\,(0 - V_2 \sin 60°)$

$$= 0{\cdot}21 \times 10^4 \times 0{\cdot}866 + \frac{1{,}000 \times 200}{9{\cdot}81 \times 1{,}000} \times 17 \times 0{\cdot}866$$

$$= 32{\cdot}2 + 450 = 482{\cdot}2 \text{ kg}$$

The resultant force, $F = \sqrt{(-643)^2 + (482{\cdot}2)^2} = 803$ kg

Direction of F, $\tan \psi = \frac{F_y}{F_x} = \frac{482{\cdot}2}{-643} = -0{\cdot}75$

$i.e.\ \psi = -37°$

Therefore, force necessary to be applied to keep the bend in position is equal and opposite in direction to F.

Impact of Jet on Hinged Surfaces

After having considered forces, due to jet, on various stationary plates and vanes, we now take up the consideration of forces

on a hinged plate which will serve as an intermediary step for taking up the final study of moving vanes.

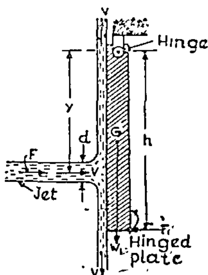


Fig. 9-9 Impact on plane hinged plate

Referring to fig. 9-9, a jet is made to strike on a plate which is hinged at one edge and whose other edge is free to move. Let

h – height of plate,

W_1 – weight of plate,

d – diameter of jet,

y – distance of the axis of jet from the hinge,

F_1 – horizontal force applied at the lower edge of the plate to prevent its movement.

Let us now find the value of force, F_1.

Change of velocity along the axis of the jet = $(V - 0)$

$\therefore$ Froce = rate of mass flow × change of velocity

$$= \frac{W}{g} \times (V - 0)$$

$$= \frac{waV^2}{g}$$

This is same as the value of force in eqn. (9 2) since the plate is not allowed to move by application of force F_1. Taking the moment of a system of forces about the top edge where the plate is hinged, we have,

$$F \times y = F_1 \times h + W_1 \times 0$$

$$\therefore \quad F_1 = F \times \frac{y}{h}$$

$$i.e.\ F_1 = \frac{waV^2}{g} \cdot \frac{y}{h} \text{ kg} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (9{\cdot}10)$$

Taking moments of all forces, acting on the plate, about the hinge point,

$$F_n \times ON = W_1 \times Z$$

$$\text{or} \quad \frac{waV^2}{g}\cos\theta \times \frac{y}{\cos\theta} = W_1 \times \frac{h}{2}\sin\theta$$

$$\therefore \quad \sin\theta = \frac{waV^2}{g} \times \frac{y}{\frac{h}{2}} \times \frac{1}{W_1} \quad \ldots (9\cdot11)$$

Problem - 7 : *A jet of 2·5 cm diameter impinges on a flat plate hinged at the centre of the upper edge. The axis of the jet is normal to and 7·5 cm below the upper edge. If velocity of jet is 9 m/sec, determine the minimum force to be applied at the lower edge of the plate to keep it in vertical plane. The plate is square of 12·5 cm side and weighs 18 kg. Determine, through what angle will the plate swing if unopposed.*

Referring to fig. 9-9 and eqn. (9·10),

area of jet, $a = \frac{\pi}{4}\left(\frac{2\cdot5}{100}\right)^2 = 0\cdot00049 \text{ m}^2$

weight of water, $W = 0\cdot00049 \times 9 \times 1{,}000 = 4\cdot42$ kg/sec

and force, $F_1 = \frac{WV}{g} \times \frac{y}{h} = \frac{4\cdot42 \times 9}{9\cdot81} \times \frac{7\cdot5}{12\cdot5} = \underline{2\cdot43 \text{ kg}}$

In absence of force F_1, the angle of swing θ is given by eqn. (9·11) *viz.*

$$\sin\theta = \frac{WV}{g} \times \frac{y}{h/2} \times \frac{1}{W_1}$$

$$= \frac{4\cdot42 \times 9}{9\cdot81} \times \frac{7\cdot5}{6\cdot25} \times \frac{1}{18} = 0\cdot27$$

$$\therefore \quad \theta = \underline{15\cdot7^\circ}$$

Problem - 8 : *A square plate 10 mm thick swings about the upper horizontal edge. A horizonal jet of water, 15 mm diameter impinges normal to the plate and 50 mm below the upper edge, keeps the plate steadily inclined at 30° to the vertical. The plate material weighs 0·0785 kg/cm³. If the velocity of jet is 10 m/sec; determine*

Further it should prove interesting to determine the angle through which the plate will swing when its movement is not

Taking moments of all forces, acting on the plate, about the hinge point,

$$F_n \times ON = W_1 \times Z$$

$$\text{or} \quad \frac{waV^2}{g} \cos\theta \times \frac{y}{\cos\theta} = W_1 \times \frac{h}{2} \sin\theta$$

$$\therefore \quad \sin\theta = \frac{waV^2}{g} \times \frac{y}{\frac{h}{2}} \times \frac{1}{W_1} \qquad \text{.. (9·11)}$$

Problem – 7 : *A jet of 2·5 cm diameter impinges on a flat plate hinged at the centre of the upper edge. The axis of the jet is normal to and 7·5 cm below the upper edge. If velocity of jet is 9 m/sec, determine the minimum force to be applied at the lower edge of the plate to keep it in vertical plane. The plate is square of 12·5 cm side and weighs 18 kg. Determine, through what angle will the plate swing if unopposed.*

Referring to fig. 9-9 and eqn. (9·10),

area of jet, $a = \frac{\pi}{4}\left(\frac{2.5}{100}\right)^2 = 0.00049 \text{ m}^2$

weight of water, $W = 0.00049 \times 9 \times 1{,}000 = 4.42$ kg/sec

and force, $F_1 = \frac{WV}{g} \times \frac{y}{h} = \frac{4.42 \times 9}{9.81} \times \frac{7.5}{12.5} = \underline{2.43 \text{ kg}}$

In absence of force F_1, the angle of swing θ is given by eqn. (9·11) *viz*.

$$\sin\theta = \frac{WV}{g} \times \frac{y}{h/2} \times \frac{1}{W_1}$$

$$= \frac{4.42 \times 9}{9.81} \times \frac{7.5}{6.25} \times \frac{1}{18} = 0.27$$

$$\therefore \quad \theta = \underline{15.7^\circ}$$

Problem – 8 : *A square plate 10 mm thick swings about the upper horizontal edge. A horizonal jet of water, 15 mm diameter impinges normal to the plate and 50 mm below the upper edge, keeps the plate steadily inclined at 30° to the vertical. The plate material weighs 0·0785 kg/cm³. If the velocity of jet is 10 m/sec; determine*

the size of the plate. If the plate is to be kept vertical, determine the minimum force to be applied at the lower edge.

Area of jet, $a = \frac{\pi}{4}\left(\frac{15}{1,000}\right)^2 = 0{\cdot}000177\ m^2$

Referring to eqn. (9·11),

$$\sin\theta = \frac{WV}{g} \times \frac{y}{h/2} \times \frac{1}{W_1} = \frac{waV^2}{g} \times \frac{2y}{h} \times \frac{1}{W_1}$$

$$i.e.\ \sin 30° = \frac{1,000 \times 0{\cdot}000177 \times 10^2}{9{\cdot}81} \times \frac{2 \times 50}{h \times 1,000} \times \frac{1}{W_1}$$

$$i.e.\quad W_1 h = 2{\cdot}825$$

$$\text{Now, } W_1 = \rho \times h^2 \times t$$

$$= (0{\cdot}0785 \times 100^3) \times h^2 \times \frac{10}{1,000}$$

$$= 785h^2$$

$$\therefore \quad 785h^2 \times h = 2{\cdot}825$$

$$\therefore \quad h^3 = \frac{2{\cdot}825}{785} = \frac{1}{278}$$

$$\therefore \quad h = \left(\frac{1}{6{\cdot}5}\right) = 0{\cdot}155\ \text{m } i.e.\ 15{\cdot}5\ \text{cm}$$

Hence the plate size is <u>15·5 cm × 15·5 cm × 10 mm</u>

Referring to eqn. (2·10),

$$F_1 = \frac{waV^2}{g} \times \frac{y}{h}$$

$$= \frac{1,000 \times 0{\cdot}000177 \times 10^2}{9{\cdot}81} \times \frac{50}{1,000} \times \frac{100}{15{\cdot}5}$$

$$= 0{\cdot}58\ \text{kg}$$

Impact of Jet on Moving Surfaces

The impact or force produced by the jet on the stationary surfaces is of academic interest only, since no useful purpose can be served as long as the surfaces remain stationary. For generation of power the force due to the impact of jet has to be employed to produce work and for this aim it is essential that moving surfaces

be used instead of stationary ones in order that they can transfer the work received by them for a useful purpose (like rotating a shaft). This is achieved by fixing the surface called vanes, blades or buckets, radially on the cylindrical periphery of large wheel so that a series of vanes in turn comes in front of the jet, each vane entering the jet gets the thrust from it and thus transfers the energy of the jet on the vanes, thus, causing rotation of the wheel.

As, now, two moving objects enter into the picture, the jet and the vane, it is desirable that the absolute velocities of these objects and also their relative velocity with respect to each other should correctly be understood in the proper perspective. Since, motion is relative, it is always specified in terms of some body that is assumed to be stationary. So long the earth was taken as the stationary body and the displacement of all objects has been measured relatively to the earth's surface. Thus the velocities of all objects are considered as absolute velocities If a jet of water be projected from a nozzle, the velocity of the jet seen by an observer in relation to the stationary earth is the absolute velocity of the jet. On the other hand, the velocity of the jet seen by an observer moving with the vane and who notes the motion relative to the moving vanes is the relative velocity of the jet. The conception of relative motion between two moving bodies is well understood by imposing that velocity on each which causes the other to come to rest, then the resultant velocity of the other is its relative velocity with respect to the one which has been brought to rest. To make this more clear let an example be illustrated. It is supposed that a jet has a velocity of 10 m per sec. and the vane is moving in the same direction away from the jet at 5 m per sec. To find the relative velocity of the jet with respect to the vane, the vane is brought to rest by imposing a velocity of opposite nature of 5 m/sec to the vane and to the jet The vane now has come to rest, and the resultant velocity of the jet is $10 - 5 = 5$ m/sec. This is its relative velocity with respect to the vane. If the vane were to approach the jet at 5 m/sec. the relative velocity of the jet with respect to the vane would be 15 m/sec.

The above results may also be obtained vectorially. Set off to scale the distance PQ to represent the velocity of the jet and PR to represent the velocity of the vane, the vector RQ represents the

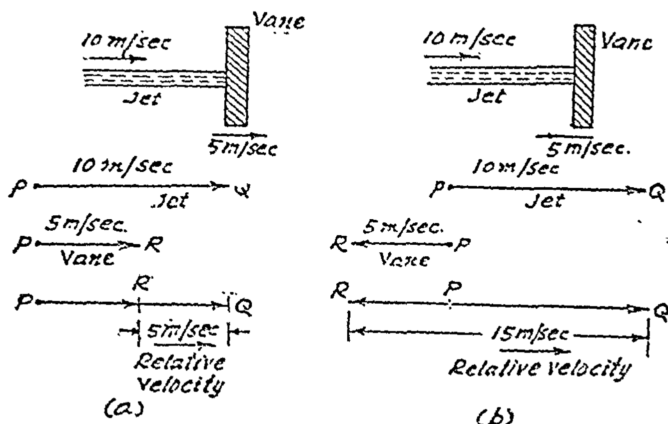


Fig. 9-11 Relative velocity

relative velocity of the jet (fig. 9-11). Thus to represent the relative motion vectorially, place the beginning of the second vector at the beginning of the first, then the line joining the end of the second vector to the end of the first represents the relative motion.

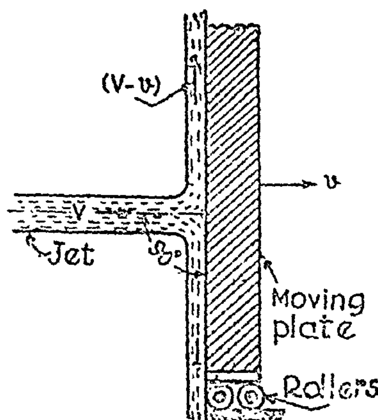


Fig. 9-12 Impact on moving flat plate

Moving flat plate: If a jet of water strikes a plate which is moving in the same direction as the jet, the velocity with which the jet strikes the plate will be the relative velocity between the jet and the plate. If v is the velocity of the plate, weight of water striking the plate per sec. is $W = wa\,(V - v)$ kg.

$$\text{Force on plate, } F = \frac{W}{g}\left\{\begin{matrix}\text{initial velocity}\\ \text{before impact in}\\ \text{the direction nor-}\\ \text{mal to the plate}\end{matrix}\right\} - \left\{\begin{matrix}\text{final velocity}\\ \text{after impact in}\\ \text{direction normal}\\ \text{to the plate}\end{matrix}\right\}$$

$$= \frac{W}{g}\left[\left(V - v\right) - 0\right]$$

$$= \frac{wa}{g}\left(V - v\right)^2 \text{kg} \qquad ..(9{\cdot}12)$$

This force is not continuously obtained from the jet as there would be a continual lengthening of jet and the distance between the plate and the nozzle which forms the jet increases by v m for every second. After some time the jet may not strike the plate at all.

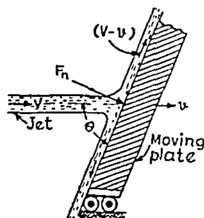


Fig. 9-13 Impact on moving inclined flat plate

Moving inclined plate : The analysis of such a case is similar to that of the fixed inclined plate except that the absolute velocity is replaced by the relative velocity of the jet. The plate moves in the direction of the jet. The weight of water reaching the plate is $W = wa\ (V - v)$ kg.

$$\left.\begin{array}{l}\text{Force normal} \\ \text{to plate, } F_n\end{array}\right\} = \frac{W}{g}\left(V_1 - V_2\right)$$

where V_1 and V_2 are the initial and final velocities of the jet in the direction normal to the plate. $V_1 = (V - v)\sin\theta$ and $V_2 = 0$. Hence,

$$F_n = \frac{wa\ (V - v)}{g}\left\{(V - v)\sin\theta - 0\right\}$$

$$= \frac{wa\ (V - v)^2}{g}\sin\theta \text{ kg} \qquad ..\ (9{\cdot}13)$$

If $\theta = 90°$, $\sin\theta = 1$ and the result obtained is the same as for a vertical moving plate (eqn. 9·12).

This particular case also does not find any practical application, on account of the fact that after some time the plate would have moved through a distance, such that the jet might not be able to reach and strike the plate.

curved vanes which are bent only in one plane. The jet comes normal to the vane. Hence,

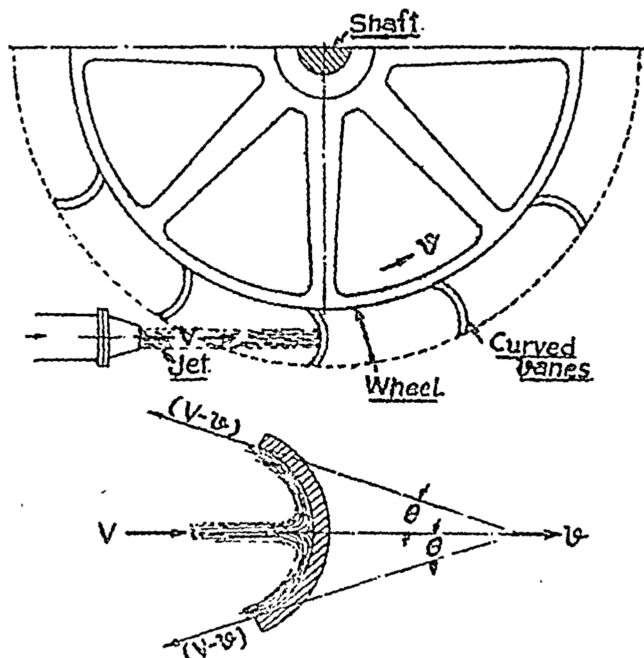


Fig. 9-15

weight of water actually striking the vane $= waV$ kg

and force, $F_x = \dfrac{waV}{g}[(V-v)-\{-(V-v)\cos\theta\}]$

$$= \frac{waV}{g}(V-v)(1+\cos\theta) \text{ kg} \qquad ..(9\cdot17)$$

Work done on the wheel per sec $\Big\} = F_x \times v$

$$= \frac{waV}{g}(V-v)(1+\cos\theta)\,v \qquad ..(9\cdot17a)$$

Kinetic energy of jet per sec. $\Big\} = \dfrac{WV^2}{2g}$

$$= \frac{waV^3}{2g}$$

$$\left.\begin{array}{l}\text{Hence, efficiency}\\ \text{of the wheel, } \eta\end{array}\right\} = \frac{\text{work done per sec.}}{\text{K.E. of jet per sec.}}$$

$$= \frac{waV(V-v)(1+\cos\theta)\,v/g}{\dfrac{waV^3}{2g}}$$

$$= \frac{2v(V-v)(1+\cos\theta)}{V^2} \quad \text{.. (9·18)}$$

For the given value of V, η will be maximum when

$$\frac{2(1+\cos\theta)}{V^2} \times \frac{d}{dv}(Vv - v^2) = 0$$

$$i.e. \quad \frac{2(1+\cos\theta)}{V^2}(V-2v) = 0$$

Hence, $\quad V = 2v$ or $v = \dfrac{V}{2}$

Hence, $\quad \eta_{max} = \dfrac{2v(2v-v)(1+\cos\theta)}{4v^2}$

$$= \frac{(1+\cos\theta)}{2} \quad \text{.. (9·19)}$$

Now, if $\theta = 0$, $\eta_{max} = 1$ or 100%. This is the theoretical maximum efficiency for wheel, when curved vanes are made semicircular.

A moving curved vane with tangential entry of water : In an attempt to increase the efficiency of the water wheel, the plane straight vane is replaced by a curved vane. In a plane vane, advantage of a part of the reaction of the jet as it leaves the vane is not available. By making the vane curved, this advantage is taken. When the jet impinges on the vane with velocity V_1, velocity of water over the vane will be equal to the relative velocity of the jet with respect to that of vane and may be found by the difference of vectors of V and v. This velocity is represented by V_r. Henceforth, the analysis is based on the construction of velocity diagrams (or velocity triangles) at entrance and exit of the vane. Since the vane is now moving, a tangentially entering jet on a fixed vane will no longer remain tangential if admitted at that original angle.

rotates in that direction; the other normal to the direction of motion of the vane and known as the *velocity of flow* V_{f1}, since the jet of water flows in and out of the vane in that direction *i.e.* it is the component of jet velocity which is responsible for flow of water along the vane. These components are obtained at the tips of the vane at entrance and exit and accordingly indicated in fig. 9-16. Further, θ represents the angle between direction of relative velocity and direction of motion of the vane at inlet and ϕ the same angle at exit. Then, if water is to enter the vane without shock, the angle of the vane at inlet must be made equal to θ. The two angles, θ and ϕ, are known as vane angles.

Force on the vane in its direction of motion is equal to the change of momentum per second of water in that direction.

Force on the vane, $F = \frac{W}{g} \{ V_{w1} - (- V_{w2}) \}$

The directions of V_{w1} and V_{w2} are opposite. Friction between water and vane is neglected. Hence relative velocities at entrance and exit are equal. *i.e.*

$$V_{r1} = V_{r2}$$

Work done on vane per sec., W.D. $= \frac{W}{g} (V_{w1} + V_{w2}) v$

If V_{w2} is in the same direction as that of V_{w1} the equation then becomes,

$$\text{W.D.} = \frac{W}{g} (V_{w1} - V_{w2}) v$$

$$\text{Hence, W.D.} = \frac{W}{g} (V_{w1} \pm V_{w2}) v \quad ..(9\text{-}20)$$

The work done, in other terms, can be expressed as change of kinetic energy of the jet per second, *i.e.*

$$\text{W.D.} = \frac{WV_1^2}{2g} - \frac{WV_2^2}{2g}$$

$$= \frac{W}{2g} (V_1^2 - V_2^2)$$

and the energy supplied to the vane is $\frac{W}{g} V_1^2$. Hence

$$\text{efficiency, } \eta = \frac{\frac{W}{2g}\left(V_1^2 - V_2^2\right)}{\frac{W}{2g} V_1^2}$$

$$= \frac{V_1^2 - V_2^2}{V_1^2}$$

$$= 1 - \left(\frac{V_2}{V_1}\right)^2 \qquad ..(9{\cdot}22)$$

It can be deduced from eqn. (9·19) that, for a given angle α at which the nozzle is set to the direction of motion of vane, the efficiency is maximum when V_2 is minimum. It may be noted that V_2 and hence V_{f_2} cannot be zero, as then the incoming water will not go out of vanes; such a condition is not acceptable for the working of a water turbine. Evidently, η will be maximum when the work done per sec. is maximum as the initial kinetic energy the jet is a known quantity. From eqn. (9·20) it can be seen work is greater when V_{w_2} is negative. This means that the + *ve* sign in the expression for work remains operative. Again work is maximum when V_2 is in a direction opposite that of v. Then directions of V_2, V_{r_2} and v become colinear; *i.e.* the exit velocity triangle reduces to a straight line. Also $V_{r_2} = V_2 + v$ and V_{r_2} will be in the direction of V_2. Such a condition suggests that angle ϕ (exit vane tip angle) is zero and $V_{w_2} = V_2$. However, such a condition for maximum possible η is only hypothetical. In practice ϕ will have to be assigned some value for the reason that the leaving jet will strike the back of the incoming vane if $\phi = 0$. However, the smaller the value of ϕ, higher is the η of the wheel. Thus for better η, V_2 and ϕ should have least possible values.

If angle α is zero,

$$V_{r_1} = V_1 - v$$

$$\text{and } \quad V_2 = V_{r_2} - v$$

Then, $V_2 = V_{r1} - v$ as $V_{r2} = V_{r1}$

$= V_1 - 2v$

Hence, $V_2 = 0$ when $v = \frac{V_1}{2}$

Thus by making $v = \frac{V_1}{2}$, V_2 can be made equal to zero. Observing this condition and substituting $V_2 = 0$ in eqn. (9·22), it can be seen that the efficiency of the wheel becomes 100%. Thus the two hypothetical conditions for making the wheel perfectly efficient ($\eta = 100\%$) are : (i) Vane should be semicircular so that jet should be made to enter vane tangentially at inlet, and leave horizontally at outlet, (ii) The vane velocity should be half the jet velocity. None of these conditions is possile for practical reason and hence no wheel has 100% efficiency.

Series of moving curved vanes on which the jet strikes at an angle at the entrance tip (Radial flow over the vanes)

In radial flow water turbines, curved vanes are fixed radially on a big wheel and the jet enters over, the vanes either from the outer periphery towards the inner periphery of the wheel or vice-versa. The jet passes over all vanes in succession and as a result, the turbine wheel is rotated, which in turn rotates the rotor of the generator. Let r_1 and r_2 be the outer (entrance) and inner (exit) radii of the wheel, ω its angular velocity, and v_1 and v_2 tangential velocities of vane tips at entrance and exit points. Conventionally velocities in direction of motion of the wheel are taken positive.

Rate of tangential momentum of water entering the vane $= \left\{ \begin{array}{l} \text{rate of} \\ \text{mass flow} \end{array} \right\} \times \left\{ \begin{array}{l} \text{velocity} \\ \text{of whirl} \end{array} \right\} = \frac{WV_{w1}}{g}$

Moment of rate of momentum at the entrance of vane $= \frac{WV_{w1} \times r_1}{g}$

Tangential rate of momentum of water leaving the vane $= \frac{WV_{w2}}{g}$

$$\left.\begin{array}{l}\text{Moment of rate of} \\ \text{momentum at exit}\end{array}\right\} = \frac{WV_{w2} \times r_2}{g}$$

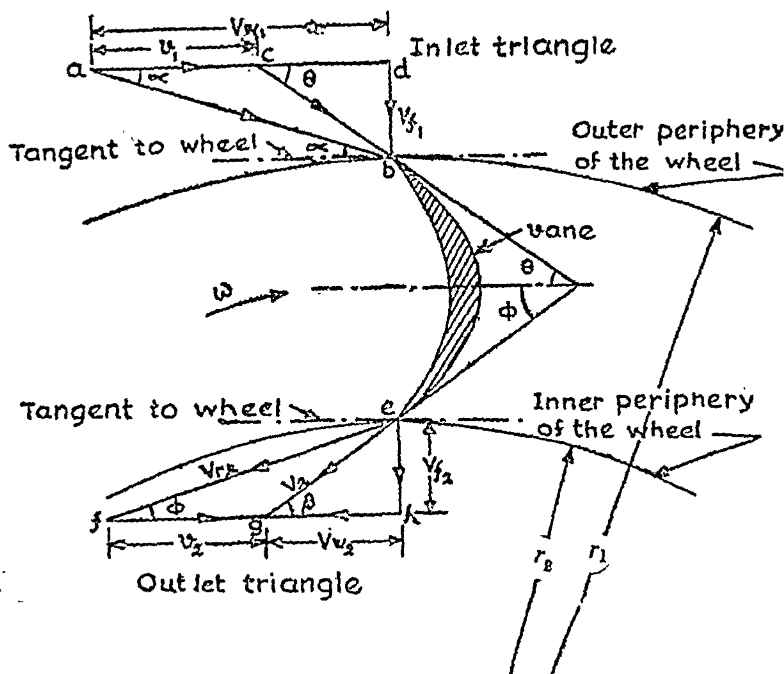


Fig. 9-17 Radial flow over curved vanes

$$\left.\begin{array}{l}\text{Rate of change of} \\ \text{moment of momentum}\end{array}\right\} = \frac{WV_{w1} \times r_1}{g} - \frac{WV_{w2} \times r_2}{g}$$

$$= \frac{W}{g} (V_{w1} \times r_1 - V_{w2} \times r_2)$$

$$= \text{torque on wheel}$$

$$\left.\begin{array}{l}\text{Rate of work done by} \\ \text{the torque}\end{array}\right\} = \text{torque} \times \text{angular velocity}$$

$$= \frac{W}{g} (V_{w1} \times r_1 - V_{w2} \times r_2)\, \omega \text{ kg·m/sec.}$$

As, $v_1 = \omega r_1$ and $v_2 = \omega r_2$.

$$\text{rate of work done on wheel} = \frac{W}{g} (V_{w1} \times v_1 - V_{w2} \times v_2) \text{ kg.m/sec.}$$

If water leaves against the direction of motion of the wheel, V_{w_2} will be negative and the above equation assumes the form

$$\frac{W}{g}\left(V_{w_1} \times r_1 + V_{w_2} \times r_2\right)$$

In a general case,

$$\text{rate of work done} = \frac{W}{g}\left(V_{w_1} \times r_1 \pm V_{w_2} \times r_2\right) \text{ kg. m/sec} \quad .. \text{ (9·23)}$$

This equation finds practical application in case of turbines.

Problem–9 : *A jet of water 5 cm diameter strikes a flat plate with a velocity of 33 m/sec. If the plate moves in the direction of the jet at 9 m/sec, estimate the force on the plate and the work done per sec. If this single plate is now replaced by a series of plates moving at the same speed, what will be the H.P. developed?*

Area of the jet, $a = \frac{\pi}{4}\left(\frac{5}{100}\right)^2 = 0{\cdot}00196 \text{ m}^2$

For a single plate,

change of velocity $= V - v = 33 - 9 = 24$ m/sec,

weight of water striking the plate $\Big\} = wa\,(V - v) = 1{,}000 \times 0{\cdot}00196 \times 24$

$= 47$ kg/sec.

force on the plate $= \left\{\begin{matrix}\text{mass of water}\\ \text{striking the}\\ \text{plate}\end{matrix}\right\} \times$ change of velocity

$= \frac{47}{9{\cdot}81} \times 24 = \underline{115 \text{ kg}}$

work done on the plate = force × plate velocity

$= 115 \times 9 = \underline{1{,}035 \text{ kg-m/sec}}$

For a series of plates,

change of velocity $= V - v = 24$ m/sec

weight of water striking the plates, $\Big\}\; W = waV = 1{,}000 \times 0{\cdot}00196 \times 33$

$= 64{\cdot}6$ kg/sec

$$\text{force on the plates} = \frac{W}{g}(V - v) = \frac{64 \cdot 6}{9 \cdot 81} \times 24 = 158 \text{ kg}$$

work done on plates = force × plate velocity

$= 158 \times 9$

$= 1{,}422$ kg–m/sec

$$\text{H. P. developed} = \frac{1{,}422}{75} = \underline{18 \cdot 9}$$

Problem–10: *A jet of water 6 cm in diameter strikes a flat plate with a velocity of 30 m/sec. The jet is inclined at an angle of 60° with the plate. If the plate is moving at 6 m/sec. away from the jet, determine the normal force on the plate and the force of the jet in its own direction. If the above flat plate is replaced by a series of inclined plates moving at the same speed, what is the H.P. developed?*

$$\text{Area of the jet} = \frac{\pi}{4}\left(\frac{6}{100}\right)^2 = 0 \cdot 002825 \text{ m}^2$$

Referring fig. 9–13 and eqn. (9–13),

$$\text{normal force on plate, } F_n = \frac{wa\,(V - v)^2 \sin\theta}{g}$$

$$= \frac{1{,}000 \times 0 \cdot 002825 \times (30-6)^2 \sin 60°}{9 \cdot 81}$$

$$= \underline{143 \cdot 5 \text{ kg}}$$

$$\left.\begin{matrix}\text{Component of } F_n \text{ in the} \\ \text{direction of jet}\end{matrix}\right\} = F_n \sin 60°$$

$$= 143 \cdot 5 \times 0 \cdot 866 = \underline{124 \text{ kg}}$$

For H.P. developed for a series of inclined plates,

discharge per sec. $= waV = 1{,}000 \times 0 \cdot 002825 \times 30 = 84 \cdot 7$ kg/sec.

$$\text{Normel force, } F_n = \frac{W}{g}(V - v)\sin\theta$$

$$= \frac{84 \cdot 7}{9 \cdot 81} \times (30 - 6) \times 0 \cdot 866 = 179 \cdot 5 \text{ kg}$$

force in the direction of jet, $F = F_n \sin 60° = 179 \cdot 5 \times 0 \cdot 866$

$= 155 \cdot 5$ kg

$$\text{H.P. Developed} = \frac{F \times v}{75} = \frac{155 \cdot 5 \times 6}{75} = \underline{12 \cdot 42}$$

Problem-11 : *A jet of water, 1·5 cm diameter, impinges on a cup, at its centre and is deflected by 170°. If the velocity of jet is 12 m/sec. and that of cup is 6 m/sec., estimate the force developed and in case a sufficient number of such cups is arranged on the periphery of a wheel such that there is at least one cup always facing the jet at all times, estimate the H. P. developed and its efficiency.*

Area of jet $= \frac{\pi}{4}\left(\frac{1\cdot5}{100}\right)^2 = 0\cdot000177 \text{ m}^2$

Weight of water striking the cup per sec. $= wa\,(V - v)$

Normal force on cup = mass per sec × change of velocity

$$= \frac{wa\,(V-v)}{g} \times (V - v)\,(1 + \cos\theta)$$

$$= \frac{wa\,(V-v)^2\,(1+\cos\theta)}{g} \text{ where } \theta = 180° - 170° = 10°$$

$$= \frac{1{,}000 \times 0\cdot000177 \times (12-6)^2\,(1+\cos 10°)}{9\cdot81}$$

$$= \underline{1\cdot29 \text{ kg}}$$

For a series of such cups,

referring to eqn. (9·17*a*)

$$\text{work developed} = \frac{waV\,(V-v)\,(1+\cos\theta)\,v}{g}$$

$$= \frac{1{,}000 \times 0\cdot000177 \times 12}{9\cdot81} \times (12-6) \times (1 + 0\cdot9848) \times 6$$

$$= 15\cdot6 \text{ kg-m/sec}$$

$$\text{H.P. developed} = \frac{15\cdot6}{75} = \underline{0\cdot207}$$

$$\text{Efficiency of the wheel} = \frac{WaV\,(V-v)\,(1+\cos\theta)\frac{v}{g}}{\frac{waV^3}{2g}}$$

$$= \frac{2v\,(V-v)\,(1+\cos\theta)}{V^2}$$

$$= \frac{2 \times 6 \times 6 \times 1\cdot9848}{12 \times 12}$$

$$= 0\cdot992 \text{ i. e. } \underline{99\cdot2\%}$$

Probelm – 12 : *A cup similar to that in a Pelton wheel deflects a jet of water through an angle of 120°. Determine the speed of the cup in terms of the velocity of the jet so that the work done by the jet on the cup shall be a maximum and express this work as a percentage of the energy of the jet.*

Show how the speed necessary for maximum efficiency would be affected if the friction of water in passing over the surface of the cup were considerable.

We have exit angle of cup, $\phi = 180 - 120 = 60°$

Work done per kg of water, W.D. $= \dfrac{V_{w1}\, v_1}{g} - \dfrac{V_{w2}\, v_2}{g}$

$$= \frac{V_1 v}{g} + \frac{v\,(V_1 - v)\cos 60° - v^2}{g} \quad (\text{since } v_1 = v_2 = v)$$

Differentiating for maximum W. D. condition,

$$\frac{d\,(W.D.)}{dv} = \frac{V_1 + V_1 \cos 60° - 2v \cos 60° - 2v}{g} = 0$$

Hence, $V_1\,(1 + \cos 60°) - 2v\,(1 + \cos 60°) = 0$

$$\therefore \quad v = \frac{V_1}{2}$$

Then, maximum work done $= \dfrac{\frac{1}{2}V_1^2 + \frac{1}{4}V_1^2 \cos 60° - \frac{1}{4}V_1^2}{g}$

$$= \frac{\frac{3}{8}V_1^2}{g}$$

Energy supplied $= \dfrac{V_1^2}{2g}$

$$\text{Efficiency, } \eta = \frac{\dfrac{\frac{3}{8}V_1^2}{g}}{\dfrac{V_1^2}{2g}} = 0{\cdot}75, \textit{ i.e. } 75\%$$

If there is no friction over the cup, the relative velocity at exit equals relative velocity at entrance. Let friction reduce relative velocity at exit to $k\, V_{r1}$

Then relative velocity at exit, $V_{r_2} = k\,(V_1 - v)$

and $V_{w_2} = k\,(V_1 - v)\cos 60° - v$

work done per kg of water, $\left.\right\}$ $\text{W.D.} = \dfrac{V_1 v}{g} + \dfrac{\{k\,(V_1 - v)\cos 60° - v\}\,v}{g}$

$$= \frac{V_1 v}{g} + \frac{kv\,(V_1 - v)\cos 60° - v^2}{g}$$

Differentiating for a maximum W.D. condition,

$$\frac{d\,(\text{W.D.})}{dv} = \frac{V_1 + kV_1 \cos 60° - 2kv \cos 60° - 2v}{g} = 0$$

i.e. $V_1\,(1 + k\cos 60°) - 2v\,(1 + k\cos 60°) = 0$

$$\therefore \quad v = \frac{V_1}{2}$$

$\therefore$ *the speed for maximum efficiency is not affected by the* friction of water passing over the cup.

Problem – 13 : *A jet of water having a velocity of 30 m/sec. impinges on a series of curved vanes moving with 10 m/sec. The jet makes an angle of 30° to the direction of motion of vanes while entering. The velocity of flow is constant from inlet to outlet. Draw the triangles of velocities for inlet and outlet and find: (i) the angles of the vane tips so that water enters without shock, (ii) the work done/kg/sec, (iii) magnitude and direction of the absolute velocity of water leaving the vane, (iv) the wheel efficiency*

We have, $V_1 = 30$ m/sec., $v = 10$ m/sec, angle $\alpha = 30°$ Hence the inlet velocity triangle is drawn and the following things are calculated.

(i) $V_{w_1} = V_1 \cos\alpha = 30 \cos 30° = 26$ m/sec.

$V_{f_1} = V_1 \sin\alpha = 30 \sin 30° = 15$ m/sec.

Since, velocities of flow are equal,

$V_{f_1} = V_{f_2} = 15$ m/sec.

$$\text{Now, } \tan\theta = \frac{V_{f_1}}{V_{w_1} - v} = \frac{15}{26 - 10} = 0{\cdot}936$$

$$\therefore \quad \theta = 43°6'$$

$$\text{Further, } V_{r_1} = \frac{V_{f_1}}{\sin\theta} = \frac{15}{0{\cdot}6833} = 22 \text{ m/sec.}$$

and $V_{r1} = V_{r2} = 22$ m/sec.

Also, $\sin\phi = \dfrac{V_{f2}}{V_{r2}} = \dfrac{15}{22} = 0{\cdot}68$

hence, $\phi = \underline{42°51'}$

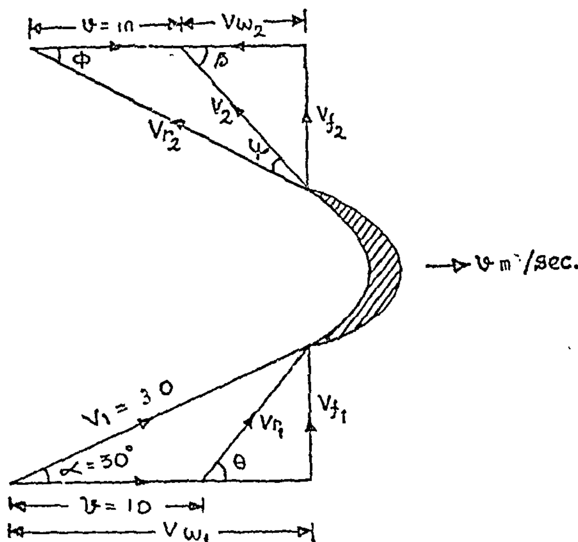


Fig. 9-18

(ii) $V_{w2} = V_{r2}\cos\phi - v = 22\cos 42°51' - 10$

$= 22 \times 0{\cdot}7329 - 10 = 6{\cdot}1$ m/sec

Work done/lb. of water/sec. $= \dfrac{(V_{w1} + V_{w2})\,v}{g}$

$= \dfrac{(26 + 6{\cdot}1) \times 10}{9{\cdot}81}$

$= \underline{32{\cdot}7 \text{ kg-m}}$

(iii) $\tan\beta = \dfrac{V_{f2}}{V_{w2}} = \dfrac{15}{6{\cdot}1} = 2{\cdot}46$

$\therefore \quad \beta = \underline{67°54'}$

$V_2 = \sqrt{V_{f2}^2 + V_{w2}^2} = \sqrt{15^2 + (6{\cdot}1)^2} = 16{\cdot}2$ m/sec.

(iv) efficiency, $\eta = \dfrac{\text{W D./kg}}{\text{K.E. at inlet/kg of water}}$

$$= \frac{32.7}{\frac{V_1^2}{2g}}$$

$$= \frac{32.7 \times 2 \times 9.81}{30^2}$$

$$= 0.714 \text{ i.e. } \underline{71.4\%}$$

Problem – 14 : *A jet of water having an absolute velocity of 30 m/sec. impinges on a series of curved vanes, making an angle of 25° to the direction of motion of the vanes and leaves at an angle of 150°. The vanes have velocity of 12 m/sec. Find : (i) the angles of the vane tips at inlet and outlet, (ii) work done per kg of water per second, (iii) the magnitude of the absolute velocity of water at outlet.*

Referring to fig. 9–18 and using the notations shown thereon, we have, $V_1 = 30$ m/sec., $v = 12$ m/sec.,
$\alpha = 25°$, $\beta = 180° - 150° = 30°$.

The triangle of velocities at inlet is now drawn. From it,

$$V_{f_1} = V_1 \sin \alpha = 30 \sin 25° = 12.7 \text{ m/sec.}$$

$$V_{w_1} = V_1 \cos \alpha = 30 \cos 25° = 27.2 \text{ m/sec.}$$

Also, $\tan\theta = \dfrac{V_{f_1}}{V_{w_1} - v} = \dfrac{12.7}{27.2 - 12} = 0.846$

$\therefore \quad \theta = \underline{40°12'}$

Since no friction loss occurs,

$$V_{r_2} = V_{r_1}$$

and $V_{r_1} = \dfrac{V_{f_1}}{\sin\theta} = \dfrac{12.7}{\sin 40°12'} = 19.7$ m/sec.

Further in the outlet velocity triangle the sides are proportional to the sines of the opposite angles, *i. e.*

$$\frac{v}{\sin \psi} = \frac{V_{r_2}}{\sin (180° - 30°)}$$

$$i.e. \quad \frac{12}{\sin\psi} = \frac{19{\cdot}7}{\sin 150°}$$

$$\therefore \quad \sin\psi = \frac{12 \times \sin 150°}{19{\cdot}7} = \frac{12 \times 0{\cdot}5}{19{\cdot}7} = 0{\cdot}305$$

$\therefore \quad \psi = 17°\,48'$ and hence angle $\phi = \beta - \psi = 30° - (17°\,48') = \underline{12°\,12'}$ ..(i)

Further, $V_{w2} = V_{r2} \cos\phi - v$

$$= 19{\cdot}7 \cos(12°\,12') - 12$$

$$= 19{\cdot}7 \times 0{\cdot}9767 - 12 = 7{\cdot}2 \text{ m/sec.}$$

$$\text{W.D./kg./sec.} = \frac{(V_{w1} + V_{w2})\,v}{g}$$

$$= \frac{(27{\cdot}9 + 7{\cdot}2)\,12}{9{\cdot}81} = \underline{42{\cdot}2 \text{ kg-m/sec.}} \quad ..(ii)$$

To find V_2, let V_{f2} be first established.

$$V_{f2} = V_{r2} \sin\phi = 19{\cdot}7 \sin 12°\,12'$$

$$= 19{\cdot}7 \times 0{\cdot}2113 = 4{\cdot}17 \text{ m/sec.}$$

Now, $V_2 = \sqrt{V_{f2}^2 + V_{w2}^2} = \sqrt{(4{\cdot}17)^2 + (7{\cdot}2)^2}$

$$= \underline{8{\cdot}32 \text{ m/sec.}} \quad ...(iii\ a)$$

Problem - 15 : *A circular jet of water delivers 60 litres per sec. with a velocity of 25 m/sec. and impinges on a vane moving in the direction of the jet with a velocity of 6 m/sec. The vane is so shaped that, if stationary, it would deflect the jet through an angle of 60°. Through what angle will it deflect the jet when it is moving in the fashion stated above ? What driving force will be exerted on the vane in its direction of motion ?*

We have, discharge, $Q = 60$ litres/sec.

weight of water, $W = w \times Q = 1 \times 60 = 60$ kg/sec.

and $V_1 = 25$ m/sec., $v = 6$ m/sec.; angles, $\phi = 60°$, $\alpha = 0$, $\theta = 0$

$$V_{r_1} = V_1 - v = 25 - 6 = 19 \text{ m/sec.}$$

$$V_{w_1} = V_1 = 25 \text{ m/sec}, V_{f_1} = 0$$

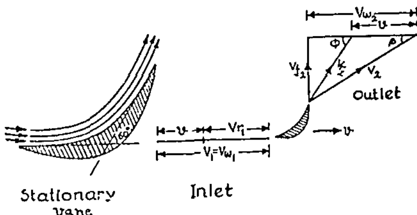


Fig. 9-19

Since there is no friction, $V_{r_2} = V_{r_1} = 19$ m/sec.

Now, $V_{w_2} = V_{r_2} \cos \phi + v = 19 \cos 60° + 6 = 15{\cdot}5$ m/sec,

and $V_{f_2} = V_{r_2} \sin\phi = 19 \times \sin 60° = 16{\cdot}4$ m/sec.

Hence, $\tan \beta = \dfrac{V_{f_2}}{V_{w_2}} = \dfrac{16{\cdot}4}{15{\cdot}5} = 1{\cdot}06$

$\therefore$ $\beta = \underline{46°7'}$ *i. e.* the jet is now deflected through 46°7'.

Driving force $= (V_{w_1} - V_{w_2}) \times \dfrac{W}{g} = (25 - 15{\cdot}5) \times \dfrac{60}{9{\cdot}81} = \underline{58{\cdot}2 \text{ kg}}$

Problem – 16 *: A jet of water impinges on a curved moving vane at an angle ∠ and leaves finally in the horizontal direction (opposite to that of the vane). The angle of the vane tip at inlet is 103°. Force exerted by the jet, per kg of water, in the direction of motion of the vane is 0·8 kg and the axial thrust 1 2 kg. The vane moves horizontally with a velocity 21 m/sec. Determine, (a) the value of angle ∠, (b) velocity of the jet, and (c) percentage reduction in the velocity of water while passing over the vane.*

Axial thrust per kg of water $= \dfrac{V_{f1} - V_{f2}}{g}$

i.e. $1{\cdot}2 = \dfrac{V_{f1}}{9{\cdot}81}$ ($V_{f2} = 0$, since water leaves horizontally)

$\therefore \; V_{f1} = 11{\cdot}8$ m/sec

Now, $\tan\theta = \dfrac{V_{f1}}{V_{w1} - v}$

From this,

$$V_{w1} = \frac{V_{f1}}{\tan\theta} + v$$

$$= \frac{11{\cdot}8}{\tan 103^\circ} + 21$$

$$= 18{\cdot}28 \text{ m/sec}$$

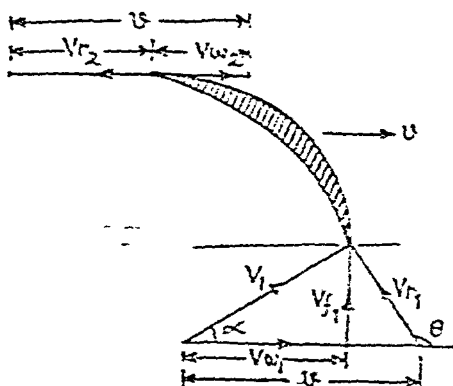


Fig. 9-20

The force in the direction of motion of the vane per kg of water

$$= \frac{V_{w1} + V_{w2}}{g}$$

$$i.e. \; 0{\cdot}8 = \frac{18{\cdot}28 + V_{w2}}{9{\cdot}81}$$

$$\therefore \; V_{w2} = 0{\cdot}8 \times 9{\cdot}81 - 18{\cdot}28 = -10{\cdot}40 \text{ m/sec}$$

Now, $V_{r2} = V_{w2} + v = -10{\cdot}40 + 21 = 10{\cdot}60$ m/sec

and $V_{r1} = \dfrac{V_{f1}}{\sin\theta} = \dfrac{11{\cdot}8}{\sin 103^\circ} = 12{\cdot}1$ m/sec

$$\tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{11{\cdot}8}{18{\cdot}28} = 0{\cdot}644$$

Hence, $\alpha = 32^\circ\, 48'$ .. .. (a)

$\therefore \; V_1 = \dfrac{V_{f1}}{\sin\alpha} = \dfrac{11{\cdot}8}{0{\cdot}542} = 21{\cdot}6$ m/sec .. .. (b)

$$\left.\begin{array}{l}\text{Reduction}\\ \text{in velocity}\end{array}\right\} = \frac{V_{r1} - V_{r2}}{V_{r1}}$$

$= \dfrac{12{\cdot}1 - 10{\cdot}60}{12{\cdot}1} = 0{\cdot}124$ *i.e.* 12·4% .. (c)

Problem-17 : *Water flows inward over a series of curved vanes which are fixed to the rim of a revolving wheel. Outer diameter of the wheel is 1 m and the inner diameter 0·5 m. Angle of the guide vane at inlet is 30°. Water leaves the wheel with a velocity of 5 m/sec. at an angle of 120° to the wheel tangent at outlet. Find angles of the moving vanes at inlet and outlet and work done per kg of water. The wheel runs at 240 rpm and velocity of the jet at inlet is 30 m/sec*

In this problem suffixes 1 and 2 respectively denote the inlet and outlet values.

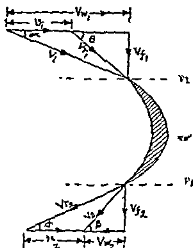


Fig. 9-21

We have, $d_1 = 1\text{m}$, $d_2 = 0{\cdot}5$ m, angle $\alpha = 30°$, $V_2 = 5$ m/sec

angle $\beta = 180° - 120° = 60°$,

$N = 240$ rpm, $V_1 = 30$ m/sec

$$v_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 1 \times 240}{60}$$

$$= 12{\cdot}5 \text{ m/sec}$$

$$v_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0{\cdot}5 \times 240}{60}$$

$$= 6{\cdot}25 \text{ m/sec}$$

Now, $V_{w1} = V_1 \cos\alpha = 30 \cos 30°$

$= 26$ m/sec

and $V_{f1} = V_1 \sin\alpha = 30° \sin 30 = 15$ m/sec

$$\text{Now, } \tan\theta = \frac{V_{f1}}{V_{w1} - v_1} = \frac{15}{26 - 12{\cdot}5} = 1{\cdot}11$$

Hence $\theta = \underline{48°}$

Further, $V_{w2} = V_2 \cos\beta = 5 \cos 60° = 2{\cdot}5$ m/sec

and $V_{f2} = V_2 \sin\beta = 5 \sin 60° = 5 \times 0{\cdot}866 = 4{\cdot}33$ m/sec

would (tend to) move the vessel in the opposite direction of the jet unless some external force is applied. This unbalanced force (F) is employed for the propulsion of ships.

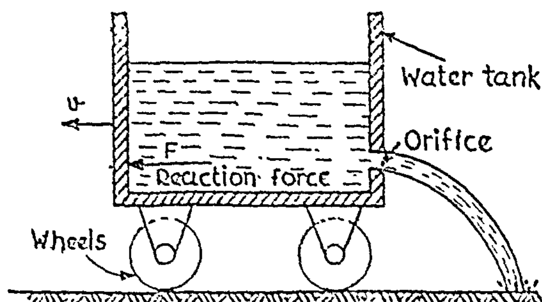


Fig. 9-23 Reaction of jet

Water is pumped from sea by a centrifugal pump into a tank carried by the ship usually through a vertical pipe provided amid

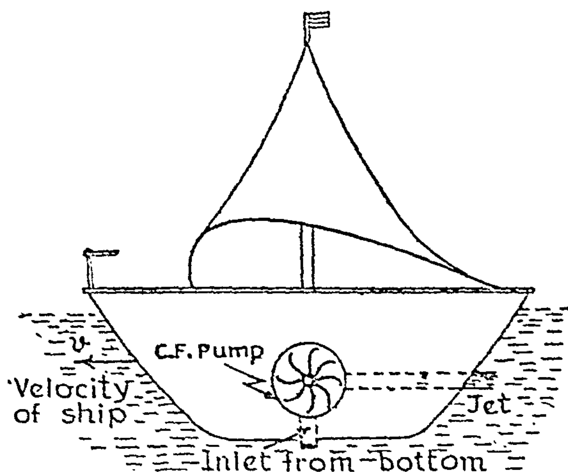


Fig. 9-24 Jet propulsion

the ship The whole of the pressure head in the tank is converted into velocity head as it flows out through a pipe from the stern (rear part) of the ship. The escaping jet of water does work in propulsion by way of reaction (fig. 9-24).

Let a – sectional area of discharging orifice, sq.m,

v – velocity of ship in m/sec.,

V – velocity of jet m/sec.,

V_r – relative velocity between jet and ship.

As the ship and jet are moving in opposite directions, the relative velocity of jet with respect to ship is the sum of two velocities i.e. $V_r = V + v$. The initial velocity of water before being pumped into the ship is zero and the final velocity of water in the direction of motion of the ship is $-V$. Hence, the rate of change of momentum of the jet relative to the surrounding water

$$= \frac{W}{g}\{0-(-V)\} = \frac{WV}{g}$$

where W, weight discharged per sec. is waV_r.

Hence, the propelling force on the ship, $F = \dfrac{WV}{g}$

Work done per sec in propulsion by the jet $= F \times$ distance covered per sec.

$$i.e.\quad W.D. = \frac{WV}{g} \times v$$

$$= \frac{W}{g}(V_r - v)\,v \qquad \text{.. (9·24)}$$

Energy supplied to the jet $= \dfrac{WV_r^2}{2g}$

$$\text{Efficiency, } \eta = \frac{\text{work got out}}{\text{work put in}}$$

$$= \frac{\frac{W}{g}(V_r - v)\,v}{\frac{W}{2g}V_r^2}$$

$$= \frac{2(V_r - v)\,v}{V_r^2}$$

For maximum efficiency, differentiate with respect to v and equate it to zero.

$$\frac{d\eta}{dv} = V_r - 2v = 0$$

$$\therefore \quad v = \frac{V_r}{2}$$

$$\eta_{max} = \frac{2(2v - v)v}{(2v)^2} = 0 \cdot 5 \text{ i.e. } 50\%$$

If the suction pipe of the pump is facing the direction of motion of the ship, water will enter the pipe with a velocity v relative to the ship. This will reduce the work to be spent on the pump by the amount $\frac{Wv^2}{2g}$. Hence, net work supplied is $\frac{W}{2g}(V_r^2 - v^2)$ and the work done by the jet remains unchanged.

Hence, the efficiency expression now becomes,

$$\eta = \frac{\frac{W}{g}(V_r - v)v}{\frac{W}{2g}(V_r^2 - v^2)}$$

$$= \frac{2(V_r - v)v}{(V_r + v)(V_r - v)}$$

$$= \frac{2v}{V_r + v} \qquad \text{.. (9·26 a)}$$

It may be observed that efficiency in this case increases as the value of v approaches V_r.

If $v = \frac{V_r}{2}$, which is the value for maximum efficiency in the foregoing case, η in this second case is

$$\eta = \frac{2v}{2v + v} = 66 \cdot 6\%$$

It may be observed in the latter case that the efficiency becomes 100%, if $V_r = v$. This cannot happen in practice as then, there is no propelling force on the ship, since $V = 0$ and jet moves with zero velocity, and no reaction is produced.

Barker's mill : Another typical example of reation of jet, leaving the pipe, against the air is *Hero's fountain* or the *Barker's mill* or *Scotch turbine*. Besides the velocity of the jet, some additional head also is utilised for the operation of such a water wheel and hence it can be said that reaction water wheels operate under pressure of water which is above that of atmosphere and the wheel is full of water. The mill consists of a vertical pipe supported in bearings. To the lower end of this pipe are connected two or more radial pipes. These pipes are either closed at end and holes are drilled across towards the end or the ends are given a right angle turn. The central pipe is fed with water under pressure and the reaction of water escaping from the holes causes them to rotate. This mill is mostly used as a rotating lawn sprinkler.

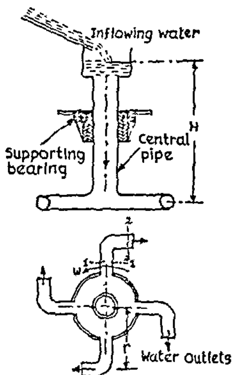


Fig. 9-25 Barker's mill

Let ω be the angular velocity of the mill, r the radius of the hole and H the head of water.

The velocity triangle for one of the nozzles are drawn in fig 9-26.

At section 1-1, (inlet to the nozzle,) the velocity (V_1) of water and peripheral velocity (v) are at right angles to each other, *i.e.* $\alpha = 90°$. Thus, velocity of whirl at inlet is zero *i.e.* $V_{w_1} = 0$

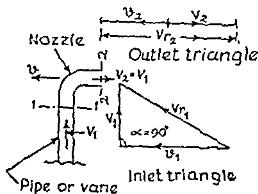


Fig. 9-26 Velocity diagram for Barker's mill

Problem-20 : *A steamboat is provided with two jet propellers one on each side, the combined area of which is 0·2 m^2. The velocity of the jet relative to the vessel is 6 m/sec. and the speed of boat is 8 knots.*

Determine the resistance to motion the boat is undergoing and also the horse-power developed in the cylinders of the engine if the work done by the jets in propulsion is 40 per cent of the work done in the cylinders. Density of sea water is 1,020 kg/ m^3.

relative velocity, $V_r = 6$ m/sec.

discharge of water, $Q = a \cdot V_r = 0.2 \times 6 = 1.2$ m^3/sec.

velocity of steam boat, $v = 8 \times 0.5144 = 4.12$ m/sec.

absolute velocity of jet, $V = V_r - v = 6 - 4.12$
$= 1.88$ m/sec.

$$\therefore \quad \text{propelling force, } P = \frac{WV}{g} = \frac{wQV}{g} = \frac{1{,}020 \times 1.2 \times 1.88}{9.81}$$
$$= 236 \text{ kg}$$

Resistance to motion = propelling force = 236 kg

$$\text{H.P. developed by the jets} = \frac{Pv}{75} = \frac{236 \times 4.12}{75} = 13 \text{ H.P.}$$

The H.P. developed by the jets is 40% of the H.P. in the cylinders of the engines.

$$\text{Hence H. P. of the engine} = \frac{100}{40} \times 13 = 32.5 \text{ H.P.}$$

Problem-21 : *In a jet-propelled ship, the water is ejected astern through four orifices each of 20 cm diameter. The efficiencies of jet, pump and engine are 65%, 80%. and 50% respectiuely. If the ship is required to sail at a speed of 15 knots in sea, find the I.H.P. required to propel it. Take the density of sea water as 1,025 kg/m^3.*

Velocity of ship, $v = 15 \times 0{\cdot}5144$

$= 7{\cdot}72$ m/sec. as 1 knot $= 0{\cdot}5144$ m/sec.

As the efficiency of the jet is greater than 50%, the advantage of scooping is taken, i.e water enters the vessel from the bow.

$$\therefore \quad \eta_j = \frac{2v}{V_r + v}$$

$$\text{or} \quad 0{\cdot}65 = \frac{2 \times 7{\cdot}72}{V_r + 7{\cdot}72}$$

$$\therefore \quad V_r = \frac{2 \times 7{\cdot}72}{0{\cdot}65} - 7{\cdot}72 = 16{\cdot}08 \text{ m/sec}$$

Weight of water discharge/sec, $W = w \times a \times V_r \times 4$

$$\therefore \quad W = 1{,}025 \times \frac{\pi}{4}\left(\frac{20}{100}\right)^2 \times 16{\cdot}08 \times 4$$

$$= 2{,}070 \text{ kg/sec.}$$

$$\eta_o = \eta_j \times \eta_p \times \eta_e = 0{\cdot}65 \times 0{\cdot}8 \times 0{\cdot}5$$

$$= 0{\cdot}26 \text{ i.e. } 26\%$$

Propelling force of jets, $F_r = \dfrac{W(V_r - v)}{g}$

$$\therefore \quad F = \frac{2{,}070 \times (16{\cdot}08 - 7{\cdot}72)}{9{\cdot}81}$$

$$= 1{,}765 \text{ kg}$$

H.P. required to propel the versel $= \dfrac{F \times v}{75 \times \eta_o}$

$$= \frac{1{,}765 \times 7{\cdot}72}{75 \times 0{\cdot}26}$$

$$= 698{\cdot}5 \text{ H.P.}$$

Problem-22 : *A jet propelled boat is sailing in sea at a speed of 20 knots, The propulsive force required is 900 kg, the resistance to motion of boat is found to be kv² kg where v is the velocity of the boat in m/sec. and k is a constant. The B H.P. of the engines driving the pumps is 215 when the pumps are working at an efficiency of 80% The arrangement of intake is such that full advantage is taken of the motion of the boat in pumping the sea water. The boat is provided with three jets. The efficiency of the engines may be taken as 25%.*

Estimate from the above data (a) the velocity of jets relative to the boat, (b) the value of the constant k, (c) the diameter of each jet, (d) efficiency of the jet, and (e) overall efficiency.

As 1 knot = 0·5144 m/sec.

Velocity of ship = 20 × 0·5144 = 10·29 m/sec.

Now, propulsive force = kv^2 kg

$$\text{or } 900 = k \times 10{\cdot}29^2$$

$$k = \underline{8{\cdot}52}$$

Again, propulsive force $= \frac{W}{g} V = \frac{W}{g}(V_r - v)$

$$\therefore \quad 900 = \frac{W}{9{\cdot}81}(V_r - 10{\cdot}29) = 8{,}800 \qquad \text{.. (i)}$$

Now, energy supplied by the pump, when the full advantage of the motion of the boat is taken is given by

$$\frac{W(V_r^2 - v^2)}{2g}$$

$$\therefore \quad \text{H.P. supplied to pump} = \frac{W(V_r^2 - v^2)}{2g \times 75 \times \eta_{PUMP}}$$

$$i.e. \quad 215 = \frac{W(V_r^2 - 10{\cdot}29^2)}{2 \times 98{\cdot}1 \times 75 \times 0{\cdot}8}$$

$$\therefore \quad W(V_r^2 - 10{\cdot}29^2) = 2{,}53{,}000 \qquad \text{.. (ii)}$$

Dividing (ii) by (i), we have,

$$\frac{W(V_r^2 - 10{\cdot}29^2)}{W(V_r - 10{\cdot}29)} = \frac{2.53.000}{8{,}800} = 28{\cdot}8$$

$$\therefore \quad \frac{(V_r + 10{\cdot}29)(V_r - 10{\cdot}29)}{(V_r - 10{\cdot}29)} = 28{\cdot}8$$

$$\therefore \quad V_r + 10{\cdot}29 = 28{\cdot}8$$

$$\therefore \quad V_r = \underline{18{\cdot}51 \text{ m/sec.}}$$

From (1) we have,

$W(V_r - 10{\cdot}29) = 8{,}800$

$\therefore \quad W = \dfrac{8{,}800}{8{\cdot}22} = 1{,}070$ kg/sec.

Now, $W = awVr$

$\therefore \quad a = \dfrac{W}{wVr} = \dfrac{1{,}070}{1{,}000 \times 18{\cdot}51} = 0{\cdot}0578$

But, $a = 3 \times \dfrac{\pi}{4} d^2$

or $0{\cdot}0578 = 3 \times \dfrac{\pi}{4} d^2$

$\therefore \quad d = \sqrt{\dfrac{0{\cdot}0578 \times 4}{3\pi}} = 0{\cdot}157$ m or 15·7 cm

Using eqn. (2·22),

efficiency of the jet, $\eta_j = \dfrac{2v}{V_r + v}$

$= \dfrac{2 \times 10{\cdot}29}{18{\cdot}51 + 10{\cdot}29}$

$= \underline{0{\cdot}358 \text{ or } 35{\cdot}8\%}$

Overall efficiency = η of jet × η of the pump × η of the engine

$= 0{\cdot}358 \times 0{\cdot}8 \times 0{\cdot}25$

$= \underline{0{\cdot}0716 \text{ i.e. } 7{\cdot}16\%}$

TUTORIAL - 9

Stationary plane and curved surfaces

1 A 8 cm diameter jet exerts a force of 100 kg in the direction of the jet which is normal to the fixed plate. Find the weight of water striking the plate per second.

[70 kg]

2 A jet of water 5 cm in diameter impinges on a fixed plate. The force exerted on it when the jet strikes perpendicularly is 65 kg. Determine, (a) the velocily of the jet, (b) the force

Estimate from the above data (a) the velocity of jets relative to the boat, (b) the value of the constant k, (c) the diameter of each jet, (d) efficiency of the jet, and (e) overall efficiency.

As 1 knot = 0·5144 m/sec.

Velocity of ship = 20 × 0·5144 = 10·29 m/sec.

Now, propulsive force = kv^2 kg

$$\text{or } 900 = k \times 10{\cdot}29^2$$

$$k = \underline{8{\cdot}52}$$

Again, propulsive force $= \dfrac{W}{g}\, V = \dfrac{W}{g}\,(V_r - v)$

$$\therefore \quad 900 = \frac{W}{9{\cdot}81}\,(V_r - 10{\cdot}29) = 8{,}800 \qquad \ldots \text{(i)}$$

Now, energy supplied by the pump, when the full advantage of the motion of the boat is taken is given by

$$\frac{W(V_r^2 - v^2)}{2g}$$

$$\therefore \quad \text{H.P. supplied to pump} = \frac{W(V_r^2 - v^2)}{2g \times 75 \times \eta_{PUMP}}$$

$$i.e.\ \ 215 = \frac{W(V_r^2 - 10{\cdot}29^2)}{2 \times 98{\cdot}1 \times 75 \times 0{\cdot}8}$$

$$\therefore \quad W(V_r^2 - 10{\cdot}29^2) = 2{,}53{,}000 \qquad \ldots \text{(ii)}$$

Dividing (ii) by (i), we have,

$$\frac{W(V_r^2 - 10{\cdot}29^2)}{W(V_r - 10{\cdot}29)} = \frac{2{,}53{,}000}{8{,}800} = 28{\cdot}8$$

$$\therefore \quad \frac{(V_r + 10{\cdot}29)(V_r - 10{\cdot}29)}{(V_r - 10{\cdot}29)} = 28{\cdot}8$$

$$\therefore \quad V_r + 10{\cdot}29 = 28{\cdot}8$$

$$\therefore \quad V_r = \underline{18{\cdot}51 \text{ m/sec.}}$$

From (1) we have,

$W(V_r - 10{\cdot}29) = 8{,}800$

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Now, $W = awVr$

$\therefore \quad a = \dfrac{W}{wVr} = \dfrac{1{,}070}{1{,}000 \times 18{\cdot}51} = 0{\cdot}0578$

But, $a = 3 \times \dfrac{\pi}{4} d^2$

or $0{\cdot}0578 = 3 \times \dfrac{\pi}{4} d^2$

$\therefore \quad d = \sqrt{\dfrac{0{\cdot}0578 \times 4}{3\pi}} = 0{\cdot}157 \text{ m or } 15{\cdot}7 \text{ cm}$

Using eqn. (2·22),

efficiency of the jet, $\eta_J = \dfrac{2v}{V_r + v}$

$$= \frac{2 \times 10{\cdot}29}{18{\cdot}51 + 10{\cdot}29}$$

$$= \underline{0{\cdot}358 \text{ or } 35{\cdot}8\%}$$

Overall efficiency = η of jet × η of the pump × η of the engine

$$= 0{\cdot}358 \times 0{\cdot}8 \times 0{\cdot}25$$

$$= \underline{0{\cdot}0716 \text{ i.e. } 7{\cdot}16\%}$$

TUTORIAL – 9

Stationary plane and curved surfaces

1 A 8 cm diameter jet exerts a force of 100 kg in the direction of the jet which is normal to the fixed plate. Find the weight of water striking the plate per second.

[70 kg]

2 A jet of water 5 cm in diameter impinges on a fixed plate. The force exerted on it when the jet strikes perpendicularly is 65 kg. Determine, (a) the velocity of the jet, (b) the force

normal to the plate when the plate is inclined at an angle of 30° to the jet, and (c) the force in the direction of motion of the jet, when the plate is inclined to the same angle.

[(a) 18·05 m/sec; (b) 32·5 kg; (c) 16·25 kg]

3 A jet of water of 1 cm diameter strikes at the centre of a fixed cup, similar to that of fig. 9–6 and is deflected by 170°. If the reaction of the cup was found to be 25 kg, when the discharge from the jet was 500 kg in 150 sec, estimate the ratio of actual to the theoretical force of the jet.

[0·876]

4 A free jet of water with a sectional area of 25 cm² and velocity of 40 m/sec. impinges tangentially on a smooth fixed vane and is deflected through (a) 60°, (b) 120°, and (c) 180°. Find the magnitude and direction of the resultant force on the vane in each case.

Resultant force, kg	Angle with horizontal, degrees
(a) 408	59·9
(b) 705	30
(c) 816	0

5 A 10 cm pipe is deflected through 60°. If the discharge of water through the pipe is 100 litres/sec., calculate the resultant force on the pipe.

[129·5 kg]

6 A 100 cm pipe line is provided with 90° bend. If the bend is fixed by 4 bolts through the flanges of the bend at each end, find the pull on each bolt, if the pipe delivers 1,800 litres/sec. of water.

[419 kg]

Hinged plate

7 A square plate, weighing 30 kg and of uniform thickness and 30 cm side, is hung so that it can swing freely about the upper horizontal edge. A horizontal jet, 2·5 cm diameter, and having velocity of 15 m/sec. impinges on the plate, and when the plate is vertical, the jet strikes the plate normally and at its centre. Find the force which must be applied at the lower edge of the plate in order to keep the plate vertical.

If the plate is allowed to swing freely, find the inclination to the vertical, which the plate will assume under the action of the jet.

[5·63 kg; 22·3°]

8 A curved plate is mounted on a slide and the slide along with the plate can be moved along the slot. It receives a jet of water at an angle of 60° with the normal to the direction of sliding and the jet is deflected through 90°. The plate and the compression spring which has stiffness of 10 kg/cm are in equilibrium, when the jet is not striking the plate. Determine the amount by which the plate will slide from its position of rest and the normal pressure on the slide. Water flows at the rate of 1·7 m^3 per minute with a velocity of 12 m/sec.

[1·27 cm; 47·3 kg]

9 A square metal plate, 6 mm thick, swings about its hrizontal edge. A horizontal jet of water 12 mm in diameter impinges with its axis perpendicular to and 5 cm below the edge of the hinge, and keeps the plate steadily inclined at 30° to the vertical. The density of the plate is 0 008 kg per cm^3. If velocity of jet is 10 m/sec.; determine the size of the plate. If this plate is to be kept vertical, by applying the normal force at its lower edge, determine the magnitude and direction of the force.

[16·48 cm; 0·342 kg]

10 A jet of 30 mm diameter impinges on a flat hinged plate with its axis perpendicular to and 80 mm below the edge of the hinge. If the velocity of jet is 10 m/sec, determine the normal force applied at the lower edge of the sqaare plate to keep it vertical. The side of the square is 200 mm. If the weight of the plate is 18 kg, determine its inclination with the jet, when the force on the lower edge is removed.

[2·87 kg; 6·9°]

Moving plane and curved surfaces

11 A jet of water 20 cm in diameter impinges normally on a flat plate moving at 1·5 m/sec, in the same direction as the jet. If the discharge from jet is 170 m^3/sec, find the force and work done per minute on the plate, (a) when a single plate is used, and (b) when a series of plates is used.

[49·25 kg; 4,430 kg–m/min; (b) 67·8 kg; 6,100 kg–m/min]

12 A jet of water 50 mm diameter, having a velocity of 20 m/sec. impinges normally on a flat plate. Find the pressure on the plate, (i) when the plate is at rest, (ii) when the plate is moving in the same direction as that of the jet with a velocity of 10 m/sec. Find also the work done per sec. in the second case.

[(i) 80 kg; (ii) 20 kg; 200 kg-m]

13 A 75 mm diameter jet, having a velocity of 25 m/sec. strikes a flat plate the normal of which is inclined at 30° to the jet. Find the normal pressure on plate, (i) when the plate is stationary, and (ii) when the plate has a velocity of 12 m/sec. away from jet.

[(i) 244 kg; (ii) 65·5 kg]

14 A jet of water, 8 cm diameter strikes a flat plate with certain velocity. The jet is inclined at an angle 60° with the plate. If the plate is stationary and the normal pressure on it is 400 kg; determine the velocity of the jet and the pressure on the plate in the direction of motion of the jet. The plate is moving at 6 m/sec., away from the jet. Determine the normal pressure on the plate, and the pressure on it in the direction of motion of the jet.

[30 m/sec; 346 kg; 255 kg; 221 kg]

15 A jet of water, 5 cm diameter, moving with a velocity of 25 m/sec. strikes horizontally a single vane which is moving in the same direction as that of jet with a velocity of 16 m/sec. The deflection angle of the vane is 130°. Fine the tangential

and axial forces exerted by the jet on the vane in the direction of the jet.Determine also the magnitude and direction of velocity of water at outlet.

[34·4 kg, 16 kg; 22·8 m/sec; 160·3°]

16 A jet of water has velocity of 30 m/sec; and it makes an angle of 30° with the tangent to the wheel circle at inlet. The vane moves with velocity of 12 m/sec. in the direction of the jet. *Find the shape of the vane to give the best results.* Find the horizontal pressure on the vane and axial thrust per kg of water striking per second.

[47° (inlet), 0° (outlet); 3·5 kg; 1·53 kg]

17 A jet of water impinges on a fixed hemispherical cup and is deflected through 180°. If velocity of jet is 30 m/sec., determine the force per kg of water, in the direction of the jet. If this cup is *replaced by* a series of *hemispherical* cups moving horizontally at 12 m/sec., find the work done per kg of water striking the cups. If the diameter of the jet is 5 cm determine the H.P developed

[6 12 kg; 49 kg-m/kg/sec.; 38·4 H.P.]

18 A jet of water having velocity of 30 m/sec. impinges on a series of vanes moving with a velocity of 15 m/sec. The jet *makes an angle of 30° to the direction of motion of the* vanes when entering and leaves at an angle of 120°. Draw the triangles of velocities for inlet and outlet and find, (a) the angles of the vane tips so that water enters and leaves without shocks (b) the work done per kg of water entering the vanes, and (c) efficiency.

[(a) 53 5°, 15 5°; (b) 43·7 kg-m (c) 95%]

Radial flow over vanes

19 Water flows outward over a series of vanes rotating at a speed of 240 rpm. The diameters of the wheel at the inner and outer periphery are 1·5 m and 2 m respectively. Water is discharged radially, at outlet, with a velocity of 5 m/sec. The work done per kg *of water is* 25 kg-m. *The velocity of flow is*

constant from inlet to outlet. Determine, (a) the angles of the vane tips at inlet and outlet, (b) guide vane angle at inlet, and (c) velocity of water at inlet.

[(a) 139·5°, 11°; (b) 21°; (c) 13·9 m/sec.]

20 Water flows radially outwards over a series of curved vanes which are fixed to the rim of a revolving wheel, having internal and external diameters 1·5 m and 2 m respectively. The wheel revolves at the speed of 300 rpm. The jet makes an angle of 30° with the wheel tangent at inlet and water leaves the vane with a velocity of 3 m/sec., making an angle of 80° with the wheel tangent at outlet. The jet issues water at the velocity of 30 m/sec. Calculate, (i) the best angles of the blades, and (ii) work done per kg per sec.

[(i) 64·5°, 6·4°; 52·5 kg–m

21 Water flows radially inward over a series of vanes at angle of 20° to the wheel tangent. At exit water is discharged radially with a velocity of 5 m/sec. and the radial velocity of flow through the wheel is constant. The inlet diameter is 1·75 times the outlet diameter. The angle of the vane tip at inlet is 60°. If the smaller diameter of the wheel is 1 m, state the correct speed of rotation of the runner and outlet vane angle. Express the gain of velocity head in the vane as a percentage of the velocity head of water leaving the guide vanes at inlet.

[119 rpm, $\phi = 38° 52'$: 14·1%]

Jet propulsion

22 A ship is run by means of jet propulsion at a speed of 55 km/hr. The advantage of the motion of ship is taken in supplying water to the tank. If the diameter of the orifice is 10 cm and discharge of water is 175 kg/sec., calculate the efficiency of jet propulsion. Derive and formula you use.

[75%]

23 A ship is propelled by four water jets, each 22·5 cm diameter. Total discharge is 3,000 kg/sec. Water is drawn through a pipe

with its open end in front of the ship and it is discharged over the stern (back side). H.P. of the pump is 600. Neglecting frictional losses, find the speed of propulsion of the ship in knots, the propelling force of the jets, and efficiency of propulsion. Take density, $w = 1{,}025$ kg/ m³. for sea water.

[3·4; 3,550 kg; 54·2%]

24 A boat sailing at a speed of 5 m/sec. draws water through orifices amidst the boat and discharges through orifices having an effective area of 600 cm² If the boat uses 600 litres per sec. of water, find the propelling force for the boat. Take the density of sea water as 1 02 kg per litre.

[312 kg]

25 In a jet-propelled ship, the resistance to motion in sea water is $20v^2$ kg, where v is the velocity of the ship in m/sec. relative to water. The ship is provided with three jets, 20 cm in diameter. Water is drawn from the front of the ship. The ship is sailing at a steady speed of the 10 m/sec. Determine, (a) quantity of water to be pumped per second, (b) relative velocity of water, (c) efficiency of jet propulsion, (d) H. P. of the pump if its efficiency is 80%, and (e) the head through which water must be lifted by the pump Take the density of sea water as 1,025 kg/m³

[(a) 1,962 kg; (b) 20 m/sec ; (c) 66·6%]
(d) 500; (e) 15·3 m.]

26 In a jet-propelled aeroplane, air enters the power unit through an opening in the front, and is discharged backwards with velocity V_r relative to the aeroplane The speed of the aeroplane is v and the weight of air used per second is W kg. Show that the horse power (for thrust) developed is

$$\frac{Wv\,(V_r - v)}{75 \times g},$$ where, v and V_r are in m/sec.

Show also that the propulsive efficiency $= \dfrac{2v}{V_r + v}$

In a certain case, the thrust required is 1,000 kg when the aeroplane is flying at 800 km/hr. Taking V_r = 600 m/sec., and the weight of air at the relevant height as 0·303 kg/m³. find the inlet area of the front opening.

[26 kg; 0·386 m²]

27 A locomotive picks up water from a trough when running at a speed of 54 km/hr. Calculate the amount of water delivered into the tender from a trough 600 m long, if the head lost in friction is $\frac{1}{4}$ the available head at entrance. The outlet of the delivery pipe having an area of 625 cm² is 3 m above the surface of water in the trough. At what speed of the locomotive delivery will commence ?

[37·5 tonnes/sec.; 31·9 km/hr]

28 The resistance of a ship is given by $0{\cdot}005v^6 + 15 \times v^{4 \cdot 9}$ kg at v m/sec. and the efficiency of the jet drive is 0·8, while that of the pump is 0·72. The vessel is to sail at 10 m/sec. Find the mass of water to be pumped astern per sec., and the H.P. required to drive the pump.

[12,100 kg; 1,430]



HYDRAULIC PRIME–MOVERS (TURBINES)

Introduction

Hydraulic prime-movers, namely turbines are the machines to convert the pressure or kinetic energy of water into mechanical work of the shaft. These turbines are installed in big hydro-electric power stations and their shafts are directly coupled to the rotors of electric generators to produce electricity which is supplied to the region around the power station.

Action of Water in Turbine

The rotation of the turbine *wheel* or *runner* or *rotor* is obtained by water flowing over curved vanes fixed to the rim of the wheel. The action of these curved vanes is to change the magnitude and direction of the water velocity. The impulse given to the wheel is the result of this change of velocity of water flowing over the vanes.

Modern water turbines may be divided in two main categories : (i) *Impulse* or *Velocity* turbines, and (ii) *Reaction* or *Pressure* turbines. In impulse turbines all the energy of water is converted into velocity or kinetic energy before water reaches the turbine vanes. The turbine, thus, operates under conditions of constant pressure, usually atmospheric. Since, the pressure fluctuations are not present, all the vanes or buckets of the turbine wheel need not be full with water at the same time. Hence, there may be one or more number of nozzles present in front of the turbine, depending upon the power to be generated.

In the reaction turbine, water enters the wheel under pressure and flows over the vanes. In passing over the vanes, the pressure head is converted to velocity head which in turn is spent in producing work and water finally leaves the wheel at atmospheric pressure. Water leaves the wheel with a large relative velocity but a small absolute velocity, practically the whole of its origina, energy having been given to the wheel. If H (metres) is the total head of entering water and V_2 m/sec is the absolute velocity of leaving water, energy imparted to the wheel per kg of water is $\left(H - \frac{V_2^2}{2g}\right)$ kg-m.

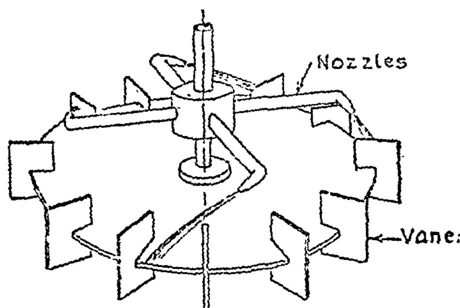
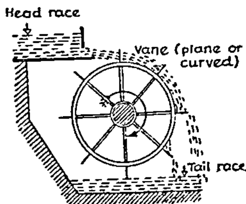


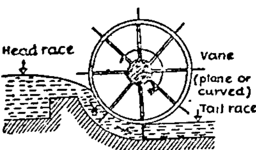
Fig. 10-1
Principle of impulse and reaction turbines

The basic principle of operation, *i.e.* generation of dynamic thrust as a result of change of momentum, is the same in both, impulse and reaction machines. This is easily observed when the two are superposed, as shown in fig. 10-1. By holding the upper ring of the nozzles stationary, and allowing the jet to strike on the lower ring of vanes, the apparatus is made to act as an impact or *impulse* machine; when the nozzles are released, the upper ring of nozzles will revolve due to reaction of the issuing jets *i.e.* it works as a *reation* machine.

During its passage through reaction turbine, the total head H of water consists partly of pressure head and partly of velocity head. As water is under pressure, the wheel must run full and may therefore, be partly or wholly submerged below the tail race. It may also discharge into the atmosphere or it may be placed about 10 m (for reasons of cavitation) above the base (foot) of the fall and discharge into a suction or a draft tube. Water is required to be admitted into the turbine over the whole circumference of the wheel.

Before the present-day turbines came into being, power on small scale was being produced by units called *water wheels*. They were revolved either by the weight of water or by the combined weight and impulse or by the impulse of the jet of water.





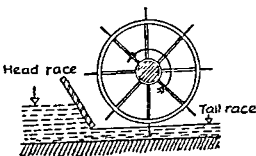


Fig. 10-2 Water wheels

These wheels were essentially fitted on horizontal shafts, and they had a number of vanes (straight) on their periphery. Under the action of water, which flows over the vanes at atmospheric pressure the wheels rotated slowly (as compared to modern turbines) and were capable of doing mechanical work. Water after doing work on the wheels fell into the tailrace. The typical examples of these wheel are (i) the overshort wheel (ii) Breast wheel, (iii) undershot wheel, which is referred to previously.

These wheels (fig. 10-2) were simple and strong in construction, and they had high and consistent efficiency even if discharge was not constant, but their power to weight ratio was very small, thereby making the machine very heavy. For small power they produced, their governing was not

very efficient and their speed was too slow to drive the present-day, directly, coupled fast running electric generators for purpose of power generation. Further, they were useful for low heads only, hence they are, in present times, very little in use.

Basic Elements

A turbine in general consists of the following basic components :

(i) A *wheel* or *runner* or *rotor* consists of the vanes fitted on periphery of a drum or wheel which is rigidly connected to a shaft supported in stationary bearings. This assembly rotates under the action of water gliding over the vanes.

(ii) A *guide apparatus* or *stator* consists of vanes fitted to the casing of the turbine and which helps the passage of water over the moving vanes in the proper direction. This may also serve as a converter of potential energy into kinetic energy of jet in case of an impulse turbine.

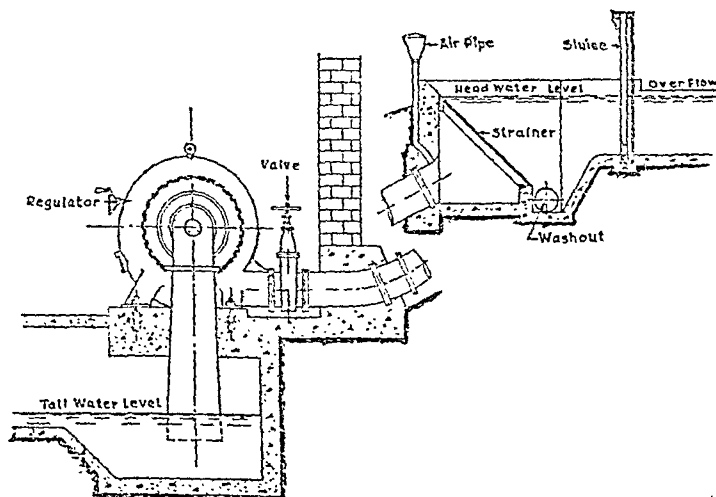


Fig. 10-3 General arrangement of turbine set

(iii) A *source of water* (dam and reservoir) and the pipe line (called the *penstock*) which leads water to the guide apparatus.

(iv) A *draft tube* and the *tailrace* in which water ultimately falls after leaving the turbine rotor and from which it flows away in the river.

The general arrangement of a turbine set with its main components is shown in fig 10-3.

Types of Turbines

Turbines are named in different fashions by looking at one or other aspect of conditions of working, availability of water or disposition of the power plant [*i.e.* position of shaft]. The following can be the basis of classification :

Head and quantity of water available : In an overall way, impulse turbines require very high head and small flow; whereas reaction turbines work on low head and large quantity of water. However, reaction turbines are made to suit conditions of medium head and medium flow and low head and large flow. Thus, *according to the head available, a turbine may be classified as high,* medium or low head turbine.

Names of inventors : These are attached to popular makes of prominent turbines in the present day use. Some of them are :

(i) *Pelton wheel* – an impulse turbine for medium head and low discharge, bears the name of the American inventor Mr. L. A. Pelton.

(ii) *Francis turbine* – a reaction turbine for medium head, medium discharge, carries the name of Mr. J B Francis, its inventor.

(iii) *Kaplan turbine* – a reaction turbine suitable for low head and great discharge assumes its name after German engineer, Dr. V. Kaplan.

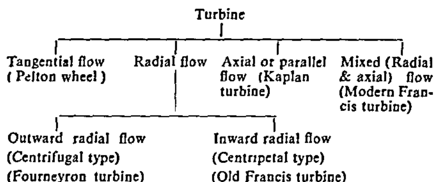
Action of water on moving vanes : Sometimes, the nature of action of water on the moving vanes, leads to naming of turbines.

Accordingly the two principal types of turbines are named : (i) The *impulse turbine*, a popular version of which is the Pelton wheel. (ii) The *reaction turbine* which is better known under two individual classes of turbines called (a) Francis turbine, and (b) Propeller or Kaplan turbines.

In case of the impulse turbine, water is led to a single nozzle through the pipes called penstock, from the reservoir. The whole pressure energy of water is converted into kinetic energy. Water, leaving the nozzle in the form of a free jet, is made to impinge upon buckets mounted on the circumference of a wheel. Water is delivered to the wheel on a part of its periphery filling or striking only a few buckets at a time. The rotor revolves in open air, *i.e.* there is no difference in water pressure at inlet to the runner and at exit from it. Hence, casing of this turbine does not perform any hydraulic function of maintaining the pressure difference or anything of the sort. Casing is provided only to prevent splashing and to guide water into the tailrace and also to safeguard against accidents. Hence this turbine is known as a *free jet turbine.*

With the reaction turbine the case is different. It operates with the rotor remaining submerged in water. Water, before entering the turbine, is under pressure, and it possesses kinetic energy too. The whole pressure energy is not transformed into kinetic energy as in the previous case. The torque on the rotor is produced by both types of energies. Water leaving the turbine is still left with some energy of both types. Pressure at entry to the turbine is greater than that at exit. This provides some possibility for water to flow through some passage other than the runner and escape without producing any useful work. This shows that a casing is absolutely essential for this type of turbine.

Direction of flow of water : At times the direction of flow of water in the runner is a good guide to understand the working and arrangement of the turbine and as such, turbine names based on this consideration also have come to stay long. Classification of turbines based on this consideration is given below.



Position of turbine shaft : Depending upon the disposition of the turbine shaft, the turbine is called either *horizontal* or *vertical turbine*. In present day practice, Pelton wheels usually have horizontal shafts whereas other turbines, especially the bigger machines have vertical shafts.

Notations : For the study of theory of turbines, adoption of uniform notations helps to form firm ideas about aspects of study and to give an insight in their merits and demerits by way of comparison. With such a view the notations listed below are followed throughout the work.

V_1 – absolute velocity of entering water (or jet), m/sec,
V_2 – absolute velocity of leaving water, m/sec,
v_1 – tangential velocity of wheel at inlet, m/sec,
v_2 – tangential velocity of wheel at outlet, m/sec,
V_{r_1} – water velocity relative to vane tip at inlet, m/sec,
V_{r_2} – water velocity relative to vane tip at exit, m/sec,
V_{w_1}, V_{w_2} – velocity of whirl of water at inlet and exit respectively, m/sec,
V_{f_1}, V_{f_2} – velocity of flow of water at inlet and exit respectively, m/sec,
r_1, r_2 – radii of the wheel at inlet and outlet vane tips respectively,
α, β – angles, entering and leaving water respectively makes with wheel's tangent, degrees,

θ – } angles of vane tips at entrance and exit respectively,
ϕ – }

Q – rate of discharge of water, m^3/sec.,

w – density of water, kg/m^3

W – rate of water flow through the wheel, kg/sec.,

H – total head of water, m,

η – hydraulic efficiency of the turbine,

N – rpm. (number of revolutions per minute),

ω – angular velocity, radians/sec ,

n – number of vanes in wheel,

t_1 – } thickness of vane tips at inlet and exit respectively,
t_2 – }

b_1 – } breadth (width) of wheel at inlet and exit respectively,
b_2 – }

P – power of the turbine, H.P. or kW.,

p – pressure of water, kg/m^2,

F_t – tangential force, kg,

F_a – axial thrust, kg,

T – turning moment or torque, kg–m,

f – coefficient of friction,

Impulse Turbines

In case of these turbines, before water enters the runner wheel of the turbine, its pressure is reduced to atmospheric value and in so doing high velocity is obtained for water. Hence, pressure of water, as it flows over the moving runner blades (or buckets) is constant. The velocity obtained is $V_1 = C_v \sqrt{2gH}$, where coefficient C_v (coefficient of velocity) takes care of losses in the nozzle or guide blades. Further the runner wheel need not run full (*i.e.* all buckets should not remain filled at the same time) in order to give scope to jet of water to act on the blades or vanes and convert the K. E. of jet into mechanical energy of the rotor.

A common feature of all varieties of impulse turbines will be that the energy is supplied in terms of jet velocity V_1 at exit from

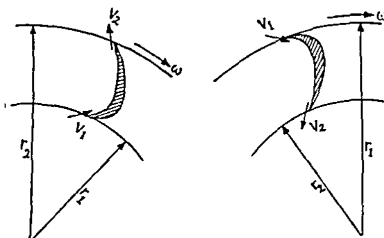
nozzle (s) and energy is discarded also in the form of velocity V_2. Hence, the hydraulic efficiency may have the form,

$$\eta_h = \frac{V_1^2 - V_2^2}{V_1^2} \quad .. (10\cdot1)$$

The hydraulic efficiency is greatest where V_2 is the least.

Radial Flow Impulse Turbine

In this type of turbine the blades or vanes are fitted radially to the rim of the rotating wheel. The flow may be either inwards



(a) Outward flow (b) Inward flow

Fig. 10-4 Radial flow turbine

or outwards. Water is made to enter through fixed guide vanes. With passage of water over moving blades, a centrifugal head is imparted to water by the rotating wheel and this head immediately gets converted into velocity head resulting in increased relative velocity of water for an outward flow and in decreased relative velocity for an inward flow.

Treating velocities in direction of motion of wheel as positive, we have,

tangential water momentum per unit mass flow rate at entrance and exit as $\frac{V_{w1}}{g}$ and $\frac{V_{w2}}{g}$ and moments of these momentums

are $\frac{V_{w1}r_1}{g}$ and $\frac{V_{w2}r_2}{g}$ respectively. Hence, change of moment of momentum, *viz.* $\frac{V_{w1}r_1}{g} - \frac{V_{w2}r_2}{g}$ is, in other words, the torque on the wheel. Hence, work done by torque is

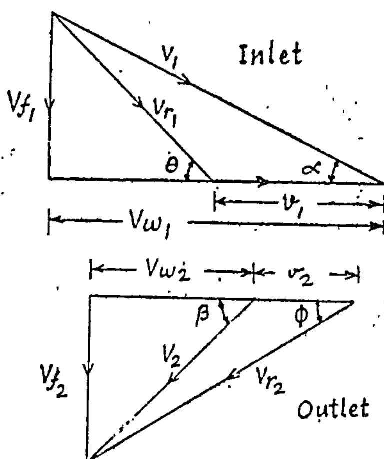


Fig. 10-5 Velocity triangles for radial flow turbine

$\left(\frac{V_{w1}r_1}{g} - \frac{V_{w2}r_2}{g}\right)\omega$. Since, $v_1 = \omega r_1$ and $v_2 = \omega r_2$,

this work becomes $\frac{V_{w1}v_1}{g} - \frac{V_{w2}v_2}{g}$. If water leaves the vanes in the direction opposite to that of rotation, then V_{w2} will have negative sign and expression for work becomes

$$\frac{V_{w1}v_1}{g} + \frac{V_{w2}v_2}{g}$$

Thus, work done per unit mass flow rate $\Big\} = \frac{V_{w1}v_1}{g} \pm \frac{V_w v_2}{g}$ kg–m .. (10·2)

The centrifugal head, referred to previously is

$$\frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

Then, $\frac{V^2_{r2}}{2g} - \frac{V^2_{r1}}{2g} = \left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right)$

i. e. $\frac{V^2_{r2}}{2g} = \frac{V^2_{r1}}{2g} + \left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right)$ .. (10·3)

For inward flow turbine, the relative velocity, as found above, decreases at exit because of the centrifugal force being negative, (since $v_2 < v_1$). This enables to control the speed of the inward flow turbine more easily than that of the outward flow turbine.

For outward flow turbines, the impressed head is positive (since $v_2 > v_1$). This increases the relative velocity at outlet and to that extent makes the task of controlling the speed of the turbine more difficult.

For outward flow radial turbines, working on the same lines as those for inward flow, the expression for work will remain the same *viz.*

$$\frac{V_{w1}v_1}{g} \pm \frac{V_{w2}v_2}{g}$$

Due to a temporary fall in load, the wheel speed increases, increasing with it the centrifugal force which reduces the V_{f2} for the inward turbine. Because of this the flow through the turbine (V_{f2}) decreases and hence the power output also falls. The turbine thus adjusts itself to the new conditions of working. For outward flow turbine, with fall in load the centrifugal force and hence the relative velocity increases and the mass flow of fluid (V_{f2}) also increases, hence the turbine tends to race up.

Radial flow impulse turbines are not suitable for very low heads, since the turbine has to be put above the foot of the fall in order to allow the turbines not to run full. This is a condition for the feasibility of working of impulse turbines. By putting them at higher level, a part of the head is not put to use, which results in a loss. If the head is greater, this loss may not be appreciable.

Axial Flow Impulse Turbines

In this turbine the moving vanes are mounted on the rotating wheel so as to remain parallel to the axis of the shaft. The moving vanes are preceeded by either nozzles, set at an angle to the plane of rotation of the wheel or by stationary guide vanes to admit flow, at the correctly desired angle, to the moving vanes. The direction

of flow of water is parallel to the axis of turbine. The two popular makes of this turbine are Jonval and Girard turbines.

Jonval turbine consists, in its simplest form, of one horizontal ring of moving blades into which water is directed by guide vanes placed above the moving blades. The flow is regulated by a horizontal sluice which closes portions of the wheel. A later type of Jonval turbine consists of several concentric rings of blades. The output is then controlled by closing one or more rings completely.

Girard turbine has two varieties : the radial flow type as well as the axial flow type. It can be used for heads upto 185 m and has an overall efficiency of about 75%. The guide vanes are not arranged over the entire circumference but are provided in two opposite sectors. The mass flow is varied by a sliding circular sluice gate which can completely shut off the flow through the vanes that are covered by it. By turning this gate, the flow may be stopped through as many guide vanes as desired. This prevents any loss of head due to contraction (throttling) when the gate is partially open. The turbine is vertical for axial flow, it is vertical or horizontal for radial flow.

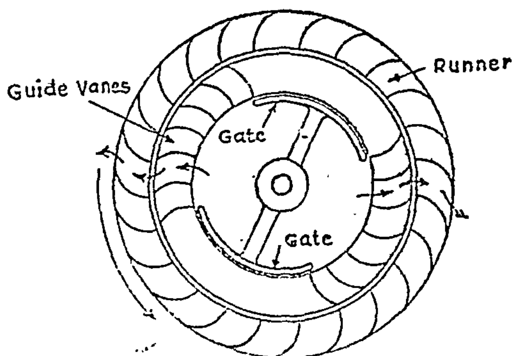


Fig. 10-6 Schematic diagram of Girard turbine

The static head of water is converted into high velocity of the jet with the help of nozzles or guide vanes acting as nozzles. Since, the flow is axial, the blade velocity at entrance and exit is the same, *viz.* $v_1 = v_2 = v$ and hence no centrifugal head is created

and imposed on the relative velocity. Hence, if no other losses are taking place, the relative velocities at entrance and exit are

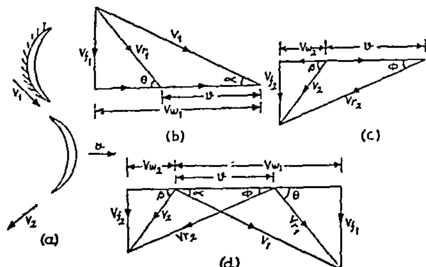


Fig. 10-7 Combined velocity diagrams

equal, *i e.* $V_{r2} = V_{r1}$. The expression for work done is obtained in a simpler form of $\frac{(V_{w1} + V_{w2})\, v}{g}$ per unit mass of water. The two velocity diagrams can be combined into one as shown in fig. 10-7. The maximum efficiency can be obtained if V_2 is as small as possible. This means that the energy going out with the jet should be the maximum.

Now considering the blade friction and assuming the blade friction coefficient as k,

$$V_{r2} = kV_{r1}$$

But, $V_{w1} = V_1 \cos\alpha$

and $V_{w2} = -(V_2 \cos\beta) = -(V_{r2} \cos\phi - v)$

$$= -(kV_{r1} \cos\phi - v)$$

$$= -\left(k \cos\phi \times \frac{V_1 \cos\alpha - v}{\cos\theta} - v\right)$$

$$\text{W. D.} = \frac{1}{g}\, v \left\{ V_1 \cos\alpha + k \cos\phi\, \frac{V_1 \cos\alpha - v}{\cos\theta} - v \right\}$$

$$= \frac{1}{g}\, v \left\{ \left(1 + \frac{k \cos\phi}{\cos\theta}\right) \left(V_1 \cos\alpha - v \right) \right.$$

Differentiating this w. r t. v and equating it to zero to obtain the wheel speed for maximum efficiency,

$$V_1 \cos\alpha - 2v = 0$$

$$i.e, \text{ speed ratio, } \frac{v}{V_1} = \frac{\cos\alpha}{2}$$

$$i\ e.\ \frac{\text{blade speed}}{\text{jet speed}} = \tfrac{1}{2}\cos\alpha \qquad \text{.. (10·4)}$$

Pelten Wheel Impulse Turbine

This is a special type of axial flow impulse turbine meant for very high heads. It is the most efficient among the impulse turbines and is the only impulse turbine in practical use to-day. Fig. 10-8 illustrates basic elements of a Pelton wheel turbine. Its overall efficiency is as high as 84%. The turbine has its present form as a result of refinement over the water wheel, which was the primitive form of the hydraulic turbine. The working blades, here, are the form of split elliptical buckets (fig. 10-9). The buckets have diver-

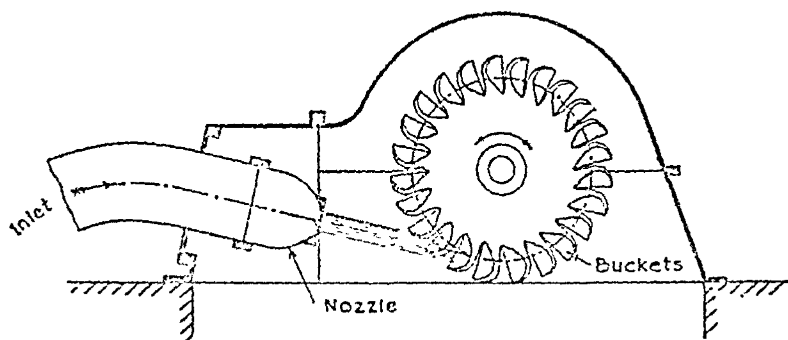


Fig. 10-8 Schematic diagram of a Pelton wheel

gent sides at the exit so that water escapes smoothly and the full force of reaction of jet, as it leaves the bucket, is utilised over and above the direct force due to the momentum of the impinging jet. The jet is deflected through 165° instead of 180° so as to avoid striking (by the leaving jet) the back of the incoming bucket and retarding the motion of the wheel.

The wheel is impinged on by one or more jets arranged along its periphery and the buckets receive the jet at their centres, and

since the jet after getting divided into two, leaves the buckets practically tangentially, as also when it enters tangentially, there is no axial thrust on the shaft of the wheel. The flow of water is governed by a throttle valve in the supply pipe or by a needle valve in the nozzle.

To consider the velocity diagrams (fig 10-10) for the Pelton wheel, let us examine the various requisites to construct them. Since the jet strikes the buckets axially, (*i e.* jet comes parallel to the plane of rotation of the wheel) $\alpha = 0$, and since the bucket entrance edge is parallel to the plane of rotation, $\theta = 0$. Then, the velocity diagram at entrance is a staright line. Hence,



Fig. 10-9 Rotor of a Pelton wheel

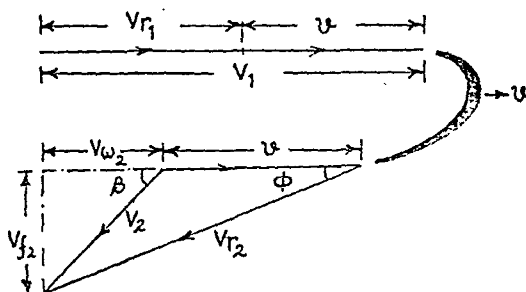


Fig. 10–10 Velocity diagrams of a Pelton wheel

$$V_{r1} = V_1 - v$$

and $$V_{w1} = V_1 = C_v \sqrt{2gH}$$

and $$V_{f1} = 0$$

From exit velocity triangle,

$$v_2 = v_1 = v$$

$$V_{r2} = V_{r1} = V_1 - v$$

$$V_{w2} = V_{r2} \cos\phi - v$$

$$= (V_1 - v) \cos\phi - v$$

$$\left.\begin{array}{l}\text{Work done per unit}\\ \text{mass of water}\end{array}\right\} = \frac{V_{w1}v_1}{g} - \frac{V_{w2}v_2}{g}$$

$$= \frac{V_1 v}{g} + \frac{\{(V_1 - v)\cos\phi - v\}\, v}{g} \text{ (since } V_{w2} \text{ is negative)}$$

$$= \frac{1}{g}\,[\,V_1 v + v\,(V_1 - v)\cos\phi - v^2\,] \qquad ..\ (10{\cdot}5)$$

$$\therefore \text{ wheel efficiency, } \eta_h = \frac{\frac{1}{g}\,[\,V_1 v + v\,(V_1 - v)\cos\phi - v^2\,]}{\frac{V_1^2}{2g}} \qquad ..\ (10{\cdot}6)$$

The optimum speed for the best efficiency or maximum work can be known if the above expression (10·6) is differentiated w.r.t. v and equated to zero.

$$\frac{d\eta}{dv} = V_1 + (V_1 - 2v)\cos\phi - 2v = 0$$

i e. $V_1 (1 + \cos\phi) - 2v\,(1 + \cos\phi) = 0$

or $(1 + \cos\phi)\,(V_1 - 2v) = 0$

But, $(1+\cos\phi)$ cannot be equal to zero.

$\therefore \quad V_1 - 2v = 0$

or $\quad v = \dfrac{V_1}{2}$

It is seen that the turbine should be rotated with that tangential velocity which is one-half of the jet velocity at entrance to the rotor. However, in practice, it is observed that the maximam efficiency is obtained when $v = 0{\cdot}46V_1$.

Substituting $v = \dfrac{V_1}{2}$, in eqn. (10·6), we have

$$\eta_{max} = \frac{2\left[\dfrac{V_1^2}{2} + \dfrac{V_1^2\cos\phi}{4} - \dfrac{V_1^2}{4}\right]}{V_1^2}$$

$$= \tfrac{1}{2}(1+\cos\phi) \qquad \text{.. (10·7)}$$

If ϕ is made zero, the efficiency is 100%. However, the deflection of the jet $(180-\phi)^\circ$ is required in order that the back of the incoming bucket does not strike the leaving jet. The proportions

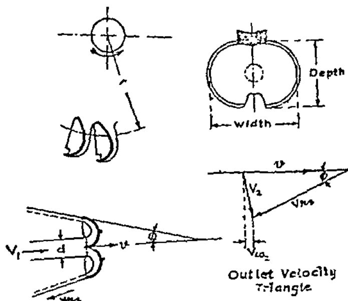


Fig. 10-11 Sections and proportions of a bucket

(fig. 10 - 11) of the buckets are fixed according to the following empirical relations : If d is diameter of jet,

depth of bucket = $1 \cdot 0d$ to $1 \cdot 2d$,
width of bucket = $3d$ to $5d$,
angle $\theta = 5°$ to $8°$,
angle $\phi = 10°$ to $20°$.

The number of buckets is obtained by making an arrangement which ensures that the buckets completely intercept the jet.

The mean radius of the bucket circle is r, and γ is the included angle between the two consecutive buckets. For the jet to be intercepted, one bucket (its centre) should be just about to move out (from the centre line of the jet) of the jet as another bucket is just moving in. E, F, G are the adjacent buckets. As jet has twice the speed of the buckets, a section of the jet moves from E to G in

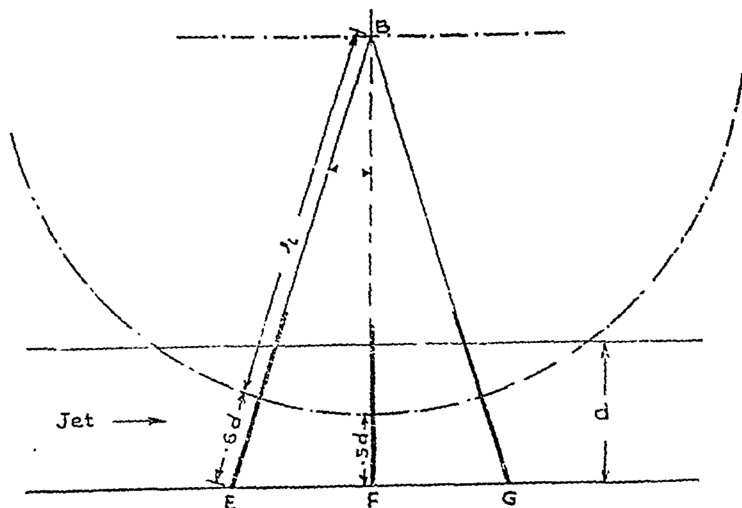


Fig. 10-12

the same time as bucket F moves to G. Hence, for jet to be always intercepted, the buckets have configuration as shown in fig. 10-12.

For triangle BEF,

$$BE = r + \tfrac{1}{2} \text{ height of bucket} = r + 0 \cdot 6d,$$

$$BF = r + \tfrac{1}{2} \text{ diameter of jet} = r + 0 \cdot 5d,$$

$$\cos \gamma = \frac{r + 0 \cdot 5d}{r + 0 \cdot 6d}$$

From it γ is obtained, and then,

$$\text{number of buckets, } n = \frac{360}{\gamma} \quad \cdots (10\ 8$$

However, this is not a practicable number since, there is not enough space available around the wheel to accomodate this number of buckets H nce, the number is brought down to about one-half of this number An empirical formula suggests the number of buckets as $0.5m + 15$, where m is the ratio of mean wheel diameter to jet diameter

Turgo impulse wheel This turbine constitutes one more example of the axial flow impulse turbine It is a revised version of the Pelton wheel. The blade shape is changed from the two-bucket

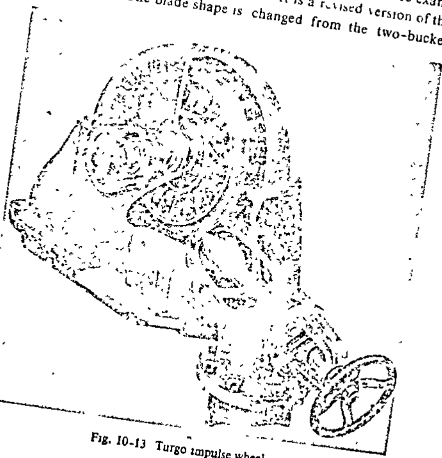


Fig. 10-13 Turgo impulse wheel

nature of Pelton wheel to a single bucket cast integral with the wheel. The bucket is held inwardly by the hub and outwardly by a peripheral band. The jet at entrance strikes the bucket at an angle (to the plane of rotation) instead of the arrangement of the Pelton wheel where it practically flows in the direction of plane of rotation. The jet is ejected by a nozzle with needle regulator resembling Pelton wheel nozzle. The striking of the jet on the buckets is similar to that in Girard impulse turbine. Water enters the buckets axially and leaves it radially outward. The number of jets to the runner is usually one or at the most two. The end thrust of the jet is taken by thrust bearings. Fig. 10-14 shows the difference between the water jets impinging the Pelton bucket and the Turgo bucket. As compared with a single jet Pelton wheel,

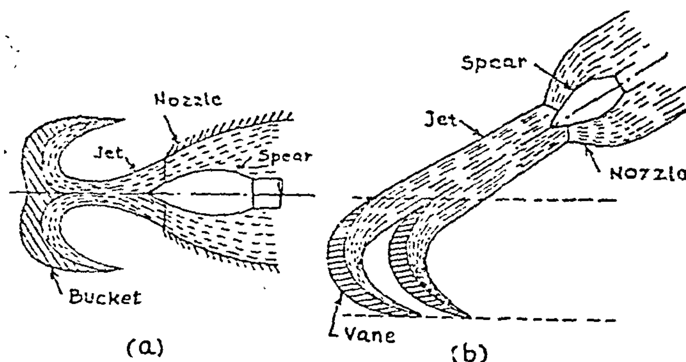


Fig. 10-14 Action of jet on (a) Pelton bucket, and (b) Turgo bucket

the Turgo turbine has higher specific speed, which means that for a given head and output, its rotational speed is higher. By having such a characteristic, it fills the gap of specific speed between the Pelton wheel and the Francis turbine. In Pelton wheel, this may be obtained by use of multi-jets. Turgo impulse wheels, are the exclusive development of Messrs Gilbert Gilkes and Gorden Ltd., England and are in use for heads from 10 m to 300 m, have diameters from 15 cm to 100 cm., speeds from 120 rpm. to 2,500 rpm. and develop power from 1·12 to 5,000 BHP. Quantity of water consumed varies between 1 and 125 m^3 per min.

Problem – 1 : *A Pelton wheel is required to work under a head of 40 m and to develop 100 H. P. at 250 rpm. Assuming an efficiency of 80% and a coefficient of velocity of 0·98, find the jet diameter, the diameter of the bucket circle, the size of the bucket, and the number of buckets required.*

We have, $V_1 = 0{\cdot}98\sqrt{2gH}$

$$= 0{\cdot}98\sqrt{2 \times 9{\cdot}81 \times 40} = 28 \text{ m/sec.}$$

For maximum efficiency,

$$v = 0{\cdot}46 v_1 \text{ (practical value)}$$
$$= 0{\cdot}46 \times 28 = 12{\cdot}9 \text{ m/sec.}$$

Further, $\text{H. P.} = \dfrac{\eta WH}{75}$ (W is mass flow rate of water)

$$\therefore \quad W = \frac{100 \times 75}{0{\cdot}8 \times 40} = 235 \text{ kg/sec.}$$

If d – diameter of jet, m

D – diameter of bucket circle, m

$$v = \frac{\pi DN}{60}$$

$$\therefore \quad D = \frac{v \times 60}{\pi N}$$
$$= \frac{12{\cdot}9 \times 60}{\pi \times 250} = 0{\cdot}985 \text{ m } i.\,e.\ 98{\cdot}5 \text{ cm}$$

Further, quantity of water flowing per sec., is

$$Q = \frac{\pi}{4} d^2 V_1 = \frac{W}{w}$$

$$\therefore \quad d^2 = \frac{W \times 4}{\pi V_1 w}$$
$$= \frac{235 \times 4}{\pi \times 28 \times 1{,}000} = 0{\cdot}0107$$

$$\therefore \quad d = 0{\cdot}103 \text{ m or } 10{\cdot}3 \text{ cm}$$

Now, depth of bucket $= 1{\cdot}2\,d$

$$= 1{\cdot}2 \times 10{\cdot}3 = 12{\cdot}4 \text{ cm, say } 12{\cdot}5 \text{ cm}$$

width $= 5d = 5 \times 10{\cdot}3 = 51{\cdot}5$, say 52 cm

For number of buckets, the included angle γ between two buckets is given by

$$\cos \gamma = \frac{r + 0.5d}{r + 0.6d} = \frac{49.25 + 0.5 \times 10.3}{49.25 + 0.6 \times 10.3} = 0.982$$

$$\text{number of buckets, } n = \frac{360}{11} = 32.7, \text{ say } 32 \text{ (even number)}$$

Problem – 2 : *A single jet Pelton wheel running at 300 rpm. developes a certain power P, with an efficiency of 80% and C_V for the nozzle is 0·98. The diameter of the jet is 100 mm. The number of buckets on the pelton wheel is 30. Taking the speed ratio as 0·46 for best results, find the bucket circle diameter, size of the buckets, weight of water used by pelton wheel, the head under which it operates and H. P. developed.*

As there are 30 buckets round the circumference of the wheel, the angle between two adjecent buckets is $\gamma = \frac{360}{30} = 12°$.

Now, for number of buckets,

$$\cos \gamma = \frac{r + 0.5d}{r + 0.6d}$$

where d = diameter of the jet = 10 cm,

and r = radius of the bucket circle.

$$\therefore \quad \cos 12° = \frac{r + 0.5 \times 10}{r + 0.6 \times 10}$$

$$0.9781 = \frac{r + 5}{r + 6}$$

$$\therefore \quad r = \frac{0.8686}{0.9219} = 39.67 \text{ cm}$$

$\therefore$ diameter of the bucket circle $= D = 39.67 \times 2 =$ <u>79·34 cm</u>

depth of bucket $= 1.2d = 1.2 \times 10 =$ <u>12 cm</u>

width of bucket $= 5d = 5 \times 10 =$ <u>50 cm</u>

Now, the bucket speed, $v = \frac{\pi DN}{60}$

$$\therefore \quad v = \frac{\pi \times 0.7934 \times 300}{60} = 12.47 \text{ m/sec.}$$

The maximum efficiency condition is

$$v = 0.46\ V_1$$

$$\therefore \quad V_1 = \frac{v}{0.46} = \frac{12.47}{0.46} = \underline{27.1\ \text{m/sec.}}$$

For a single jet, weight of water used is

$$W = \frac{\pi}{4}\ d^2 \times V_1 \times w$$

$$= \frac{\pi}{4}\left(\frac{10}{100}\right)^2 \times 27.1 \times 1{,}000 = \underline{212.8\ \text{kg/sec}}$$

Jet velocity, $V_1 = C_v\ \sqrt{2gH}$

where H is the head under which the Pelton wheel works.

$$\therefore \quad 27.1 = 0.98\ \sqrt{2 \times 9.81 \times H}$$

$$\therefore \quad H = \frac{\left(\frac{27.1}{0.98}\right)^2}{2 \times 9.81} = \underline{38.96\ \text{m}}$$

$$\text{Now,} \quad \text{H.P.} = \frac{WH}{75} \times \eta$$

$$= \frac{212.8 \times 38.96 \times 0.8}{75}$$

$$= \underline{88.5\ \text{H.P.}}$$

Problem – 3 : *The buckets of a Pelton wheel deflect the jet through an angle of 170° and the relative velocity of water is reduced to 88% by friction. Calculate from first principles the value of hydraulic efficiency for a speed ratio of 0.47. Bucket circle diameter (mean) is 100 mm and there are 2 jets for which C_v is 0.98. Actual efficiency of the wheel is 0.9 times the theoretical value. Under a gross head of 600 m the wheel develops 1,675 H.P. The loss of head due to pipe friction is 50 m. Calculate, (a) the speed of rotation in rpm, and (b) diameter of nozzles. Sketch the shape of suitable bucket indicating the method of attachment to the runner.*

Jet speed, $V_1 = C_v\ \sqrt{2gH}$

$$= 0.98\ \sqrt{2 \times 9.81 \times (600 - 50)} = 102\ \text{m/sec.}$$

Wheel speed, $v_1 = 0.47 \times V_1 = 0.47 \times 102 = 48$ m/sec.

$$\text{Again,} \quad v_1 = \frac{\pi DN}{60}$$

Problem - 5: *A Pelton wheel is to be designed for the following data :*

H. P. to be developed	.. *8,010 H.P.*
Net head available	.. *300 m*
Speed of the wheel	.. *600 rpm.*
Speed ratio	.. *0·46*
Ratio of jet diameter to wheel diameter	.. $\frac{1}{8}$
Mechanical efficiency	.. *90%*
Velocity coefficient of the nozzle	.. *0·98*

Find the number of jets, the diameter of the jets and wheel, and the quantity of water required.

$$\text{W.H.P.} = \frac{\text{H.P. developed}}{\eta_{mech}} = \frac{8{,}010}{0.9} = 8{,}900 \text{ H.P.}$$

$$\text{W.H.P.} = \frac{WH}{75}$$

$$\therefore \quad W = \frac{8{,}900 \times 75}{300} = 2{,}225 \text{ kg/sec}$$

$$\therefore \quad \text{Discharge per sec, } Q = \frac{W}{w} = \frac{2{,}225}{1{,}000} = \underline{2.225 \text{ m}^3/\text{sec}}$$

$$\text{Velocity of the jet, } V_1 = C_v \sqrt{2gH}$$

$$= 0.98\sqrt{19.62 \times 300} = 75 \text{ m/sec.}$$

$$\text{Speed ratio} = \frac{v}{V_1} \text{ i.e. } 0.46 = \frac{v}{75}$$

$$\therefore \quad v = 0.46 \times 75 = 34.5 \text{ m/sec}$$

$$\text{But, } v = \frac{\pi DN}{60}$$

$$\therefore \quad \text{diameter of the wheel, } D = \frac{60\,v}{\pi N} = \frac{60 \times 34.5}{\pi \times 600} = \underline{1.1 \text{ m}}$$

$$\text{Diameter of the jet, } d = \frac{D}{8} = \frac{1.1}{8} = 0.1375 \text{ m i.e. } \underline{13.75 \text{ cm}}$$

$$\text{Discharge, } Q = \text{jet area} \times \text{jet velocity} \times \text{No. of jets}$$

$$\therefore \quad \text{No. of jets} = \frac{Q}{\text{jet area} \times \text{jet velocity}}$$

$$= \frac{2.225}{\frac{\pi}{4}\left(\frac{13.75}{100}\right)^2 \times 75} = \frac{2.225 \times 40{,}000}{\pi \times (13.75)^2 \times 75} = \underline{2}$$

Problem – 6 : *A Pelton wheel turbine is supplied with water through a pipe 35 km long from a reservoir in which the level of water is 600 m above that of wheel. The turbine runs at 600 rpm. and develops 6,000 H.P. Calculate diameter of pipe, if transmission efficiency of pipe is 83·3% taking value of f = 0·005. Calculate cross-sectional area of jet and mean diameter of the bucket circle. Assume C_v = 0·98 for jet, speed ratio = 0·46, and hydraulic efficiency = 85%.*

Transmission efficiency of pipe $= \frac{H - H_l}{H}$ *i.e.* $0{\cdot}833 = \frac{600 - H_l}{600}$

$\therefore \quad H_l = 600 - 0{\cdot}833 \times 600 = 100$ m

$\therefore$ available head, $H = 600 - 100 = 500$ m

Now, velocity of jet, $V_1 = C_v \sqrt{2gH}$

$= 0{\cdot}98 \sqrt{19{\cdot}62 \times 500} = 96$ m/sec

Velocity of bucket, $v = 0{\cdot}46\ V_1 = 0{\cdot}46 \times 96 = 44$ m/sec.

Also, $v = \frac{\pi DN}{60}$

$\therefore$ Diameter of the bucket circle, $D = \frac{60\,v}{\pi N} = \frac{60 \times 44}{\pi \times 600} = 1{\cdot}4$ m

Output of turbine = H.P. of two jets × η_{hyd} of turbine

i.e. $6000 = 2\,\frac{waV_1^3}{2g \times 75} \times \eta_{hyd}$.

$\therefore \quad a = \frac{6{,}000 \times 2 \times 9{\cdot}81 \times 75}{2 \times 1{,}000 \times (96)^3 \times 0{\cdot}85} = \underline{0{\cdot}00588\ \text{m}^2}$

Discharge of water, $Q = 2 \times a \times V_1$

$= 2 \times 0{\cdot}00588 \times 96 = 1{\cdot}13$ m³/sec

Head lost in friction, $H_l = \frac{flQ^2}{3D^5}$

$\therefore \quad 100 = \frac{0{\cdot}005 \times 35{,}000 \times (1{\cdot}13)^2}{3D^5}$

$\therefore \quad D = \sqrt[5]{\frac{0{\cdot}005 \times 35{,}000 \times (1{\cdot}13)^2}{300}} = 0{\cdot}9428$ m *i.e.* 94·28 cm

Problem – 7 : *A single jet Pelton wheel is supplied with water from a reservoir 300 m above the centre of the nozzle through a pipe of 60 cm diameter and 5 km long. The friction coefficient for the pipe is 0·008. The jet has diameter of 8 cm and its velocity coefficient is 0·97. The wheel bucket has a speed of 0·47 of the jet speed and deflects the water through 160°, the relative velocity at outlet is 80% of that at inlet. The mechanical efficiency of the wheel is 80%. Determine B. H. P. of wheel, its hydraulic efficiency and overall efficiency.*

Let, V = velocity of water in the pipe in m/sec, and

V_1 = velocity of jet in m/sec.

$\therefore \quad AV = A_1 V_1$

$$\therefore \quad V = \frac{A_1}{A} V_1 = \left(\frac{d}{D}\right)^2 \times V_1$$

$$\therefore \quad V = \left(\frac{8}{60}\right)^2 \times V_1 = 0{\cdot}0177\ V_1 \text{ m/sec}$$

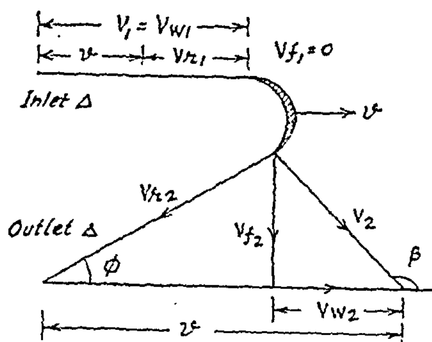


Fig. 10-16

$$\text{Frictional loss in the pipe} = \frac{4fl}{D} \cdot \frac{V^2}{2g}$$

$$= \frac{4 \times 0{\cdot}008 \times 5{,}000}{0{\cdot}6} \times \frac{(0{\cdot}0177)^2\, V_1^2}{19{\cdot}62}$$

$$= 0{\cdot}00426\ V_1^2 \text{ m of water}$$

Frictional loss is nozzle $= \left\{\frac{1}{C_v^2} - 1\right\}\frac{V_1^2}{2g}$

$$= \left\{\frac{1}{(0.97)^2} - 1\right\}\frac{V_1^2}{2g}$$

$$= 0.00326\ V_1^2 \text{ m of water}$$

Energy of the jet at outlet from nozzle

$$= \frac{V_1^2}{2g} = \frac{1}{19.61} V_1^2 = 0.05\ V_1^2$$

According to Bernoulli's equation,

$$\text{Available head} = \begin{Bmatrix}\text{friction losses}\\ \text{in pipe}\end{Bmatrix} + \begin{Bmatrix}\text{friction losses}\\ \text{in nozzle}\end{Bmatrix} + \begin{Bmatrix}\text{energy}\\ \text{in jet}\end{Bmatrix}$$

$$\therefore \quad 300 = 0.00426\ V_1^2 + 0.00326\ V_1^2 + 0.05\ V_1^2$$

$$\therefore \quad V_1 = \sqrt{\frac{300}{0.05852}} = 71.5 \text{ m/sec}$$

Net available head at the nozzle $= 300 -$ friction loss in pipe

$$= 300 - 0.00426\ (71.5)^2$$

$$= 278.2 \text{ m of water}$$

Now, speed of the bucket of the Pelton wheel,

$$v = 0.47\ V_1 = 0.47 \times 71.5 = 33.6 \text{ m/sec}$$

From inlet triangle of velocity, $V_{r1} = V_1 - v$

$$= 71.5 - 33.6 = 37.9 \text{ m/sec}$$

and relative velocity of water at outlet,

$$V_{r2} = 0.8\ V_{r1} = 0.8 \times 37.9 = 30.32 \text{ m/sec}$$

Angle of blade tip at outlet, $\phi = 180° - 160° = 20°$

From outlet triangle of velocity,

$$V_{w2} = V_{r2} \cos\phi - v = 30.32 \times \cos 20° - 33.6 = -5.2 \text{ m/sec}$$

($-ve$ sign means that the water leaves in the direction of motion of vane at outlet)

$$\therefore \quad \text{W. D./kg} = \frac{(V_{w1} + V_{w2})\,v}{g} = \frac{(71.5 - 5.2)\,33.6}{9.81} = 228 \text{ kg-m}$$

$$\text{Hydraulic efficiency} = \frac{228}{278.2} \times 100 = \underline{82.3\%}$$

Quantity of water supplied, $= waV_1$

$$= 1{,}000 \times \frac{\pi}{4}\left(\frac{8}{100}\right)^2 \times 71{\cdot}5$$

$$= \underline{359 \text{ kg/sec}}$$

$$\text{B. H. P.} = \eta_{mech} \frac{\text{W.D./kg} \times W}{75}$$

$$= \frac{0{\cdot}8 \times 359 \times 228}{75} = 872$$

$$\text{Overall efficiency} = \frac{\text{B.H.P.} \times 100}{W \times 300/75} = \frac{872 \times 75}{359 \times 300} \times 100 = \underline{60{\cdot}6\%}$$

Problem – 8 : *A Pelton wheel is supplied by four pipes in parallel connected to a short common pipe leading to the nozzle. If the losses in the short pipe and nozzle are small in comparison with the friction losses in the four pipes, show that the power delivered to the wheel by the jet is a maximum when* $d = \sqrt[4]{\frac{2D^5}{fl}}$, *in which* d *is the nozzle diameter,* D *the diameter,* l *the length of each pipe, and* f *the friction coefficient.*

Hence find the maximum horse-power which can be delivered to the Pelton wheel by a nozzle of suitable diameter, if $D = 90$ *cm,* $l = 20$ *km; when level of water in the reservoir is 400 m above the centre of the jet. What is the diameter of the nozzle and what is the flow ? Take for the pipe* $f = 0{\cdot}008$.

A part of available head is used in friction and rest will be converted into K.E. of jet. Let V m/sec be the velocity of water in the pipe line and V_1 m/sec the velocity of jet.

From mass continuity law,

$$4AV = aV_1$$

$$\therefore \quad 4 \times \frac{\pi}{4} D^2 \times V = \frac{\pi}{4} d^2 V_1$$

$$\therefore \quad V = \frac{d^2}{4D^2} \times V_1$$

$$\therefore \quad V^2 = \frac{d^4}{16D^4} \times V_1^2$$

Total head lost in friction of each pipe,

$$H_f = \frac{4fl}{D} \cdot \frac{V^2}{2g} = \frac{4f \times l}{2gD} \times \frac{d^4}{16D^4} \times V_1^2 = \frac{4fl\,d^4\,V_1^2}{32g\,D^5}$$

K.E. of the jet at outlet from nozzle $= \dfrac{V_1^2}{2g}$

$$\therefore \quad H = \frac{4fl\,d^4\,V_1^2}{32g\,D^5} + \frac{V_1^2}{2g} \text{ (assuming no loss in the nozzle).}$$

$$\therefore \quad H = \frac{V_1^2}{2g}\left(\frac{fl\,d^4 + 4D^5}{4D^5}\right)$$

$$\therefore \quad V_1 = \left\{\frac{8gH\,D^5}{fld^4 + 4D^5}\right\}^{1/2}$$

Now power of jet, $P = \dfrac{waV_1^3}{2g \times 75} = \dfrac{w}{2g} \times \dfrac{\pi\,d^2}{4 \times 75}\left\{\dfrac{8HgD^5}{fl\,d^4 + 4D^5}\right\}^{3/2}$

$$= \frac{w\pi\,\{8HgD^5\}^{3/2}}{2g \times 300} \times \frac{d^2}{\{fld^4 + 4D^5\}^{3/2}}$$

$\therefore \quad P = K\dfrac{d^2}{\{fld^4 + 4D^5\}^{3/2}}$ where $K = \dfrac{w\pi\,\{8HgD^5\}^{1/2}}{2g \times 300}$

For maximum power, $\dfrac{dP}{dd} = 0$

$$\therefore \quad \frac{dP}{dd} = \frac{K[\,2d\,(fld^4 + 4D^5\,)^{3/2} - \frac{3}{2}d^2\,(fld^4 + 4D^5\,)^{1/2} \times (4\,fld^3)\,]}{(fld^4 + 4D^5\,)^3} = 0$$

$$\therefore \quad 2d\,(fld^4 + 4D^5\,)^{3/2} - 6\,fld^5\,(fld^4 + 4D^5\,)^{1/2} = 0$$

$$\therefore \quad 2d\,(fld^4 + 4D^5) = 6\,fld^5$$

$$\therefore \quad fld^4 + 4D^5 = 3fld^4 \;\; i.e. \;\; 4D^5 = 2\,fld^4$$

$$\therefore \quad d = \sqrt[4]{\frac{2D^5}{fl}}$$

Hence, $d = \sqrt[4]{\dfrac{2 \times (0{\cdot}9)^5}{0{\cdot}008 \times 20{,}000}} = \underline{0{\cdot}294\text{ m}}$

Now, $V_1 = \left\{\dfrac{8HgD^5}{fld^4 + 4D^5}\right\}^{\frac{1}{2}} = \left\{\dfrac{8 \times 400 \times 9{\cdot}81 \times (0{\cdot}9)^5}{0{\cdot}008 \times 20{,}000 \times (0{\cdot}294)^4 + 4\,(0{\cdot}9)^5}\right\}^{\frac{1}{2}}$

$= 73$ m/sec.

$$\text{Power} = \frac{waV_1^3}{2g \times 75} = \frac{1{,}000 \times \frac{\pi}{4}\,(0{\cdot}294)^2 \times (73)^3}{19{\cdot}62 \times 75} = \underline{18{,}050 \text{ H.P.}}$$

$$\text{Discharge, } W = waV_1 = 1{,}000 \times \frac{\pi}{4}\,(0{\cdot}294)^2 \times 73 = \underline{4{,}960 \text{ kg/sec}}$$

Problem - 9 : *An outward flow impulse wheel has an internal diameter of 1·5 m and an external diameter of 1·8 m and it rotates at 250 r.p.m. The wheel has 30 vanes, 2 cm thick at inlet and 3 cm thick at outlet. The head is 42 m. The effective widths of the wheel face at inlet and outlet are 25 cm and 20 cm respectively. The quantity of water supplied per second is 6 m³/sec. Determine the angle of the tips of the vanes at inlet and outlet so that water will leave radially.*

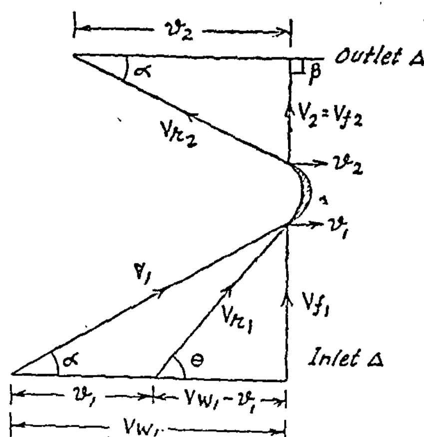


Fig. 10-16

$$\text{Vane tip velocity at inlet, } v_1 = \frac{\pi\, d_1\, N}{60}$$

$$= \frac{\pi \times 1{\cdot}5 \times 250}{60} = 19{\cdot}65 \text{ m/sec}$$

Vane tip velocity at outlet, $v_2 = \frac{\pi d_2 N}{60}$

$$= \frac{\pi \times 1{\cdot}8 \times 250}{60} = 23{\cdot}6 \text{ m/sec}$$

Absolute velocity of water at inlet,

$$V_1 = \sqrt{2gH} = \sqrt{19{\cdot}62 \times 42} = 28{\cdot}7 \text{ m/sec}$$

Discharge of water at inlet, $Q = (\pi d_1 - nt_1)\, b_1 \times V_{f_1}$

$$\therefore \quad 6 = \left(\pi \times 1{\cdot}5 - \frac{30 \times 2}{100}\right) \times \frac{25}{100} \times V_{f_1}$$

$$\therefore \quad V_{f_1} = \frac{6}{1{\cdot}03} = 5{\cdot}84 \text{ m/sec}$$

Discharge of water at outlet, $Q = (\pi d_2 - nt_2)\, b_2 V_{f_2}$

$$\therefore \quad 6 = \left(\pi \times 2{\cdot}8 - \frac{30 \times 3}{100}\right) \times \frac{20}{100} \times V_{f_2}$$

$$\therefore \quad V_{f_2} = \frac{6}{0{\cdot}95} = 6{\cdot}32 \text{ m/sec}$$

Since the water leaves radially at outlet, $\beta = 90°$.

Hence, $V_{w_2} = O$, $V_2 = V_{f_2} = 6{\cdot}32$ m/sec.

$$\therefore \quad \tan \phi = \frac{V_{f_2}}{V_2} = \frac{6{\cdot}32}{23{\cdot}6} = 0{\cdot}269$$

$$\therefore \quad \phi = 15° \, 4'$$

Now, $\sin \alpha = \frac{V_{f_1}}{V_1} = \frac{5{\cdot}84}{28{\cdot}7} = 0{\cdot}203$

$$\therefore \quad \alpha = 11° \, 46'$$

Thus, $V_{w1} = v_1 \cos \alpha = 28{\cdot}7 \times \cos 11°46' = 28{\cdot}7 \times 0\ 979 = 28{\cdot}1$ m/sec

and $\tan \theta = \frac{V_{f_1}}{V_{w1} - v_1} = \frac{5{\cdot}84}{28{\cdot}1 - 19{\cdot}65} = \frac{5{\cdot}84}{8{\cdot}45} = 0{\cdot}69$

$$\therefore \quad \theta = \underline{34°36'}$$

Problem – 10 : *An outward flow Girard impulse turbine has the following dimensions: inlet guide vane angle of 16°; moving blade outlet angle of 165°; inlet and outlet diameter of the runner is 1·2 m and 1·5 m respectively. The speed of the runner is 270 rpm. and it works under a head of 120 m, when the flow of water is 3·4 m³/sec. If the friction losses in flow through the runner* $= \frac{0.5V_{r2}^2}{2g}$, *find the magnitude and direction of absolute velocity at outlet, hydraulic efficiency, angle of the blade tip at inlet and power developed.*

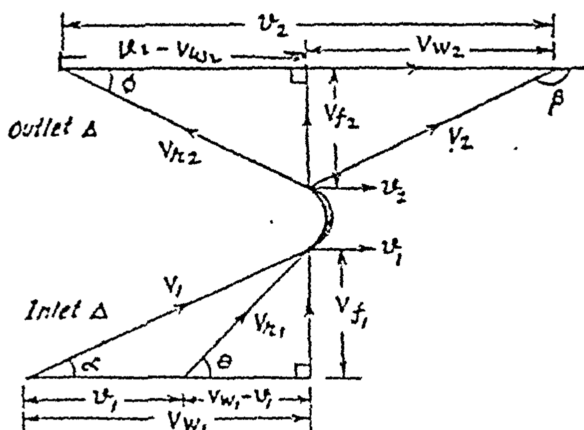


Fig. 10-17

Velocity of vane tip at inlet, $v_1 = \frac{\pi d_1 N}{60}$

$$= \frac{\pi \times 1{\cdot}2 \times 270}{60} = 16{\cdot}95 \text{ m/sec}$$

Velocity of vane tip at outlet, $v_2 = \frac{\pi d_2 N}{60}$

$$= \frac{\pi \times 1{\cdot}5 \times 270}{60} = 21{\cdot}2 \text{ m/sec}$$

Velocity of water at inlet, $V_1 = \sqrt{2gH} = \sqrt{19{\cdot}62 \times 120}$

$$= 48{\cdot}5 \text{ m/sec}$$

Velocity of flow at inlet, $V_{f1} = V_1 \sin \alpha = 48.5 \sin 16°$
$= 48.5 \times 0.2756 = 13.35$ m/sec.

Velocity of whirl at inlet, $V_{w1} = V_1 \cos \alpha = 48.5 \times 0.9613$
$= 46.5$ m/sec.

From inlet triangle,

$$\tan\theta = \frac{V_{f1}}{V_{w1} - v_1} = \frac{13.35}{46.5 - 16.95} = 0.453$$

$$\therefore \quad \theta = 24°22'$$

Now, $\phi = 180° - 165° = 15°$

From outlet triangle we have,

$$V_{w2} = V_{r2}\cos\phi - v_2 = V_{r2}\cos 15° - 21.2 = V_{r2} \times 0.9659 - 21.2$$

$$V_2^2 = V_{r2}^2 + v_2^2 - 2v_2 V_{r2} \cos 15°$$

$$\therefore \quad V_2 = \sqrt{V_{r2}^2 + 450 - 41V_{r2}}$$

As this is an impulse turbine the pressure throughout is atmospheric i.e. $p_1 = p_2$ and since the flow is in a horizontal plane, $Z_1 = Z_2$.

$$\therefore \quad \frac{p_1}{w} + \frac{V_1^2}{2g} - \left(\frac{p_2}{\omega} + \frac{V_2^2}{2g}\right) - \frac{0.5\, V_{r2}^2}{2g} = \frac{V_{w1}\, v_1 + V_{w2}\, v_2}{g}$$

$$\therefore \quad \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{0.5\, V_{r2}^2}{2g} = \frac{V_{w1}\, v_1 + V_{w2}\, v_2}{g}$$

$$\therefore \quad V_1^2 - V_2^2 - 0.5\, V_{r2}^2 = 2\,(V_{w1}\, v_1 + V_{w2}\, v_2)$$

$$\therefore \quad (48.5)^2 - (V_{r2}^2 + 450 - 41\, V_{r2}) - 0.5\, V_{r2}^2$$
$$= 2\,(46.5 \times 16.95 + 0.9659\, V_{r2} - 21.2)\, 21.2$$

$$\therefore \quad 2{,}350 - V_{r2}^2 - 450 + 41V_{r2} - 0.5\, V_{r2}^2 = 1{,}580 + 41\, V_{r2} - 900$$

$$\therefore \quad 1.5\, V_{r2}^2 = 1{,}220$$

$$\therefore \quad V_{r2} = 28.5 \text{ m/sec}$$

$$\therefore \quad V_2 = \sqrt{V_{r2}^2 + 450 - 41\,V_{r2}} = \sqrt{(28.5)^2 + 450 - 41 \times 28.5} = 9.74 \text{ m/sec}$$

$$\text{Then, } \tan\beta = \frac{V_{f2}}{V_{w2}} = \frac{V_{r2}\sin\phi}{V_{r2}\cos\phi - v_2} = \frac{28.5 \sin 15^\circ}{28.5 \cos 15^\circ - 21.2}$$

$$= \frac{28.5 \times 0.2588}{28.5 \times 0.9659 - 21.2} = 1.165$$

$$\therefore \quad \beta = 49^\circ 21'$$

Weight of water discharged/sec. $= w \times Q = 1{,}000 \times 3.4 = 3{,}400$ kg/sec

$$V_{w2} = V_{r2}\cos\phi - v_2 = 28.5 \times \cos 15^\circ - 21.2 = 6.3 \text{ m/sec.}$$

$$\therefore \quad \text{W.D./kg/sec.} = \frac{V_{w1}v_1 + V_{w2}V_2}{g} = \frac{46.5 \times 16.95 + 6.3 \times 21.2}{9.81}$$

$$= 94 \text{ kg-m}$$

$$\therefore \quad \text{Power developed} = \frac{W \times \text{W.D./kg/sec}}{75} = \frac{3{,}400 \times 94}{75} = 4{,}250 \text{ H.P.}$$

$$\text{Hydraulic, } \eta = \frac{\text{W.D./kg/sec.}}{H} = \frac{94}{120} \times 100 = 78\%$$

Problem – 11 : *The mean blade circle diameter of an axial flow impulse turbine is 2 m. The guide blade angle at inlet is 25°, the vane angle at inlet and outlet are 54° and 26° respectively and the breadth of the moving blades at inlet is 12 cm. If the working head is 100 m, find the speed of the turbine. Also find the H.P. developed if, with full circumferential admission, the passages are 88% full at inlet.*

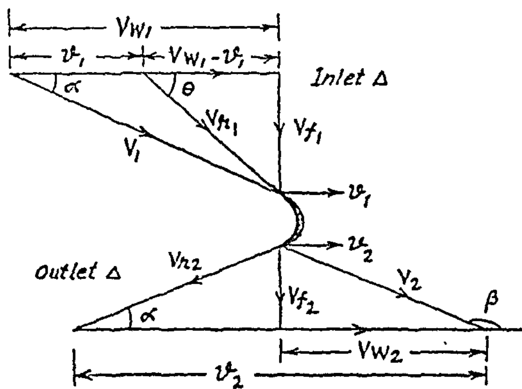


Fig. 10-18

As the turbine is an impulse one and the pressure through out is atmospheric, the velocity of the water at inlet,

$$V_1 = \sqrt{2gH} = \sqrt{19{\cdot}62 \times 100} = 44{\cdot}2 \text{ m/sec}$$

$$V_{w_1} = V_1 \cos\alpha = 44{\cdot}2 \times \cos 25° = 44{\cdot}2 \times 0{\cdot}9063 = 40 \text{ m/sec}$$

$$V_{f_1} = 44{\cdot}2 \times \sin 25° = 44{\cdot}2 \times 0{\cdot}4226 = 18{\cdot}6 \text{ m/sec}$$

and $\theta = 54°$ and $\phi = 26°$

$$\therefore \quad \tan\theta = \frac{V_f}{V_{w_1} - v_1}$$

$$\therefore \quad \tan 54° = \frac{18{\cdot}6}{40 - v_1}$$

$$\therefore \quad v_1 = 40 - \frac{18{\cdot}6}{\tan 54°} = 40 - \frac{18{\cdot}6}{1{\cdot}3764} = 26{\cdot}48 \text{ m/sec}$$

But, $v_1 = \dfrac{\pi d_1 N}{60}$

$$\therefore \quad N = \frac{60 v_1}{\pi d_1} = \frac{60 \times 26{\cdot}48}{\pi \times 2} = \underline{253 \text{ rpm.}}$$

As the turbine has an axial flow, $v_2 = v_1 = v = 26{\cdot}48$ m/sec

and $V_{r1} = V_{r2}$

But, $V_{r1} = \dfrac{V_{f1}}{\sin\theta} = \dfrac{18{\cdot}6}{\sin 54°} = \dfrac{18{\cdot}6}{0{\cdot}8090} = \underline{22{\cdot}8 \text{ m/sec}}$

$\therefore \quad V_{r2} = 22{\cdot}8$ m/sec

As V_{r2} is less than v_2 in magnitude, the outlet triangle will be as shown in fig. 10·23 and the water will leave in the same direction as the motion of the blade.

$$V_{w_2} = v_2 - V_{r2} \cos\phi = 26{\cdot}48 - 22{\cdot}8 \times \cos 26° = 5{\cdot}98 \text{ m/sec.}$$

$$\text{W.D /kg} = \frac{(V_{w_1} - V_{w_2})\, v}{g} = \frac{(40 - 5{\cdot}98) \times 26{\cdot}48}{9{\cdot}81} = 92 \text{ kg-m/sec}$$

Discharge of water, $Q = \pi Db\, V_{f_1} \times 0{\cdot}88 = \pi \times 2 \times \dfrac{12}{100} \times 18{\cdot}6 \times 0{\cdot}88$

$$= 12{\cdot}3 \text{ m}^3\text{/sec.}$$

$$\text{H.P.} = \frac{W \times \text{W.D./kg}}{75} = \frac{w \times Q \times \text{W.D./kg}}{75} = \frac{1{,}000 \times 12{\cdot}3 \times 92}{75} = 15{,}100$$

Problem – 12 : *The mean vane circle diameter of an axial flow impulse turbine is 2 m. The guide vane outlet angle is 25°, radial height of vanes is 20 cm and velocity coefficient is 0·97. The wheel vane outlet angle is 30° and the relative velocity at outlet is 80% of the relative velocity at inlet. At normal power, the supply head is 100 m and admission takes place over* $\frac{1}{8}$th *of the circumference. Determine the wheel speed and correct inlet vane angle if outflow is axial. Evaluate the power developed and hydraulic efficiency under these conditions. Neglect vane thickness effect.*

Absolute water velocity at guide vane outlet, V_1 $\Big\} = C_v \sqrt{2gH}$

$$= 0{\cdot}97\sqrt{2 \times 9{\cdot}81 \times 100}$$

$$= 43 \text{ m/sec.}$$

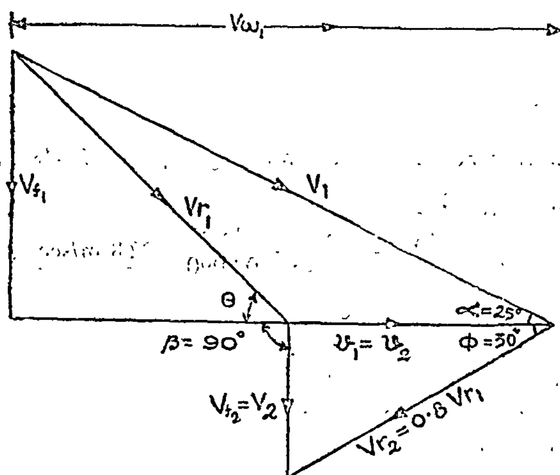


Fig. 10–19

From outlet velocity triangle, $= v_2 = V_{r2} \cos 30°$

$$= 0{\cdot}8V_{r1} \times 0{\cdot}866 = 0{\cdot}6928V_{r1}$$

From inlet velocity triangle,

$$\frac{V_{r1}}{\sin 25°} = \frac{v_1}{\sin(\theta - 25°)}$$

$$\therefore \quad \sin(\theta - 25°) = \frac{v_1 \sin 25°}{V_{r_1}} = \frac{0.6928 V_{r_1} \times 0.423}{V_{r_1}} \text{ (since } v_2 = v_1 = v\text{)}$$

$$\therefore \quad \sin(\theta - 25°) = 0.294$$

$$\therefore \quad \theta - 25° = 17.1° \qquad i.e.\ \theta = 25° + 17.1° = \underline{42.1°}$$

Further, $v = v_1 = V_1 \cos 25° - V_{r_1} \cos 42.1°$

$$= 43 \cos 25° - \frac{v_1 \times \cos 42.1°}{0.6928} = 39 - 1.07\, v_1$$

$$\therefore \quad v_1 = \frac{39}{2.07} = 18.8 \text{ m/sec.}$$

Now, $v_1 = \dfrac{\pi d N}{60}$ $\quad$ i.e. $N = \dfrac{18.8 \times 60}{\pi \times 2} = \underline{180 \text{ rpm.}}$

Work done/kg $= \dfrac{V_{w1} v_1}{g} = \dfrac{43 \cos 25° \times 18.8}{9.81} = 75$ kg-m/sec

Discharge/sec $= \pi d t \times \frac{1}{8} \times V_{f_1}$

$$= \pi \times 2 \times \frac{20}{100} \times 0.125 \times 43 \sin 25° = 9 \text{ m}^3\text{/sec.}$$

$\therefore$ H.P. developed $= \dfrac{wQ \times \text{W.D./kg}}{75}$

$$= \frac{1{,}000 \times 9 \times 75}{75} = \underline{9{,}000}$$

Further, $\eta_h = \dfrac{\text{W.D./kg}}{H} = \dfrac{75}{100} = 0.75$ i.e. 75%

Reaction Turbine

This type of the turbine constitutes a separate class of turbines by itself. In contrast to the impulse turbine, this turbine works under pressure, *i.e.* water, while it flows over the vanes of the runner, *has pressure above that of atmosphere. Hence this turbine* is sometimes called *pressure turbine.* Only a small part of the pressure energy of water is converted into kinetic energy, sufficient to enable water to flow over the guide vanes and enter the runner; as such no nozzles are to be found. Water, while it flows over the

runner vanes, spends its pressure energy which is first converted into kinetic energy which is ultimately imparted to the runner. In order that water enters the runner in the correct direction, a fixed wheel carrying the guide vanes is put around the inner or outer periphery of the runner and this guide wheel is enveloped by a spiral casing which receives water from the penstocks. Since air should not enter the runner and destory the pressure of water, the wheel is required to be run full, *i. e.* simultaneously all the vane passages of the runner are full of water. This is not the case with impulse turbine as seen earlier. At times, the pressure of water as it leaves the runner is reduced to a value below atmospheric to extract maximum work from water, and then to enable water to flow out in the tail race, draft tubes are used to convert the exhaust (outlet) velocity into pressure head which equalises the atmospheric value. The free end of the draft tube is submerged deep in the tail water, making, thus, the entire water passage, right from the head race upto the tail race, totally enclosed.

Reaction turbines are suitable for medium heads, and they are smaller in size compared to the Pelton wheel for the same power output. The popular makes of these turbines are (i) Francis turbine, (ii) Propeller turbine, and (iii) Kaplan turbine.

Francis Turbine

The present-day, Francis turbine is an inward mixed flow reaction turbine. Water, under pressure enters the runner from the guide vane towards the centre in radial direction and leaves the runner axially. It operates under medium heads and requires medium quantity of water. There is a difference in pressure of water between the exit tips of guide vanes and working vanes. This pressure difference is termed as *reaction pressure* and is partly responsible for the motion of the runner and the turbine is referred to as a *reaction turbine*. The overall movement of the runner is obtained as a result of change of both the potential and kinetic energies of water.

The Francis turbine has two varieties. In the *closed type*, (fig. 10–20) water is led to the turbine through the penstock whose

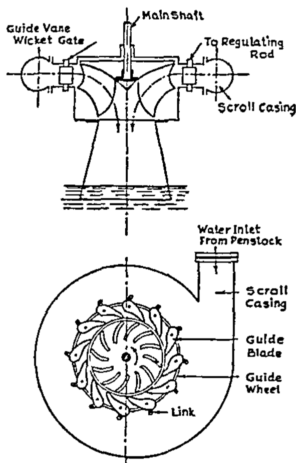


Fig. 10–20 Closed type Francis turbine

end is connected to the spiral casing of the turbine. This spiral casing directs water evenly to the guide vanes. Water then passes through the runner and finally goes to the tail race through the draft tube. The closed type Francis turbine may be of two types, horizontal and vertical. The horizontal type is used for medium and high heads and the vertical type for medium and low heads. Larger units are, however, mostly, of the vertical type.

Components of the Francis turbine : The essential components of this reaction turbine are (i) spiral casing, (ii) guide mechanism (iii) runner and (iv) draft tube. The *spiral casing* facilitates the smooth flow of water over the guide vanes and it also ensures that all the guide vanes receive full quantity of water. It helps to avoid loss of efficiency by preventing the eddies being formed during the flow from penstock to the runner. The spiral casing or the scroll casing, as it is sometimes called, is made with its cross sectional area reducing uniformly around the circumference, maximum at entrance and nearly zero at the tip. Sometimes, *stay vanes* are provided inside the casing which serve to support the casing and direct the water to guide vanes. The position of the inlet to the spiral casing depends on the direction of water flowing out from the penstock which may vary according to the site. Sometimes these casings are fitted with pressure-gauges and inspection windows. They are made of concrete without steel plate lining, welded rolled steel plate or cast steel.

Guide mechanism consists of two annular parallel plates (circular) between which the guide vanes of the aerofoil section are located. The guide vanes can be rotated about its pivot centre through mechanism of regulating ring, link, lever and governor and thus they can be closed or opened, thus allowing variable quantity of water according to the needs. Guide vanes are usually made of cast steel.

The runner of the turbine may be cast in one piece or may be made of separated steel plates welded together. Small runners are made from cast iron, bigger ones from cast steel and sometimes from stainless steel or bronze when water is chemically impure and there is danger of corrosion. Flow through the runner is of the radial-axial (mixed) type and hence the vanes are suitably shaped. Runners may be classified as slow, medium or fast depending upon the specific speed. The passage area between the vanes is kept enlarging in direction of flow in case of radially inward and outward flow turbines, since the velocity decreases as the flow advances and the same mass of water should flow over all section of the passage.

Draft tube is a riveted steel plate pipe or concrete tunnel of gradually increasing cross-section towards the outlet and is attached axially to the turbine at a point where water leaves the runner. Its other end is kept immersed about 1 m below the lowest tail race level. By providing the draft tube the working head of the turbine is increased by the height between the runner exit and tail race level. In the absence of this tube, discharge from the turbine would be at the runner exit level. The draft tube thus permits the turbine to be installed above the tail race without loss of available head.

Further, an indirectly helping factor is the high velocity accompanying the leaving water. This kinetic energy of water which is normally lost by not being utilised in any way, is converted into equivalent pressure head by adopting a draft tube of increasing area. This helps in making the discharge at much reduced velocity, thus resulting in a gain of kinetic head. This increases the negative pressure head at turbine runner exit with which the net working head on the turbine is increased. To summarize this it may be said that the use of the draft tube makes it possible to install the turbine at a level higher then that of tail race without loss of available head and to regain a part of the velocity head (kinetic energy) at exit from the turbine runner.

This exit velocity energy varies from 3 to 15% of the net working head of Francis turbines, depending upon the specific speed and increases with higher specific speeds. Hence, the recovery of this considerable bulk of energy constitutes a very *important problem and sufficient attention has to be paid for the* proper design of draft tubes.

The draft tube should be made conical, as there are less losses taking place in it. The exit area of the cone can be increased either by increasing the cone angle or by increasing the length of the tube. The cone angle cannot be made greater than 10° to 18°, as with greater angle, the flow breaks away from the wall, causing greater hydraulic losses. Hence, tubes are required to be made longer.

The best form of the draft tube, giving the minimum losses is the straight conical tube that can be installed for a small turbine or for horizontal turbine. For big, turbines the draft tubes are bent out. If the tube is kept vertical, its length will necessitate the con struction of the hydro-station to be more elaborate and deep.

To suit particular conditions of installation, (horizontal and vertical) various draft tubes are schematically illustrated in fig. 10-21. Out of these it will be naturally expected that the straight type, fig. 10-21 a & b, will be the most efficient. The difference between (b) and (a) is that, (b) has a bell mouthed outlet and an internal conical core. This is known as the *Moody Hydraucone draft tube*; it can work efficiently even with helical flow. It may be noted that the helical flow is a result of the whiral component of velocity at the exit of runner.

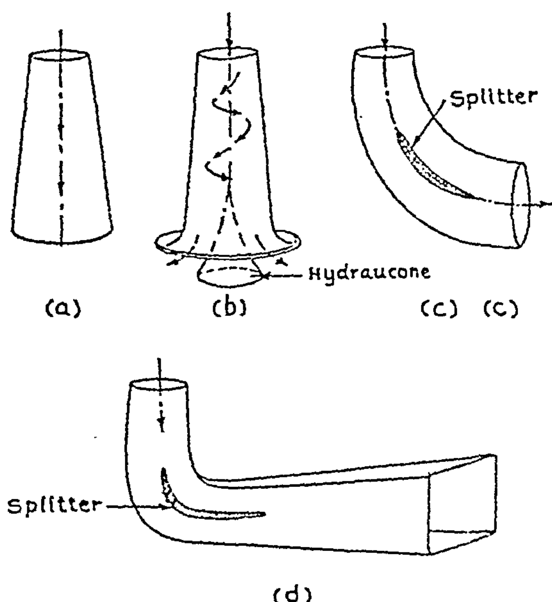


Fig. 10-21 Types of draft tubes

For efficient working of the draft tube the cone angle is limited. The necessary length of the draft tube, sometimes, cannot be acco-

modated in a vertical manner for want of sufficient head room in low head installations. In that case, a 90° bend is essential (fig. 10-21 c & d). Water enters vertically and leaves horizontally into the tail race. For such bent draft tube, the efficiency is naturally lower than that for straight tubes. In order to minimize the losses a deflecting vane, termed as *splitter* is provided at the elbow in bent tubes, as shown in c & d of fig. 10-21.

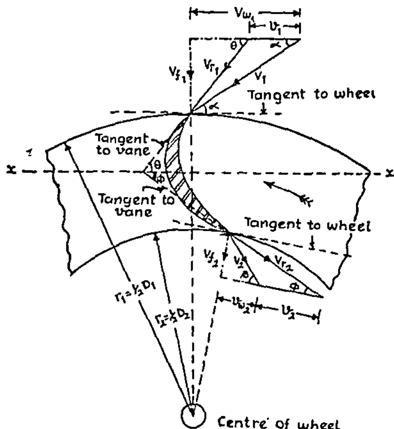


Fig. 10-22 Velocity triangles for reaction turbine

Work done : As in case of impulse turbine, the work developed by a reaction turbine is estimated with the help of velocity diagrams constructed for the inlet and outlet tips of the working vane. As far as construction of these triangles goes, the pattern

remains the same in the sense that the wheel velocities are v_1, v_2 (diameters are different) and absolute velocities at inlet and exit are different, so also the relative velocities since the pressure is continuously falling over the vane. Sometimes it is observed that V_{r2} is greater than V_{r1}.

$$\left.\begin{array}{l}\text{Tangential momentum}\\ \text{of water/sec. at inlet tip}\end{array}\right\} = \frac{W}{g} V_{w1}$$

$$\left.\begin{array}{l}\text{Moment of this momentum}\\ (i.e.\text{ angular momentum})\end{array}\right\} = \frac{W}{g} V_{w1} r_1$$

Similarly,

$$\left.\begin{array}{l}\text{Angular momentum}\\ \text{of water/sec. at exit tip}\end{array}\right\} = \frac{W}{g} V_{w2} r_2$$

$$\therefore \left.\begin{array}{l}\text{rate of change of}\\ \text{angular momentum}\end{array}\right\} = \frac{W}{g} V_{w1} r_1 - \frac{W}{g} V_{w2} r_2$$

$$= \frac{W}{g} (V_{w1} r_1 - V_{w2} r_2)$$

$$= \text{torque applied on runner}$$

$$\therefore \text{ work done/sec. on runner} = \text{torque} \times \text{angular velocity}$$

$$= \frac{W}{g} (V_{w1} r_1 - V_{w2} r_2)\,\omega$$

$$= \frac{W}{g} (V_{w1} v_1 - V_{w2} v_2) \qquad \text{.. (10·9)}$$

If discharge at outlet is radial, $\beta = 90°$ and hence,

$V_{w2} = 0.$ Then,

$$\text{W.D./sec.} = \frac{W}{g} V_{w1} v_1 \qquad \text{..(10·9a)}$$

If V_{w2} has direction opposite to that of V_{w1}, we have

$$\text{W.D./sec.} = \frac{W}{g} (V_{w1} v_1 + V_{w2} v_2)$$

Hence, in general, $\text{W.D./sec} = \frac{W}{g} (V_{w1} v_1 \pm V_{w2} v_2)$ .. (10·10)

An approximation to the conditions of the axial flow reaction machine may be obtained by assuming $V r_2 = V_1$, $\phi = \alpha$, and $V_{f1} = V_{f2}$.

Then, $V_{w1} = V_1 \cos \alpha$

$$V_{2w} = -(V r_2 \cos\phi - v) = -(V_1 \cos \alpha - v)$$

$\therefore \quad v(V_{w_1} - V_{w_2}) = v(2V_1 \cos \alpha - v)$

This is maximum when $\frac{v}{V_1} = \cos\alpha$

This indicates that in reaction turbine, the ratio of vane speed to jet speed is greater than that in similar impulse turbine for maximum efficiency; while V_1 is smaller for a given head in the reaction tubine (about $1/\sqrt{2}$ times that in impulse turbine), the rotational speed of a reaction turbine remains higher, *i.e.* about $\sqrt{2}$ times greater.

The discharge, W through the radial turbine can be estimated in the following manner.

$$W = wQ$$
$$= w \times \text{radial area at inlet} \times \text{radial velocity at inlet}$$
$$= w \times \pi D_1 \times b_1 \times V_{f_1}$$

where, b_1 is the width of vane wheel at inlet.

If there are n number of vanes and the vane tip thickness at inlet is t, the real discharge is

$$W = w(\pi D_1 - nt_1) b_1 V_{f_1} \quad .. (10\cdot11)$$

The discharge will be the same at outlet from the runner also *i.e.*

$$W = w(\pi D_2 - nt_2) b_2 V_{f_2} \quad .. (10\cdot11a)$$

Thus, the discharge through any turbine runner can be estimated by assessing correctly the flow area and knowing the velocity of flow.

Heads at inlet and exit of runner : Let a vertical turbine be considered and p_1 and p_2 be pressures of water at inlet and exit of the runner vane. The datum plane is assumed to be horizontal passing through the centre of the runner.

The total head at inlet consists of the velocity (at exit from guide vanes) head and pressure head, the former being smaller than the latter, *i e.*

$$H_1 = \frac{p_1}{w} + \frac{V_1^2}{2g}$$

Similarly, total head at outlet from vane is

$$H_2 = \frac{p_2}{w} + \frac{V_2^2}{2g}$$

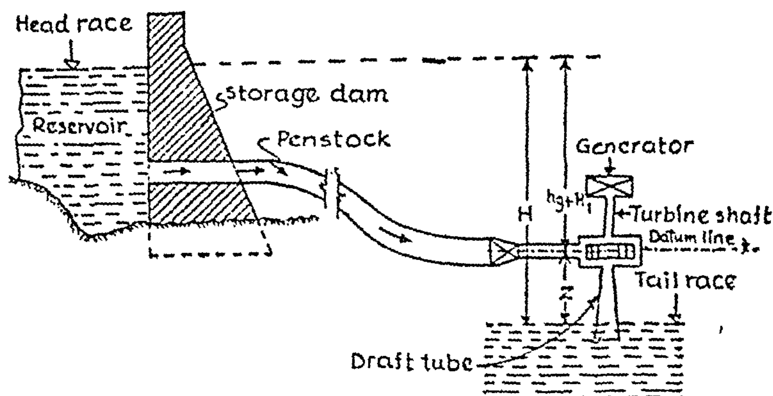


Fig. 10-23

Referring Bernoulli's theorem to inlet and exit of this horizontal runner, we have,

$H_1 = H_2$ + total losses between inlet and exit of runner

$= H_2$ + friction loss + head lost due to work done on runner

Similar relationship can be stated for total energies also *viz.*

$$E_1 = E_2 + \left\{ \begin{array}{c} \text{energy loss due to} \\ \text{friction in runner} \end{array} \right\} + \left\{ \begin{array}{c} \text{energy loss due to} \\ \text{work done on runner} \end{array} \right\}$$

If h_g be the loss of head in guide vanes and Z is the height of the inlet point above the tail race, the total head H on turbine is

$$H = h_g + H_1 + Z$$

This head, H is shown in fig. 10-23. From it, one may see that head H is measured from tail water level upto the head race.

Efficiency : The hydraulic efficiency, η_h is given as

$$\eta_h = \frac{\text{hydraulic output}}{\text{hydraulic input}}$$

$$= \frac{\text{head lost on W.D. for unit mass of water}}{\text{total head of the turbine}}$$

$$= \frac{\frac{1}{g}(V_{w_1}v_1 \pm V_{w_2}v_2)}{H}$$

In other terms, it is

$$\eta_h = \frac{\text{W.D. by runner per sec.}}{\text{total energy input to the turbine per sec.}}$$

$$= \frac{\frac{W}{g}(V_{w_1}v_1 \pm V_{w_2}v_2)}{WH} \quad \ldots \ldots \ldots (10\cdot12)$$

The overall efficiency, η_o of turbine is the product of the mechanical and hydraulic efficiencies of the turbine, viz.

$$\eta_o = \eta_h \times \eta_{mech} \quad \ldots \ldots \ldots (10\cdot13)$$

and the mechanical efficiency may be expressed as

$$\eta_{mech} = \frac{\text{B.H P. of turbine}}{\text{Water H. P. of turbine}}$$

$$= \frac{\text{B.H.P.}}{\frac{W}{g}(V_{w_1}v_1 \pm V_{w_2}v_2)/K} \quad \ldots (10\cdot14)$$

where, $K = 75$ is a conversion constant for H.P. from work units. It may be noted that for better efficiency the outflow from the runner should be radial, *i.e.* angle $\beta = 90°$ or $V_{w_2} = 0$ and $V_2 = V_{f_2}$. The η_h for Francis turbine has an average value varying from 80 to 90%.

Kaplan and Propeller Turbines

The modern Francis turbine is a mixed-flow turbine in the sense that the inflow is radial whereas water leaves the turbine in

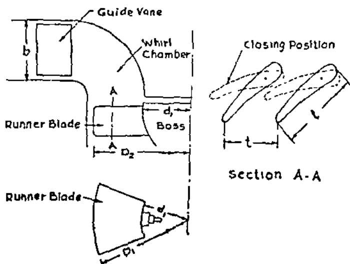
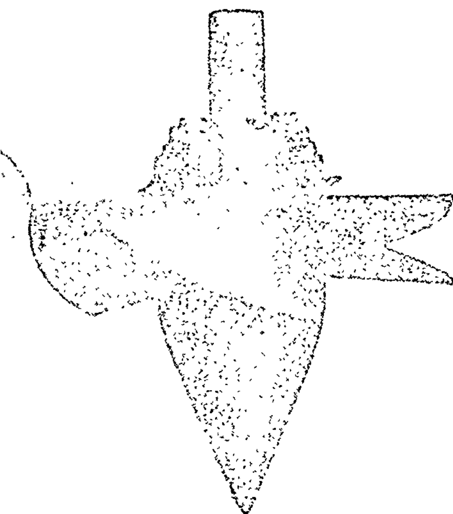


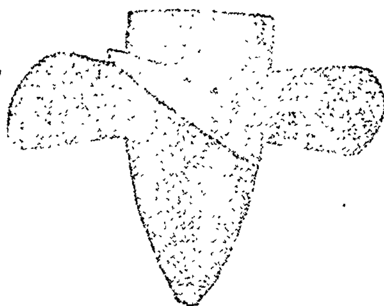
Fig. 10-24 Arrangement of a Kaplan turbine

the axial direction. The Kaplan turbine goes one step forward in this respect and in it flow is entirely axial, hence it can be looked upon as a development of the Francis turbine. It has been seen that with increase in specific speed of the Francis turbine, the tendency for the turbine is to become an axial flow type. The number of vanes (working) also greatly reduces to about 4 to 8 with the result that quantity of water handled for same diameter increases with lower losses on friction. It, therefore, becomes more compact in construction and has higher rotational speed. Because of these characteristics, this turbine is used at such places where comparatively low head and large quantity of water is available.

The spiral casing guide mechanism and draft tube of the Francis turbine are retained for this turbine. But the Kaplan runner is different from the Francis runner. Kaplan vanes are attached to the boss, nullifying the necessity of the ring (band). Kaplan turbines have been constructed to develop, 23,000 H.P. under a head of 25 m and 66,000 H.P. under a head of 15 m running at 75 rpm. Smallest head upto 5 m has also been employed.



(a) Kaplan turbine rotor



(b) Propeller turbine rotor

Fig. 10-25

Blade adjustment : Contrary to the Francis turbine in which the guide vanes are rotated about the pivots to adjust the load and speed, here in the Kaplan turbine the working vanes can be moved and adjusted to vary passage area between them (fig. 10-25). This

can be done, when the turbine is rotating, with the help of a servomotor mechanism operating inside the hollow coupling of the turbine and generator shaft.

If the adjustable blades of the Kaplan turbine are rigidly fixed to the boss, the turbine becomes the Propeller turbine. This turbine is economical in operation, but can only work under full load for the best efficiency. This type of turbine has also been constructed to a very big capacity *e. g.* 70,000 H. P. at 120 rpm. under a head of 30 m.

Water enters the turbine casing in a radial fashion. Space is kept between the guide vanes outlet and runner inlet. This space is called as *whirl chamber*. It is in this whirl chamber that water changes its direction from radial to axial, before entry to working vane. The outer curve of this chamber is an ellipse.

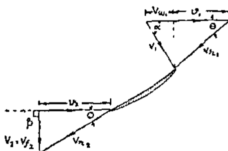


Fig. 10-26 Velocity diagrams of Kaplan turbine

The various proportions of the turbine are ·

Semi-major axis of ellipse, $a \simeq 0.13\ D_1$

Semi-minor axis of ellipse, $b \simeq 0.16\ D_1$ to $0.2\ D_1$

Diameter of boss, $d_1 \simeq 0.35\ D_1$ to $0.6\ D_1$

Section of the blade is aerofoil.

Pitch, $t = \frac{\pi D_2}{n}$; ratio $\frac{l}{t} \simeq 0.9$ to 1.05

Problem-13 : *In an outward flow reaction turbine the rim speed at inlet is 12 m/sec and the ratio of the radii is 0·8. The turbine is placed one metre below the water surface in the tail race, and the wheel vane angles are 90° and 20° at inlet and outlet respectively. The radial velocity of flow at inlet is 4·2 m/sec. Neglecting frictional losses, and taking velocity of outflow as radial, find the guide vane angle, pressure head at inlet to the wheel, speed of flow from guides at inlet, the total head, and hydraulic efficiency.*

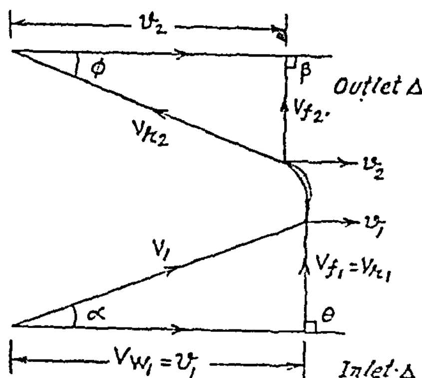


Fig. 10-27

Velocity of vane tip at inlet, $v_1 - 12$ m/sec.

$$\therefore \quad V_{w1} = v_1 = 12 \text{ m/sec}$$

Velocity of vane tip at outlet, $v_2 = \dfrac{12}{0{\cdot}8} = 15$ m/sec

Velocity of flow at inlet, $V_{f1} = 4{\cdot}2$ m/sec

$$\therefore \quad \tan\alpha = \frac{V_{f1}}{v_1} = \frac{4{\cdot}2}{12} = 0{\cdot}35$$

$\therefore$ guide vane angle, $\alpha = \underline{19{\cdot}3°}$

Now, $V_1 = \sqrt{V_{w1}^2 + V_{f1}^2} = \sqrt{(12)^2 + (4{\cdot}2)^2} = \underline{12{\cdot}7 \text{ m/sec}}$

$$\text{Hydraulic } \eta = \frac{\text{head utilized}}{\text{head supplied}} = \frac{82.3 \times 100}{90} = \underline{92.6\%}$$

Difference of pressure, $\frac{p_1}{w} - \frac{p_2}{w}$

$$= \frac{V_{w1}v_1 - V_{w2}v_2}{g} - \left(\frac{V_1^2}{2g} - \frac{}{2g}\right) + \frac{0.2V_{r2}^2}{2g}$$

$$= 82.3 - \left(\frac{1520 - 181}{19.62}\right) + \frac{0.2 \times (33.4)^2}{19.62}$$

$$= \underline{25.7 \text{ m}}$$

$$\text{H.P. developed} = \frac{1{,}000 \times 0.42 \times 82.3}{75} = 461$$

Problem-15 : *An inward flow turbine having an overall efficiency of 75% is required to give 180 H.P. The head H is 9 m. The velocity of the periphery of the wheel is 3·47 $\sqrt{H}$, and the radial velocity of flow is 1·15$\sqrt{H}$. The wheel is to make 120 r.p.m. The hydraulic losses in the turbine are 20% of the available energy. Determine (a) the guide blade angle at inlet; (b) the wheel vane angle at inlet; (c) the diameter of the wheel; and (d) the width of the wheel at inlet. Assume the turbine is a reaction one with radial discharge at outlet.*

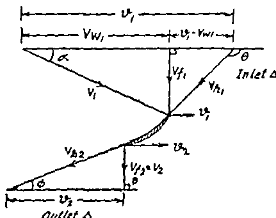


Fig. 10-29

Radial velocity of flow at inlet, $V_{f_1} = 1{\cdot}15\sqrt{H}$

$= 1{\cdot}15\sqrt{9} = 3{\cdot}45$ m/sec

Velocity of the vane tip at inlet, $v_1 = 3{\cdot}47\sqrt{H}$

$= 3{\cdot}47\sqrt{9} = 10{\cdot}41$ m/sec

Now, $v_1 = \dfrac{\pi d_1 N}{60}$

$\therefore \quad d_1 = \dfrac{60 v_1}{\pi N} = \dfrac{60 \times 10{\cdot}41}{\pi \times 120} = \underline{1{\cdot}66 \text{ m}}$

As the discharge is radial, $\beta = 90°$.

$\therefore \quad V_2 = V_{f_2}$ and $V_{w2} = 0$

as hydraulic losses are 20%, $\eta_{hyd.} = 80\%$

$$\eta_{hyd.} = \frac{V_{w1} v_1}{gH} = \frac{H - \dfrac{V_2^2}{2g}}{H}$$

$$\therefore \quad H - \frac{V_2^2}{2g} = \eta_{hyd.} \times H$$

$$\therefore \quad 9 - \frac{V_2^2}{2g} = 0{\cdot}8 \times 9$$

$V_2 = \sqrt{19{\cdot}62 \times 1{\cdot}8} = 5{\cdot}94$ m/sec

Also $\quad V_{w1} = \dfrac{gH \times \eta_{hyd}}{v_1} = \dfrac{9{\cdot}81 \times 9 \times 0{\cdot}8}{10{\cdot}41} = 6{\cdot}8$ m/sec

Thus, $\tan\alpha = \dfrac{V_{f_1}}{V_{w1}} = \dfrac{3{\cdot}45}{6{\cdot}8} = 0{\cdot}507$

$\therefore$ inlet guide vane angle, $\underline{\alpha = 26{\cdot}9°}$

Further, $\tan\theta = \dfrac{V_{f_1}}{V_{w1} - v_1} = \dfrac{3{\cdot}45}{6{\cdot}8 - 10{\cdot}41} = -\,0{\cdot}955$

$\therefore \quad \theta = \underline{136{\cdot}3°}$

Overall $\quad \eta = \dfrac{\text{output H.P.}}{WH/75}$

$\therefore \quad 0{\cdot}75 = \dfrac{180 \times 75}{W \times 9}$

$\therefore \quad W = \dfrac{180 \times 75}{9 \times 0{\cdot}75}$ 2,000 kg/sec.

$\therefore \quad Q = \dfrac{W}{w} = \dfrac{2{,}000}{1{,}000} = 2$ m³/sec.

But, $\quad Q = \pi d_1 b_1 V_{f_1}$

$\therefore$ width of the wheel at inlet, $b_1 = \dfrac{Q}{\pi d_1 V_{f_1}}$

$$\therefore \quad b_1 = \frac{2}{\pi \times 1{\cdot}66 \times 3{\cdot}45} = 0{\cdot}111 \text{ m}$$

$$\therefore \quad b_1 = \underline{11{\cdot}1 \text{ cm}}$$

Problem-16 : *An inward flow reaction turbine has a wheel 75 cm and 50 cm diameter at inlet and outlet respectively, the width at inlet being 8 cm. The wheel blade angles at inlet and outlet are 105° and 12° respectively. Net head under which the turbine works is 72 m. Velocity of flow is uniform throughout from inlet to outlet. The blade thickness blocks 8% of the circumferential area of the runner. Hydraulic efficiency and mechanical efficiency of the turbine is 90% and 82% respectively. Calculate the speed and output of the turbine and the quantity of water it would require. The discharge at outlet is free and radial.*

From outlet triangle of velocity which is a right angled triangle as the discharge at outlet is radial as shown in fig 10-30,

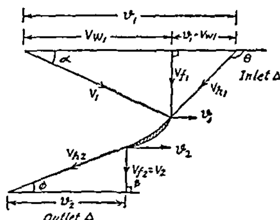


Fig. 10-30

$$\tan\phi = \frac{V_{f_2}}{v_2}$$

$$\therefore \quad v_2 = \frac{V_{f_2}}{\tan\phi} = \frac{V_{f_1}}{\tan 12^\circ} = \frac{V_{f_1}}{0{\cdot}2126}$$

But, $\frac{v_2}{v_1} = \frac{d_2}{d_1}$

$$\therefore \quad v_2 = v_1 \times \frac{d_2}{d_1} = v_1 \times \frac{75}{50} = 1{\cdot}5\, v_1$$

$$\therefore \quad 1{\cdot}5\, v_1 = \frac{V_{f_1}}{0{\cdot}2126}$$

$$\therefore \quad v_1 = \frac{V_{f_1}}{1{\cdot}5 \times 0{\cdot}2126}$$

$$\therefore \quad V_{f_1} = 0{\cdot}32\, v_1$$

But, $\tan\theta = \frac{V_{f_1}}{V_{w_1} - v_1}$

$$\therefore \quad \tan 105^\circ = \frac{V_{f_1}}{V_{w_1} - v_1}$$

$$V_{w_1} = v_1 + \frac{V_{f_1}}{\tan 105^\circ} = v_1 - \frac{V_{f_1}}{3{\cdot}7321} = v_1 - \frac{0{\cdot}32 v_1}{3{\cdot}7321}$$

$$= v_1 - 0{\cdot}0857\, v_1 = 0{\cdot}9143\, v_1$$

Hyd. $\eta = \frac{V_{w_1} v_1}{gH}$

$$\therefore \quad 0{\cdot}9 = \frac{0{\cdot}9143\, v_1 \times v_1}{9{\cdot}81 \times 72}$$

$$\therefore \quad v_1 = \sqrt{\frac{0{\cdot}9 \times 9{\cdot}81 \times 72}{0{\cdot}9143}} = 26{\cdot}3 \text{ m/sec.}$$

But, $v_1 = \frac{\pi d_1 N}{60}$

$$\therefore \quad N = \frac{60 v_1}{\pi d_1} = \frac{60 \times 26{\cdot}3}{\pi \times 0{\cdot}75} = \underline{670 \text{ r.p.m.}}$$

Velocity of flow at inlet, $V_{f_1} = 0{\cdot}32 \times 26{\cdot}3 = 8{\cdot}4$ m/sec

Velocity of whirl at inlet, $V_{w_1} = 0{\cdot}9143 \times 26{\cdot}3 = 24$ m/sec.

Quantity of water, $Q = \pi d_1 b_1\, V_{f_1} \times k$

$$= \pi \times 0{\cdot}75 \times \frac{8}{100} \times 8{\cdot}4 \times 0{\cdot}92$$

$$= \underline{1{\cdot}455 \text{ m}^3/\text{sec.}}$$

$$\text{Output of turbine} = \frac{wQ\left\{\frac{V_{w_1} v_1}{g}\right\}}{75} \times \eta_{mech}$$

$$= \frac{1{,}000 \times 1{\cdot}455 \left\{\frac{24 \times 26{\cdot}3}{9{\cdot}81}\right\}}{7} \times 0{\cdot}82$$

$$= \frac{0{\cdot}82 \times 1{\cdot}455 \times 64{\cdot}5}{75} = \underline{1{,}025 \text{ H. P.}}$$

Problem – 17 : *Calculate the main dimensions and the blade angles for an inward flow reaction turbine from the following specifications. Velocity of flow is constant from inlet to outlet. Discharge at outlet is radial.*

Net head supplied to the turbine	.. *61·5 m*
Speed of the turbine	.. *700 rpm.*
Flow ratio	.. *0·15*
Ratio of wheel width to diameter at inlet	.. *0·1*
Area of flow blocked by the thickness of vanes is	.. *5%*
Ratio of diameters at inlet and outlet	.. *2*
Output of the turbine	.. *450 B.H.P.*
Hydraulic efficiency of the turbine	.. *94%*
Mechanical efficiency of the turbine	.. *85%*

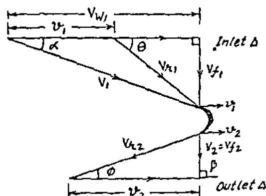


Fig. 10-31

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{W.H.P.}} = \frac{450}{W \times 61{\cdot}5/75} = 0{\cdot}85$$

$\therefore$ weight of water discharged/sec, $W = \dfrac{450 \times 75}{61{\cdot}5 \times 0{\cdot}85} = 645$ kg/sec

$\therefore$ discharge, $Q = \dfrac{W}{w} = \dfrac{645}{1{,}000} = 0{\cdot}645 \text{ m}^3/\text{sec}$

$$\text{Flow ratio} = \frac{V_{f1}}{\sqrt{2gH}} = \frac{V_{f1}}{\sqrt{19{\cdot}62 \times 61{\cdot}5}} = 0{\cdot}15$$

$\therefore \quad V_{f1} = 0{\cdot}15\sqrt{19{\cdot}62 \times 61{\cdot}5} = 5{\cdot}2$ m/sec

$\therefore \quad V_{f2} = 5{\cdot}2$ m/sec.

Now, $Q = \pi D_1 b_1 V_{f1} \times k$

i.e. $0{\cdot}645 = \pi \times d_1 \times 0{\cdot}1 d_1 \times 5{\cdot}2 \times 0{\cdot}95$ As $k = \dfrac{100-5}{100} = 0{\cdot}95$

$\therefore$ inlet diameter of the wheel is,

$$d_1 = \sqrt{\frac{0{\cdot}645}{\pi \times 0{\cdot}52 \times 0{\cdot}95}} = \underline{0{\cdot}645 \text{ m}}$$

Outlet diameter of the wheel, $d_2 = \dfrac{d_1}{2} = \underline{0{\cdot}3225 \text{ m}}$

Velocity of vane tip at inlet, $v_1 = \dfrac{\pi d_1 N}{60} = \dfrac{\pi \times 0{\cdot}645 \times 700}{60}$

$= 23{\cdot}6$ m/sec

Velocity of vane tip at outlet, $v_2 = \dfrac{\pi d_2 N}{60} = \dfrac{\pi \times 0{\cdot}3225 \times 700}{60}$

$= 11{\cdot}8$ m/sec

Hyd. $\eta = \dfrac{V_{w1} v_1}{gH}$ as $V_{w2} = 0$ due to radial discharge

$\therefore \quad 0{\cdot}94 = \dfrac{V_{w1} \times 23{\cdot}6}{9{\cdot}81 \times 61{\cdot}5}$

$\therefore \quad V_{w1} = \dfrac{0{\cdot}94 \times 9{\cdot}81 \times 61{\cdot}5}{23{\cdot}6} = 24$ m/sec

From inlet triangle of velocity,

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} = \frac{5.2}{24} = 0.217$$

$$\therefore \quad \underline{\alpha = 12°15'}$$

$$\text{Again, } \tan \theta = \frac{V_{f1}}{V_{w1} - v_1} = \frac{5.2}{24 - 23.6} = \frac{5.2}{0.4} = 13$$

$$\therefore \quad \underline{\theta = 85.6°}$$

From outlet triangle of velocity,

$$\tan \phi = \frac{V_{f2}}{v_2} = \frac{5.2}{11.8} = 0.441$$

$$\therefore \quad \underline{\phi = 23° 48'}$$

Problem-18 : *In an inward flow reaction turbine operating under a total head of 14 m, the guide vane discharge is inclined at 20° to the wheel tangent. At exit from the runner, pressure is atmospheric. Discharge is radial with a speed 4 m/sec. and the radial speed through the runner is constant. The ratio of inlet and outlet diameters is 3 to 2. The wheel vane inlet angle is 90°. State the correct peripheral speed of the runner and outlet vane angle. Express the fluid friction loss in the machine as a percentage of the total head.*

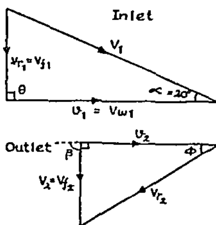


Fig. 10-32

From inlet velocity triangle (fig. 10-32),

$$v_1 = \frac{V_{f1}}{\tan \alpha} = \frac{4}{\tan 20°} = 11 \text{ m/sec.}$$

and $V_{w1} = v_1 = 11$ m/sec.

The blade tip velocity at outlet,

$$v_2 = v_1 \times \frac{d_2}{d_1} = 11 \times \frac{2}{3}$$

$$= \underline{7.33 \text{ m/sec.}}$$

From outlet velocity triangle,

$$\tan \phi = \frac{V_{f2}}{v_2} = \frac{4}{7.33} = 0.545$$

$$\therefore \quad \text{angle } \phi = 28.6°$$

$$\text{W.D./kg} = \frac{V_w v_1}{g} = \frac{11 \times 11}{9{\cdot}81} = 12{\cdot}33 \text{ m-kg}$$

$\therefore$ energy loss as fluid friction $= H - \text{W.D./kg} - \dfrac{V_2^2}{2g}$

$$= 14 - 12{\cdot}33 - \frac{4^2}{2 \times 9{\cdot}81}$$

$$= 0{\cdot}854 \text{ m-kg/kg}$$

Therefore, fluid friction loss as the percentage of total head is

$$\frac{0{\cdot}854}{14} \times 100 = \underline{6{\cdot}1\%}$$

Problem-19 : *An inward flow reaction turbine has a degree of reaction 0·6. The peripheral velocity of the blades at entry is 12 m/sec. and velocity of flow is constant at 2·5 m/sec. The rotor diameter at entry is twice that at exit. Find the blade angles at entry and exit, assuming that water leaves the rotor without whirl and that friction losses may be neglected. Sketch the shape and arrangement of the blades.*

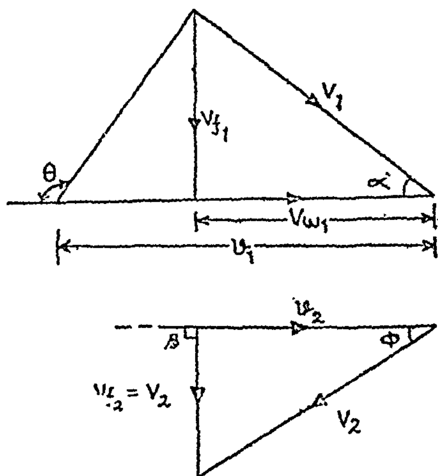


Fig. 10-33

$$\text{Degree of reaction} = \frac{\text{pressure drop across the runner}}{\text{total head}}$$

$\therefore$ pressure drop across runner $= 0{\cdot}6H$

$\therefore$ pressure drop across guide vane $= H - 0.6H = 0.4H = \dfrac{V_1^2}{2g}$

From inlet velocity triangle,

$$V_1^2 = V_{f1}^2 + V_{w1}^2$$

$$\therefore \quad 0.4\, H = \frac{V_{f1}^2 + V_{w1}^2}{2g} \qquad \text{.. (i)}$$

Again, neglecting friction losses,

$$\text{total head} = \text{W.D./kg} + \frac{V_2^2}{2g}$$

$$i.e. \quad H = \frac{V_{w1} v_1}{g} + \frac{V_{f2}^2}{2g} \qquad \text{.. (ii)}$$

Substituting value of H from (i) and (ii),

$$\frac{2.5\,(V_{f1}^2 + V_{w1}^2)}{2g} = \frac{V_{w1} \times v_1}{g} + \frac{V_{f2}^2}{2g}$$

But, $V_{f1} = V_2 = 2.5$ m/sec. and $v_1 = 12$ m/sec.

$$\therefore \quad \frac{2.5}{2}\,(2.5^2 + V_{w1}^2) = V_{w1} \times 12 + \frac{2.5^2}{2}$$

$$\therefore \quad V_{w1}^2 = \frac{12}{1.25} V_{w1} + \frac{3.125}{1.25} - 6.25$$

$$\therefore \quad V_{w1}^2 - 9.6\, V_{w1} = -3.75$$

$$\therefore \quad (V_{w1} - 4.8)^2 = -3.75 + 4.8^2$$

$$\therefore \quad V_{w1} = 9.2 \text{ m/sec.}$$

Now, from inlet velocity triangle,

$$\tan(180 - \theta) = \frac{V_{f1}}{v_1 - V_{w1}} = \frac{2.5}{12 - 9.2} = 0.895$$

$\therefore$ $180° - \theta = 41.8°$ *i.e.* $\theta = \underline{138.2°}$

From outlet velocity triangle,

$$v_2 = v_1 \times \frac{d_2}{d_1} = 12 \times \tfrac{1}{2} = 6 \text{ m/sec.}$$

and, $\tan \phi = \dfrac{V_{f2}}{v_2} = \dfrac{2.5}{6} = 0.416$

$\therefore$ outlet blade angle, $\phi = \underline{22.6°}$

Problem-20 : *An inward flow reaction turbine has guide vane angle at inlet of 22°. The external diameter of the runner is 60 cm and internal diameter is 40 cm. The velocity of flow through the runner is constant and the velocity of whirl at exit may be taken as zero. Calculate the angles of the moving vane at inlet and exit, assuming that velocity head at entrance to the runner is half the total head. If the width of the runner at entrance is 8 cm and the effect of vane thickness is to reduce the circumference by 8% calculate also the speed of rotation of the runner and the H.P. developed under a supply head of 24 m. Neglect friction losses.*

Velocity head at entrance to runner, $\dfrac{V_1^2}{2g} = 0{\cdot}5H = 0{\cdot}5 \times 24 = 12$ m

$\therefore \quad V_1 = \sqrt{2g \times 12} = 15{\cdot}34$ m/sec.

Velocity of flow, $V_{f_1} = 15{\cdot}34 \times \sin 22°$
$= 5{\cdot}748$ m/sec.

Velocity of whirl, $V_{w_1} = 15{\cdot}34 \times \cos 22°$
$= 14{\cdot}22$ m/sec.

Now, neglecting friction losses,

H = W.D./kg + velocity head at exit

i.e. $$24 = \frac{V_{w_1} v_1}{g} + \frac{V_{f_2}^2}{2g} \quad (\because \quad V_{w_2} = 0)$$

or $$24 = \frac{14{\cdot}22 \times v_1}{9{\cdot}81} + \frac{V_{f_2}^2}{2g}, \quad (\because \quad V_2 = V_{f_2} = V_{f_1})$$

or $$24 = 1{\cdot}45 v_1 + \frac{5{\cdot}748^2}{2 \times 9{\cdot}81}$$

$\therefore \quad v_1 = 15{\cdot}39$ m/sec.

But, $$v_1 = \frac{\pi D_1 N}{60}$$

i.e. $$15{\cdot}39 = \frac{\pi \times 2 \times N}{60}$$

$\therefore \quad N = \underline{490 \text{ rpm.}}$

Further, $$\text{W.D./kg} = \frac{V_{w_1} v_1}{g} = \frac{14{\cdot}22 \times 15{\cdot}39}{9{\cdot}81} = 22{\cdot}32 \text{ kg–m}$$

Now, discharge, $Q = (\pi D_1 b_1)(1 - 0{\cdot}08) \times V_{f_1}$

$$= \left(\pi \times 0{\cdot}6 \times \frac{8}{100}\right) \times 0{\cdot}92 \times 5{\cdot}478$$

$$= 0{\cdot}76 \text{ m}^3/\text{sec.}$$

Further, H.P. developed $= \dfrac{wQ \times \text{W.D./kg}}{75}$

$$= \frac{1{,}000 \times 0{\cdot}76 \times 22{\cdot}32}{75} = \underline{226}$$

To find out the angles of the runner blade, we have,

$$\tan\theta = \frac{V_{f_1}}{V_{w1} - v_1} = \frac{5{\cdot}748}{14{\cdot}22 - 15{\cdot}39} = -4{\cdot}912$$

$$\therefore \quad \tan(180 - \theta)^\circ = 4{\cdot}952$$

$$\therefore \quad \theta = 180 - 78{\cdot}5 = \underline{11{\cdot}5^\circ}$$

The peripheral velocity at exit, $v_2 = v_1 \times \dfrac{D_2}{D_1} = 15{\cdot}39 \times \dfrac{0{\cdot}4}{0{\cdot}6} = \underline{10{\cdot}26 \text{ m/sec}}$

$$\text{and} \quad \tan\phi = \frac{V_{f_2}}{v_2} = \frac{5{\cdot}478}{10{\cdot}26} = 0{\cdot}5602$$

$\therefore$ outlet angle of vane, $\phi = \underline{29^\circ 15'}$

Problem-21 : *An inward flow reaction turbine has a runner of 45 cm diameter and 5 cm wide at the outer diameter. The inner diameter is 30 cm and the width here is 8 cm. The vanes occupy 8% of the periphery. The guide vane angle is 24°, the inlet runner vane angle is 95° (vanes inclined forward in the direction of rotation), and the outlet angle is 30°.*

If the hydraulic friction losses amount to 12 percent and mechanical friction to 6% of supply head, calculate (a) the speed for no shock at inlet when the pressure in the outer casing is 54 m above the pressure at discharge from the runner, (b) the output horse-power.

From mass continuity, $Q = \pi\, d_1 b_1 V_{f_1}\, k = \pi d_2 b_2 V_{f_2}\, k$

$$\therefore \quad \pi \times \frac{45}{100} \times \frac{5}{100} \times V_{f_1} \times 0.92 = \pi \times \frac{30}{100} \times \frac{8}{100} \times V_{f_2} \times 0.92$$

$$\therefore \quad 225 V_{f_1} = 240 V_{f_2}$$

$$\therefore \quad V_{f_1} = 1.068 V_{f_2} \text{ or } V_{f_2} = 0.936 V_{f_1}$$

But, $\tan\alpha = \dfrac{V_{f_1}}{V_{w_2}}$

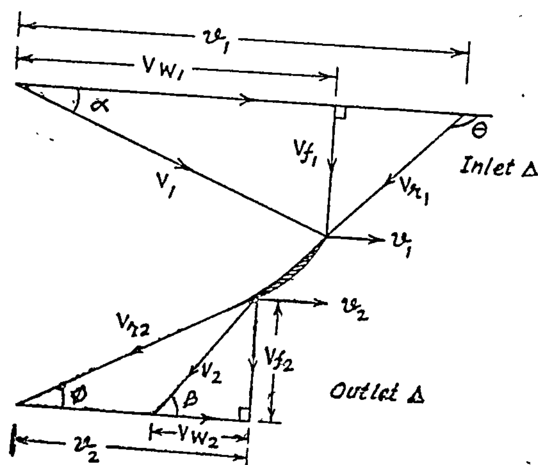


Fig. 10–34

$$\therefore \quad V_{w_1} = \frac{V_{f_1}}{\tan 24^\circ} = \frac{V_{f_1}}{0.4452} = 2.24\ V_{f_1}$$

$$\therefore \quad V_{w_1} = 2.24 V_{f_1}$$

Further, $\tan\theta = \dfrac{V_{f_1}}{V_{w_1} - v_1}$

$$\therefore \quad \tan 95^\circ = \frac{V_{f_1}}{V_{w_1} - v_1} \qquad \therefore \quad V_{w_1} - v_1 = \frac{Vf_1}{\tan 95^\circ}$$

$$\therefore \quad v_1 = V_{w_1} - \frac{V_{f_1}}{-11.43} = V_{w_1} + 0.0875 V_{f_1} = 2.24 V_{f_1} + 0.0871 V_{f_1}$$

$$\therefore \quad v_1 = 2.328 V_{f_1}$$

But, $\dfrac{v_2}{v_1} = \dfrac{d_2}{d_1}$

$$\therefore \quad v_2 = v_1 \times \frac{d_2}{d_1} = 2.3275 V_{f_1} \times \frac{30}{45} = 1.552 V_{f_1}$$

From outlet triangle

$$\tan\phi = \frac{V_{f_2}}{v_2 + V_{w_2}}$$

$$\therefore \quad v_2 + V_{w_2} = \frac{V_{f_2}}{\tan\phi}$$

$$\therefore \quad V_{w_2} = \frac{V_{f_2}}{\tan\phi} - v_2 = \frac{V_{f_2}}{\tan 30^\circ} - 1{\cdot}552V_{f_1}$$

But, $V_{f_2} = \dfrac{V_{f_1}}{1{\cdot}068}$

$$\therefore \quad V_{w_2} = \frac{V_{f_1}}{1{\cdot}068 \times 0{\cdot}5774} - 1{\cdot}552V_{f_1} = 0{\cdot}073V_{f_1}$$

But, $\tan\beta = \dfrac{V_{f_2}}{V_{w_2}} = \dfrac{0{\cdot}936V_{f_1}}{0{\cdot}073V_{f_1}} = 12{\cdot}8$

$$\therefore \quad \underline{\beta = 85{\cdot}6^\circ}$$

Absolute velocity of water at outlet, $V_2 = \dfrac{V_{f_2}}{\sin\beta} = \dfrac{0{\cdot}936V_{f_1}}{0{\cdot}9971}$

$$= 0{\cdot}94V_{f_1}$$

Now, Supply head = W.D./kg + K.E. at outlet

$$\therefore \quad 0{\cdot}88 \times 54 = \frac{V_{w_1}v_1 + V_{w_2}v_2}{g} + \frac{V_2^2}{2g}$$

$$\therefore \quad 47{\cdot}5 = \frac{(2{\cdot}24V_{f_1} \times 2{\cdot}328V_{f_1}) + 0{\cdot}073V_{f_1} \times 1{\cdot}552V_{f_1}}{9{\cdot}81} + \frac{(0{\cdot}94V_{f_1})^2}{2 \times 9{\cdot}81}$$

$$\therefore \quad 9{\cdot}81 \times 47{\cdot}5 = 5{\cdot}21V_{f_1}^2 + 0{\cdot}113V_{f_1}^2 + 0{\cdot}441V_{f_1}^2 = 5{\cdot}764V_{f_1}^2$$

$$\therefore \quad V_{f_1} = \sqrt{\frac{9{\cdot}81 \times 47{\cdot}5}{5{\cdot}764}} = 8{\cdot}95 \text{ m/sec}$$

$$\therefore \quad v_1 = 2{\cdot}328V_{f_1} = 2{\cdot}328 \times 8{\cdot}95 = 20{\cdot}8 \text{ m/sec.}$$

$$\therefore \quad N = \frac{60v_1}{\pi d_1} = \frac{60 \times 20{\cdot}8}{\pi \times 0{\cdot}45} = \underline{885 \text{ rpm.}}$$

$$\text{W.D./kg.} = \frac{V_{w_1}v_1 + V_{w_2}v_2}{g}$$

$$= \frac{5.323\ V_{f1}^2}{g} = \frac{5.323 \times (8.95)^2}{9.81} = 43.7 \text{ kg-m}$$

Discharge, $W = wQ = w\ \pi\ d_1 b_1\ V_{f1} \times k$

$$= 1{,}000 \times \pi \times 0.45 \times \frac{5}{100} \times 8.95 \times 0.92$$

$$= 581 \text{ kg}$$

$$\therefore \quad \text{W.H.P.} = \frac{581 \times 43.7}{75} = 338 \text{ H.P.}$$

$$\text{B.H.P.} = \eta_{mech} \times \text{W.H.P.} = 0.94 \times 338 = \underline{318 \text{ H.P.}}$$

Problem-22 : *An inward flow pressure turbine has runner vanes which are radial at inlet and inclined backward at 45° to the tangent at discharge. The guide vanes are inclined at 15° to the tangent at inlet and the velocity of water leaving the guides is 24 m/sec. Determine the correct speed for the runner and the absolute velocity of water at the point of discharge, if the diameter at entry is twice that at discharge and the width at entry is 0·6 times that at discharge.*

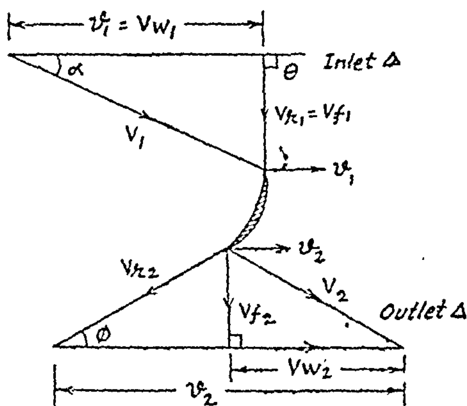


Fig. 10-35

As the water enters radially at inlet, $\theta = 90°$.

$\therefore \quad v_1 = V_{w1}$ and $V_{f1} = V_{r1}$

and $\quad \alpha = 15°$ and $V_1 = 24$ m/sec

$\therefore \quad v_1 = V_{w1} = V_1 \cos\alpha = 24 \cos 15°$

$= 24 \times 0.9659 = \underline{23.2 \text{ m/sec}}$

But, $\quad V_{f1} = V_1 \sin\alpha = 24 \times 0.2588 = 6.2$ m/sec

$Q = \pi d_1 b_1 V_{f1} = \pi d_2 b_2 V_{f2}$

since, $\quad d_1 = 2d_2$ and $b_1 = 0.6 b_2$

$2d_2 \times 0.6 b_2 \times 6.2 = d_2 b_2 V_{f2}$

$\therefore \quad V_{f2} = 1.2 \times 6.2 = 7.44$ m/sec

From outlet triangle,

$$\tan\phi = \frac{V_{f2}}{v_2 - V_{w2}}$$

$$\therefore \quad v_2 - V_{w2} = \frac{V_{f2}}{\tan\phi} = \frac{7.44}{\tan 45°} = \frac{7.44}{1} = 7.44 \text{ m/sec}$$

Further, $\dfrac{v_1}{v_2} = \dfrac{d_1}{d_2}$

$$\therefore \quad v_2 = v_1 \times \frac{d_2}{d_1} = 23.2 \times \frac{d_2}{2d_2} = 11.6 \text{ m/sec}$$

$\therefore \quad V_{w2} = v_2 - 7.44 = 11.6 - 7.44 = 4.16$ m/sec

Absolute velocity of water at outlet, $V_2 = \sqrt{V_{f2}^2 + V_{w2}^2}$

$= \sqrt{(7.44)^2 + (4.16)^2}$

$= \underline{8.52 \text{ m/sec}}$

Problem-23 : *A vertical shaft inward flow reaction turbine works with a net head of 30 m. The external diameter of the runner is 39 cm and the inlet width is 3.8 cm. The runner blade angle at entry is 100° measured from the tangent at the runner periphery drawn in the direction of runner rotation. Effect of blade thickness on inlet area can be neglected. Water enters the runner from the guide blades at an angle of 155° to the tangent at the rxnner periphery. The velocity of flow through the runner is constant, water*

enters the straight draft tube from the runner without whirl, and discharge takes place from the draft tube into the tailrace with a velocity 2·5 m/sec. The loss of head in the turbine due to fluid resistance is 3·9 m.

Determine : (a) the runner blade angle at a point on the outlet edge where the radius of rotation is 9 cm, (b) the specific speed of the turbine and (c) the approximate inlet diameter of the draft tube (d) speed of turbine.

From inlet triangle of velocity

$$\tan \alpha = \frac{V_{f1}}{V_{w1}}$$

$$\therefore \quad V_{w1} = \frac{V_{f1}}{\tan\alpha} = \frac{V_{f1}}{\tan 25^\circ} = 2{\cdot}144\, V_{f1}$$

$$\text{and} \quad v_1 = V_{w1} - \frac{V_{f1}}{\tan 80^\circ} = 2{\cdot}144 V_{f1} - 0{\cdot}176\, V_{f1}$$

$$= 1{\cdot}968\, V_{f1}$$

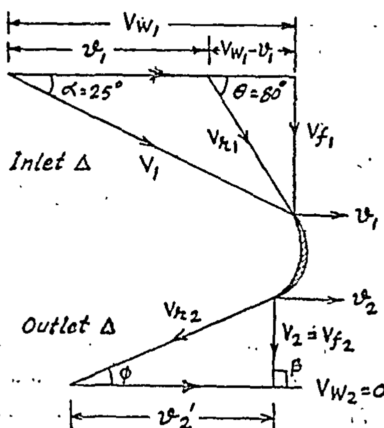


Fig. 10-36

As discharge is in radial direction, $V_{w2} = 0$.

$$\therefore \quad \text{W·D./kg/sec.} = \frac{V_{w1} v_1}{g} = \frac{2{\cdot}144 V_{f1} \times 1{\cdot}96 V_{f1}}{9{\cdot}81} = 0{\cdot}43 V_{f1}^2$$

But, available head = W.D./kg + losses + K.E. at exit

$$\therefore \quad \text{W.D./kg} = 30 - 3{\cdot}9 - \frac{(2{\cdot}5)^2}{19{\cdot}62} = 25{\cdot}78 \text{ kg-m}$$

$$\therefore \quad 0{\cdot}43 \ V_{t_1}^2 = 25{\cdot}78$$

$$\therefore \quad V_{t_1} = \sqrt{\frac{25{\cdot}78}{0{\cdot}43}} = 7{\cdot}7 \text{ m/sec}$$

and $\quad v_1 = 1{\cdot}968 \times 7{\cdot}7 = 15{\cdot}15$ m/sec

Now, $\quad V_{t_1} = V_{t_2} = V_2 = 7{\cdot}7$ m/sec

Velocity of the outer edge at 9 cm radius is $v_2 = \frac{v_1}{r_1} \times 9$

$$\therefore \quad v'_2 = \frac{15{\cdot}15 \times 9}{19{\cdot}5} = 7 \text{ m/sec.}$$

From outlet triangle,

$$\tan \phi = \frac{V_{t_2}}{v'_2} = \frac{7{\cdot}7}{7} = 1{\cdot}1$$

$$\therefore \quad \phi = 47{\cdot}7°$$

Now, $v_1 = \frac{\pi d_1 N}{60}$

$$\therefore \quad N = \frac{60 v_1}{\pi d_1} = \frac{60 \times 15{\cdot}15}{\pi \times 0{\cdot}39} = \underline{742 \text{ rpm.}}$$

Discharge, $W = \pi d_1 b_1 V_{t_1} \times w = \pi \times 0{\cdot}39 \times \frac{3{\cdot}8}{100} \times 7{\cdot}7 \times 1{,}000$

$$= 357 \text{ kg/sec}$$

$$\therefore \quad Q = 0{\cdot}357 \text{ m}^3\text{/sec}$$

$$\therefore \quad \text{H.P.} = \frac{\text{W.D./kg} \times \text{flow/sec}}{75} = \frac{25{\cdot}78 \times 357}{75} = 123 \text{ H.P.}$$

Now, specific speed, $N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{742\sqrt{123}}{(30)^{5/4}}$

$$= \underline{137 \text{ rpm.}}$$

Let the inlet diameter of the draft-tube be d.

$$\text{Then, } Q = \frac{\pi}{4} d^2 \times V_2 \qquad \text{as } V_2 = V_{f2} = 7{\cdot}7 \text{ m/sec.}$$

$$\therefore \qquad d = \sqrt{\frac{4Q}{\pi V_2}} = \sqrt{\frac{4 \times 0{\cdot}357}{\pi \times 7{\cdot}7}} = 0{\cdot}242 \text{ m or } 24{\cdot}2 \text{ cm}$$

Problem-24 : *A vertical shaft inward flow reaction turbine runner developes 16,800 H.P. and uses 12·2 m³/sec of water, when the net head is 114 m. The runner has a diameter of 1·5 m and rotates at 430 rpm. Water enters the runner without shock with a velocity of flow of 9·6 m/sec and passes from the runner to the draft tube without whirl with a velocity of 7·2 m/sec. The difference between the sum of the pressure and potential heads at the entrance to the runner and at entrance to the draft tube is 60 m.*

Determine : (a) the velocity and direction of the water entering the runner from the fixed guide blades, (b) the entry angle of the runner blades, and (c) the loss of head in the runner.

As the water leaves without velocity of whirl at outlet,

$$V_{w2} = 0, \quad \beta = 90°, \quad V_2 = V_{f2}$$

$$\text{Velocity of vane tip at inlet, } v_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 1{\cdot}5 \times 430}{60}$$

$$= 33{\cdot}8 \text{ m/sec}$$

$$\text{H.P.} = \frac{\text{W.D /kg} \times \text{flow in kg/sec.}}{75}$$

$$\therefore \quad \text{W.D./kg} = \frac{16{,}800 \times 75}{12{\cdot}2 \times 1{,}000} = 103{\cdot}5 \text{ kg-m}$$

$$\text{But, W.D./kg} = \frac{V_{w1} v_1}{g}$$

$$\therefore \qquad V_{w1} = \frac{g \times 103{\cdot}5}{v_1} = \frac{9{\cdot}81 \times 103{\cdot}5}{33{\cdot}8} = 30 \text{ m/sec}$$

$$\therefore \qquad \tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{9{\cdot}6}{30} = 0{\cdot}32$$

$$\therefore \qquad \underline{\alpha = 17°45'}$$

Absolute velocity at entrance, $V_1 = \frac{V_{f1}}{\sin \alpha} = \frac{9.6}{0.3048} = \underline{31.6 \text{ m/sec}}$

$$\therefore \quad \tan\theta = \frac{V_{f1}}{V_{w1} - v_1} = \frac{9.6}{30 - 33.8} = -\frac{9.6}{3.8} = -2.53$$

$$\therefore \quad \theta = 180 - 68°\,26' = \underline{111°34'}$$

K.E. of the jet at entrance $= \frac{V_1^2}{2g} = \frac{(31.6)^2}{19.62} \approx 51$ kg-m

K.E. of the leaving water at outlet $\frac{V_2^2}{2g} = \frac{(7.2)^2}{19.62} = 3.64$ kg-m

Loss of head in runner = difference in pressure and potential head + K.E. at inlet − K.E at outlet − W.D./kg/sec.

$$\therefore \quad \text{Loss of head in runner} = 60 + 51 - 3.64 - 103.5$$
$$= 3.86 \text{ m of water}$$

Problem - 25 : *Two inward flow turbine runners have the same diameter 0·6 m and the same efficiency. They work under the same head with a velocity of flow of 6 m/sec. One runner runs at 600 rpm. and has an inlet blade angle of 60°. Determine the speed at which the other turbine should run if its inlet blade angle is 115°. Assume the radial discharge at outlet in both turbines.*

First runner

Velocity of vane tip at inlet, $v_1 = \frac{\pi d_1 N}{60}$

$$= \frac{\pi \times 0.6 \times 600}{60} = 18.8 \text{ m/sec.}$$

Now, $\tan\theta = \frac{V_{f1}}{V_{w1} - v_1}$

$$\therefore \quad V_{w1} = v_1 + \frac{V_{f1}}{\tan\theta} = 18.8 + \frac{6}{\tan 60°} = 18.8 + \frac{6}{1.7321}$$
$$= 22.27 \text{ m/sec}$$

As the discharge is radial at outlet,

$$\text{W.D./kg} = \frac{V_{w1} \times v_1}{g} = \frac{22.27 \times 18.8}{9.81} \approx 43.6 \text{ kg-m}$$

Second runner

As both turbines run under the same head with the same efficiency, the work done in the second runner is also same and is equal to 43·6 kg–m.

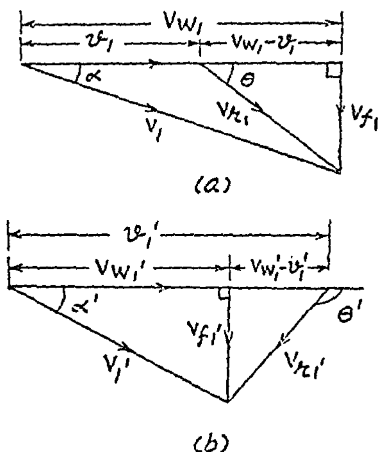


Fig. 10–37

Also the velocity of flow is the same equal to 6 m/sec.

$$\therefore \quad \frac{V'_{w1} \times v_1'}{g} = 43{\cdot}6 \text{ and } V'_{f1} = 6 \text{ m/sec}$$

But, $\theta' = 115°$

$$\therefore \quad \tan(180° - 115°) = \frac{V'_{f1}}{v_1' - V'_{w1}}$$

$$\therefore \quad v_1' - V'_{w1} = \frac{V'_{f1}}{\tan 65°} = \frac{6}{2{\cdot}1445} = 2{\cdot}79 \text{ m/sec}$$

$$\therefore \quad V'_{w1} = v_1' - 2{\cdot}79 \text{ m/sec}$$

Also $\quad V'_{w1}v_1' = 43{\cdot}6 \times 9{\cdot}81 = 428$

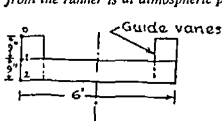
$$\therefore \quad (v_1' - 2{\cdot}79)\, v'_1 = 428$$

$$\therefore \quad v'^2_1 - 2{\cdot}79 v_1' - 428 = 0$$

$$\therefore \quad v_1' = 19{\cdot}8 \text{ m/sec}$$

$$\therefore \quad N' = \frac{60 v_1'}{\pi d'_1} = \frac{60 \times 19{\cdot}8}{\pi \times 0{\cdot}6} = \underline{630 \text{ rpm.}}$$

Problem-26 : *In a parallel flow reaction (Jonval) turbine, the shaft axis is vertical and water flow is downwards. The vertical depth of guide and wheel passages is 22·5 cm in each case. The mean diameter is 2 m. The guide outlet angle is 30° and vane thickness coefficient at entrance is 0·88. At outlet, the wheel vane angle is 25° and vane thickness coefficient is 0·9. The radial height of the wheel vanes and guide vanes is 30 cm. The loss in the runner vanes is 8% of the total head. The wheel speed is 90 rpm. and the final discharge is axial. Estimate the wheel vane angles, pressure drop across the runner, power developed and hydraulic efficiency, assuming that exit from the runner is at atmospheric pressure.*



Mean peripheral velocity of wheel vanes is

$$v = \frac{\pi D N}{60} = \frac{\pi \times 2 \times 90}{60}$$

$$= 9·42 \text{ m/sec}$$

From outlet velocity triangle

$$V_{f_2} = v \tan\phi = 9·42 \tan 25°$$

$$= 4·38 \text{ m/sec}$$

$$\text{Now, } V_{f_1} = \frac{V_{f_2} \times 0·9}{0·88} = \frac{4·38 \times 0·9}{0·88}$$

$$= 4·48 \text{ m/sec}$$

From inlet velocity triangle,

$$V_{w_1} = \frac{V_{f_1}}{\tan\alpha} = \frac{4·48}{\tan 30°}$$

$$= \frac{4·48}{0·5774}$$

$$= 7·75 \text{ m/sec}$$

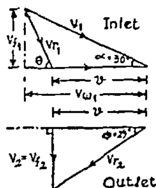


Fig. 10-38

$$\text{and } \tan\theta = \frac{V_{f_1}}{V_{w_1} - v} = \frac{4·48}{7·75 - 9·42} = -2·68$$

$$\therefore \quad \theta = 180 - 70·35 = \underline{109·65°}$$

$$\text{and } \quad V_1 = \sqrt{V_{f_1}^2 + V_{w_1}^2} = \sqrt{4·48^2 + 7·75^2} = 8·93 \text{ m/sec}$$

$$\text{Lose of head in guide vanes} = 0·08 \frac{V_1^2}{2g} = \frac{0·08 \times 8·93^2}{19·62} = 0·3255 \text{ m}$$

$$\text{W.D. per kg} = \frac{V_{w_1}v_1}{g} = \frac{7.75 \times 9.42}{9.81} = 7.45 \text{ kg-m}$$

Applying Bernoulli's theorem to inlet and outlet of runner and taking the outlet point as datum,

$$\frac{V_1^2}{2g} + \frac{p_1}{w} + Z_1 = \text{W.D./kg} + \text{losses} + \frac{V_2^2}{2g} + \frac{p_2}{w}$$

Substituting H_1 for $\left\{\frac{V_1^2}{2g} + \left(\frac{p_1}{w} - \frac{p_2}{w}\right) + Z_1\right\}$

$$H_1 = \text{W.D./kg.} + 0.08\,H_1 + \frac{V_2^2}{2g}$$

$$i.e.\ 0.92H_1 = 7.45 + \frac{4.38^2}{19.62} = 8.435 \text{ m}$$

$$\therefore \quad H_1 = \frac{8.435}{0.92} = \underline{9.18 \text{ m}}$$

$$\therefore \quad \frac{V_1^2}{2g} + \frac{p_1 - p_2}{w} + Z_1 = 9.18 \text{ m}$$

$$\therefore \quad \frac{8.93^2}{19.62} + \frac{p_1 - p_2}{w} + \frac{22.5}{100} = 9.18$$

$$\therefore \quad \frac{p_1 - p_2}{w} = 4.875$$

i.e. pressure head across the runner is $\underline{4.875 \text{m}}$

Further, discharge, $Q = \pi D_1 b_1 k_1 V_{f_1}$

$$= \pi \times 2 \times \frac{30}{100} \times 0.9 \times 4.48$$

$$= 7.58 \text{ m}^3/\text{sec.}$$

$$\therefore \quad \text{H.P. developed} = \frac{wQ \times \text{W.D./kg.}}{75}$$

$$= \frac{1{,}000 \times 7.58 \times 7.45}{75} = \underline{755 \text{ H.P.}}$$

Supply head, $H = H_1$ (since exit from turbine is atmospheric)

$$\therefore \quad \text{hydraulic efficiency, } \eta_h = \frac{\text{W.D./kg}}{H} = \frac{7.45}{9.18} \times 100 = \underline{81.5\%}$$

Problem-27 : *An inward flow reaction turbine works under a total head of H = 30 m. The centre-line is 3 m above the tail water level. The discharge from the wheel vanes is radial and the outlet diameter is half the inlet diameter. The rim velocity at inlet is* $0.6V_1$,

the radial velocity of flow at inlet is $0{\cdot}15V_1$, the velocity at outlet from the draft tube is $0{\cdot}1V_1$, where $V_1\ \sqrt{2gH}$.

Assume, there is no shock loss at entry to wheel. The losses in the runner passages are 7% of H, friction losses in the draft tube are 3% of H, losses in penstock and guides 4% of H, velocity of flow over the runner is constant.

Find, (a) pressure head at inlet to and outlet from the runner, (b) suitable angles for runner and guide vanes, and (c) ratio of rim speed at inlet to water speed at inlet to runner.

$$\text{W.D./kg} = H - \begin{Bmatrix}\text{losses in penstock, guides,}\\ \text{runner, and draft tube}\end{Bmatrix} - \begin{Bmatrix}\text{velocity head}\\ \text{lost to tail race}\end{Bmatrix}$$

$$= 30 - 30\,(0{\cdot}04 + 0{\cdot}07 + 0{\cdot}03) - \frac{\{0{\cdot}1\sqrt{2gH}\}^2}{2g}$$

$$= 30\,(1 - 0{\cdot}14 - 0{\cdot}01)$$

$$= 25.5 \text{ kg-m}$$

Now, wheel velocity, $v_1 = 0{\cdot}6\ \sqrt{2g\times 30} = 14{\cdot}5$ m/sec

Inlet flow velocity, $V_{f1} = 0{\cdot}15\ \sqrt{2g\times 30} = 3{\cdot}62$ m/sec

Draft tube outlet velocity, $V_o = 0{\cdot}1\ \sqrt{2g\times 30} = 2{\cdot}42$ m/sec.

$$\text{Again W.D./kg} = \frac{V_{w1}v_1}{g}$$

$$i.e.\ 25{\cdot}5 = \frac{V_{w1}\times 14{\cdot}5}{9{\cdot}81}$$

$$\therefore\quad V_{w1} = \frac{25{\cdot}5\times 9{\cdot}81}{14{\cdot}5} = 17{\cdot}3 \text{ m/sec.}$$

$$\text{Hence, } \tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{3{\cdot}62}{17{\cdot}3} = 0{\cdot}21$$

i.e guide vane angle, $\underline{\alpha = 11{\cdot}9^\circ}$

$$\text{Now, } V_1^2 = V_{w1}^2 + V_{f1}^2$$

$$= 17{\cdot}9^2 + 3{\cdot}62^2$$

$$\therefore\quad V_1 = 17{\cdot}5 \text{ m/sec.}$$

$$\text{and } \tan\theta = \frac{V_{f1}}{V_{w1} - v_1} = \frac{3{\cdot}62}{17{\cdot}3 - 14{\cdot}5} = 1{\cdot}315$$

$$\therefore\quad \theta = 52{\cdot}7^\circ$$

$$\text{Further, } \tan\phi = \frac{V_{f2}}{v_2} = \frac{V_{f1}}{0{\cdot}5v_1} = \frac{3{\cdot}62}{0{\cdot}5\times 14{\cdot}5} = 0{\cdot}5$$

$$\therefore\quad \underline{\phi = 26{\cdot}51^\circ}$$

The ratio, $\frac{\text{rim speed}}{\text{water speed}}$ at inlet $= \frac{v_1}{V_1} = \frac{14.5}{17.5} = \underline{0.827}$

Pressure head at inlet to runner is

$$\frac{p_1}{w} = H - \text{losses in penstock and guides} - \frac{V_1^2}{2g}$$

$$= 30 - 0.04 \times 30 - \frac{18.2^2}{2 \times 9.81}$$

$$= \underline{13.15 \text{ m}}$$

Pressure head at outlet from runner is found out with help o Bernoulli's theorem applied to exits from runner and draft tube.

$$\text{or } \frac{p_2}{w} + h_2 + \frac{V_{f2}^2}{2g} = \frac{p_0}{w} + \frac{V_0^2}{2g} + \text{losses in draft tube}$$

$$\text{or } \frac{p_2}{w} = \frac{V_0^2}{2g} - \frac{V_{f2}^2}{2g} + \text{losses} - h_2$$

$$= \frac{2.42^2}{2 \times 9.81} - \frac{3.62^2}{2 \times 9.81} + 0.03 \times 30 - 3$$

$$= 2.46 \text{ m}$$

i.e. pressure head at outlet from runner is 2.46 m below atmosphere

Problem–28 : *A vertical spindle inward flow reaction turbin operates under a head of H m. From the following data, estimat the pressure head at runner inlet in terms of H, the wheel vane angle and velocity coefficient for the guide vanes.*

Guides : outlet angle, 18°; velocity head generated 0.49H

Runner : oulet diameter is ½ inlet diameter; outlet area is 0· inlet area; loss in wheel vanes, 0.12H rim speed a inlet, 2.75 $\sqrt{H}$; discharge is radial.

Draft tube : inlet area is equal to runner outlet area; outlet area i 2 times inlet area; height above tail race, 0.12H losses, 0.048 of inlet velocity head.

As $\frac{V_1^2}{2g} = 0.49H$, $V_1 = 3.1\sqrt{H}$ m/sec.

and $v_1 = 2.75\sqrt{H}$, $v_2 = \frac{1}{2}v_1 = 1.375\sqrt{H}$ m/sec.

Now, $V_{f_1} = V_1 \sin \alpha = 3\cdot1\sqrt{H} \times \sin 18^\circ = 0\cdot96\sqrt{H}$ m/sec.

and $V_{f_2} = \dfrac{V_{f_1}}{0\cdot7} = \dfrac{0\cdot96\sqrt{H}}{0\cdot7} = 1\cdot35\sqrt{H}$ m/sec.

Also, $V_{w_1} = V_1 \cos \alpha = 3\cdot1\sqrt{H} \times \cos 18^\circ = 2\cdot95\sqrt{H}$ m/sec.

Now, total head at inlet to runner is found out by applying Bernoulli's equation to inlet to runner and outlet from draft tube points.

$$\text{or } \frac{p_1}{w} + \frac{V_1^2}{2g} + h_1 = \text{W.D}/kg. + \begin{Bmatrix}\text{losses in runner} \\ \text{and draft tube}\end{Bmatrix} + \begin{Bmatrix}\text{velocity head at} \\ \text{draft tube outlet}\end{Bmatrix}$$

$$i.e.\ \frac{p_1}{w} + 0\cdot49H + 0\cdot12H = \frac{2\cdot95\sqrt{H} \times 2\cdot75\sqrt{H}}{9\cdot81} + \left(0\cdot12H + 0\cdot048\frac{V_{f_2}^2}{2g}\right) + \frac{(\frac{1}{2}V_{f_2})^2}{2g}$$

$$\text{or } \frac{p_1}{w} + 0\cdot61H = 0\cdot828H + 0\cdot12H + (0\cdot048 + 0\cdot25)\frac{(1\cdot37\sqrt{H})^2}{2\times9\cdot81}$$

$$\therefore \quad \frac{p_1}{w} = \underline{0\cdot3665H}$$

$$\text{Further, } \tan\theta = \frac{V_{f_1}}{V_{w_1} - v_1} = \frac{0\cdot96\sqrt{H}}{2\cdot93\sqrt{H} - 2\cdot75\sqrt{H}} = 4\cdot8$$

$$\therefore \quad \theta = \underline{78\cdot2^\circ}$$

$$\text{and } \tan\phi = \frac{V_{f_2}}{v_1} = \frac{1\cdot37\sqrt{H}}{1\cdot375\sqrt{H}} = 0\cdot999$$

$$\therefore \quad \phi = \underline{45^\circ}$$

Head available for conversion into kinetic energy at the guide vane inlet is

$$H - h_1 - \frac{p_1}{w} = H - 0\cdot12\,H - 0\cdot3665\,H = 0\cdot5135H$$

$$\text{and } V_1 = C_v\sqrt{2g \times 0\cdot5135H}$$

$$\text{But, } V_1 = 3\cdot1\sqrt{H}$$

$$\therefore \quad 3\cdot1\sqrt{H} = C_v\sqrt{2g \times 0\cdot5135H}$$

$$\therefore \quad C_v = \underline{0\cdot98}$$

Governing of Turbines

The main function of a governor is to maintain constant speed of rotation when load on the turbine fluctuates. Governing of water turbines is obtained by controlling the rate of flow of water to the moving vanes. In case of Pelton wheel, the needle valve is closed or opened, for Francis turbine the wicket gates are opened or closed and for Kaplan turbine the runner blades are rotated in addition to operating the wicket gates.

Two issues arise while thinking about the governing action of a turbine. The governor should act quickly to restore the speed to normal as fast as possible after it has been upset. Again, it should not close the pipe line very quickly to prevent the occurrence of a serious hammer blow in it. This means that the governor is to be so designed as to keep the speed rise of turbine and pressure rise of pipe within limits. Hence, governors are fitted up with additional safety arrangements against hammer blow. A deflector or diffusor in Pelton wheels and a relief valve or pressure regulator in reaction turbines are the units under reference.

Compared to the governors of diesel engines and steam turbines, the governor of a water turbine requires a very strong mechanism to exert forces necessary to divert or stop the flow of water into a turbine runner. Even in a small hydraulic governor, the energy required for an operating stroke is of the order of 5,000 kg-m while in large turbines it is of the order of 30,000 kg-m. As in other governors, the governor of the hydraulic turbine also depends on the action of revolving masses. Since the energy required is great, the force developed by the sensitive fly-balls of the centrifugal governor can only be used to control a relay, which, in turn, will control the regulating mechanism. The governor merely actuates a pilot valve which admits oil under pressure to one or the other side of the piston in a cylinder. This piston and the cylinder together are known *as servomotor* and the movement of the piston brings about changes in the set up of the gates or nozzles.

A schematic diagram of fig. 10-39 illustrates the working of the governor for a hydraulic turbine. The change in speed of the turbine is communicated to the fly-balls X by means of belt gearing or direct gearing from the turbine main shaft. (Synchronous electrical drive is also used instead of mechanical drive). If the wheel speed increases, for instance, the sleeve, to which is attached

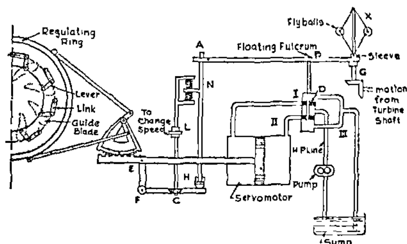


Fig. 10-39 Servomotor governing mechanism

the floating lever, is raised. This lever, for the moment, pivots about A and the relay valve D is raised through the link at B. The passage I is now open to oil supply line and admits the high pressure oil to the left hand side of the servomotor piston. The piston is forced towards the right and decreases the turbine gate opening. Duriug this movement, the oil on the right hand side of the servomotor exhausts through the passage II and III to the oil sump.

Simultaneonsly with the movement of the piston of the servomotor to the right, the bell crank lever EFC which is attached to the piston rod at E, is rotated downwards about F and the arm C is lowered. This pulls down the pivot A and causes the fulcrum B to be lowered. Thus, the relay port I is closed again and stops the motion. The governor is, therefore, prevented from over-

travelling. This device is called *compensating device*; without the compensating device the servomotor piston would continue to move in the same direction and would advance far beyond the proper point. Finally, when the hydraulic conditions become readjusted, the governor would then move back to the other side, in its proper place. Thus, the governor would continually hunt and maintain oscillations. This would happen if the lever AG were pivoted permanently at A.

By making the lever floating, as shown, it is seen that any new position due to change of load is decisively established without the danger of hunting. The floating fulcrum moves up or down, as the case may be, and prevents over-travelling of the piston and establishes the new position firmly. Such a governor is called a *dead beat type governor*. H is a dash pot. On any rapid motion of the gates, following a sudden change of load, the dash pot acts as a rigid connection and moves the pivot A. But, if the governor acts slowly due to gradual change of load, there is no need for compensation and hence the dash pot does not transmit any motion to pivot A. A second rod from C, connected to the springs as shown, will move up or down and will serve to stop the motion of pivot A during slow movement of the servomotor piston. The wheel at L has right-hand and left-hand threads in it. By turning it, the length of the rod between C and N will change. Thus, the relay valve is raised or lowered, thereby, varying the turbine speed.

Governing of Pelton wheel: Various methods have been adopted to regulate the speed of a Pelton wheel.

Use of a throttle valve in the pipe line : By using a throttle valve or sluice valve in the pipe line, the supply of water may be adjusted to demand. But, this is a very crude and wasteful method. By throttling, a part of the available head is destroyed. Also, this reduction in head means a smaller jet velocity and hence, an alteration of the speed ratio. Hence, there is a drop in efficiency at partial loads. This is indicated in the graph of fig. 10-40.

Spear or needle regulation : The ideal method of governing should not affect head, it should merely vary the quantity of water used in direct proportion to the power demanded.

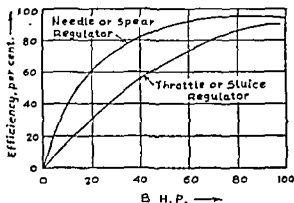


Fig. 10-40 Comparison of needle and throttle regulation

The spear or needle method accomplishes this result very nearly. The spear or needle is slided axially in the nozzle. As the needle is moved to and fro, it alters the annular area of flow

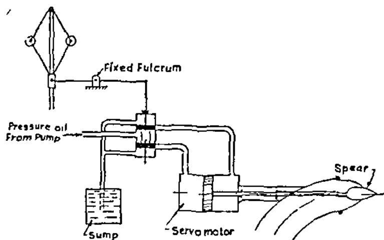


Fig. 10-41 Spear or needle regulator of a Pelton wheel

and thus varies the amount of water to be discharged. The velocity of the jet is very nearly the same as head remains unaltered, but

the diameter of the jet changes for different positions of the spear. As a result, the ratio of bucket speed to jet speed is kept constant over a wide range of loads. Hence, the efficiency is independent of the load for a wide range. This is shown by the curve of fig. 10-40. This arrangement is illustrated in fig. 10-41. In it, the movement of the needle is obtained from the piston of the servo-motor. A small difference in the governor mechanism here is that, instead of a floating fulcrum, the lever connecting the centrifugal governor to the control valve moves about a fixed fulcrum. As the needle moves slowly, there is no necessity for compensating device.

Regulation by deflection of jet : The chief disadvantage of the needle regulator is its unability to be used to close the nozzle swiftly without incurring the risk of dangerous water hammer. Its use may be satisfactory if the pipe line is not long, the velocity of flow is low, and the changes of load are small and gradual. In case it is used, the penstock is provided with a surge tank. However, if

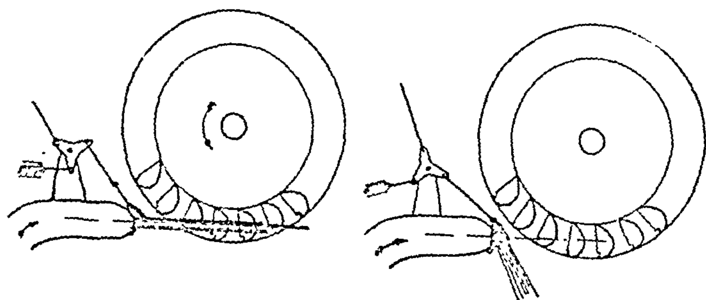


Fig. 10-42 Regulation by deflection of jet for Pelton wheel

the pipe line is long, velocity of flow very high, and the load is variable over a wide range, a deflecting nozzle or hood provides a satisfactory method. When the load decreases, the deflector is moved into position across the jet thus deflecting the jet partly or wholly clear of the buckets (fig. 10-42). Instead of providing a hood for deflecting, the nozzle itself may be made with a ball and socket joint so that the entire jet can be deflected below the wheel if necessary. The flow in the pipe line is independent of the load,

and very close and rapid regulation is possible. But, this method involves lot of waste of water.

Combined needle and deflector regulation: This method claims the advantages of both the previous methods. On reduction of load, the servomotor rod R, operates the link L about the fulcrum F. This is achieved independently of the needle N, since the lower end of the link L can move freely in the slot M of the needle. The forward movement of the link L interposes the deflector H in the way of the jet by means of the rod P. The jet is thus partly deflected from the wheel as in previous method.

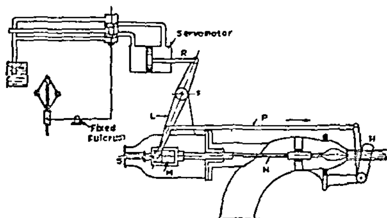


Fig. 10-13 Combined needle and deflector regulation for Pelton wheel

In the meanwhile, the needle N which is under the pressure of spring S moves forward slowly, thus reducing the jet diameter as in the case of needle governing till the hood is again tangential to the jet. At this position the slot M would again be in contact with the lower end of the link L. On increase of load, both the deflector and the spear move back together.

Governing of reaction turbines : The quantity of water supplied to a reaction turbine is regulated with the help of guide vanes which are pivoted and connected by levers and links to the regulating ring. Two long, regulating rods connect the regulating ring to a lever, which is keyed to a regulating shaft which is turned

by deriving motion from the piston of the servomotor. By altering the positions of all the guide vanes, the area of passage for water flow is changed and hence the quantity of water (refer fig. 10-39).

Beside this, the penstock pipe feeding the turbine is fitted with a *relief valve* or what is known as a *pressure regulator*. When guide vanes are suddenly closed, the relief valve opens and diverts the water direct to the tail race. It, as such, resembles the deflector of the Pelton wheel. The simultaneous operation of guide vanes and relief valve is termed as *double regulation* and is accomplished by the governor. This is necessary in case, where the pipe line is long and no surge tank is provided.

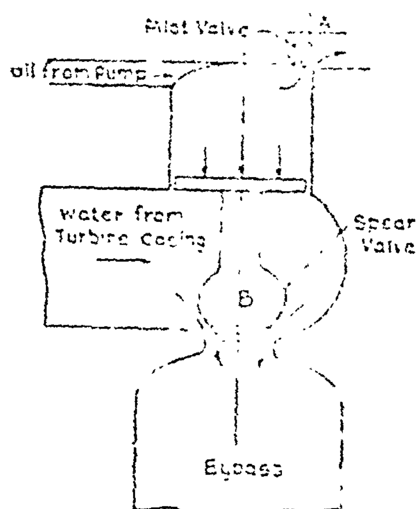


Fig. 10-44
Relief valve

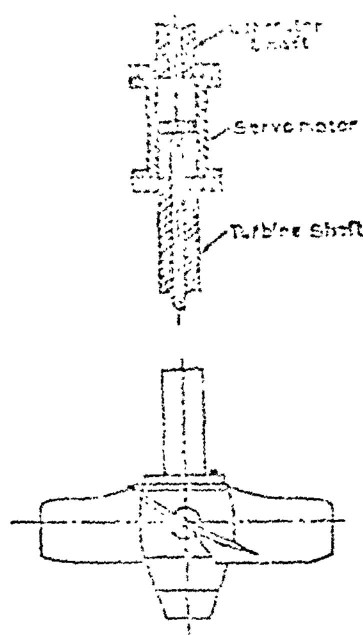


Fig. 10-45
Governing of Kaplan turbine

The relief valve (fig. 10-44) consists of a spear valve, which, when in open position, connects the turbine casing to the by pass, through which water will flow out to the tail race without entering the runner. Under normal working, the spear valve is kept in

closed position by oil under pressure from the pump of the servo-motor governor. The motion of the piston of the servomotor is utilised to operate the pilot valve which releases the oil pressure. Water from turbine casing lifts the spear valve and flows into the by-pass.

Governing of Kaplan turbine : Kaplan turbine has a sort of double regulation which comprises the movement of guide vanes and rotation of runner blades (fig. 10-45). This is arranged with a view to obtain a balanced and most satisfactory relationship between the relative positions of guide and working vanes. The servomotor operates in the hollow shaft of the turbine and the movement of the piston is employed to twist the blades through suitable linkages. There is another servomotor to operate on the guide vanes. Both governors are actuated simultaneously so as to maintain very high efficiency.

In the Francis turbine, the correct disposition of the guide and moving blades is obtained at full load only. At partial loads, there is shock at entry and eddy currents are formed, since the guide vanes only are moved. In Kaplan turbine, since both sets of vanes are moved at a time, high efficiency is obtaind even at partial loads. Because of this fact, Kaplan turbine is particularly suitable for low head projects where the load and head vary frequently.

Performance of turbines

Turbines are required to work under varying conditions of head, discharge, power and speed. In the study of turbine so far made, conditions for best performance were established on the basis of constancy of the above stated variables. In practice, however, such uniformity rarely prevails. As such, it is necessary now, to review the nature of variations that may occur.

(i) The head on the turbine may change, and in order to maintain constant design efficiency, the output and the speed may be adjusted accordingly, the discharge (gate opening) being kept

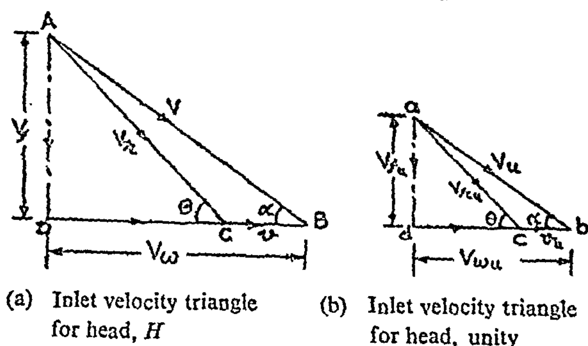
constant. These conditions are very useful in testing of models and manipulating design data for geometrically similar machines.

(ii) The discharge may be varied according to the required power output by changes in gate opening and needle movement, the head and speed remaining constant. These are the *normal operating conditions* for most of the turbines.

(iii) The speed may be varied according to power output desired, the head and discharge being kept constant. These conditions are not found in practice, but they are very useful in laboratories for testing power units.

In predicting the performance of a turbine under different operating conditions, the concept of *unit power, unit speed*, and *unit quantity* are introduced. These are the quantiteis when the head supplied is reduced to unity – one metre – the speed being adjusted to (i) above, so that the efficiency is normally unaffected.

If the head varies, it is assumed that the same efficiency is maintained by keeping the shape of velocity triangles unaltered, *i.e.* wheel and jet speeds are altered in the same proportion. In other words, the angles of jet and vane are kept the same.



(a) Inlet velocity triangle for head, *H*

(b) Inlet velocity triangle for head, unity

Fig. 10-46

In fig. 10-46, ABC represents the inlet velocity triangle under its working head, *H*, and abc represents velocity triangle for the same turbine for unit head. The subscript *u* (unity) is used to

denote unit conditions. Angles α and θ are same in both cases, and the two triangles become similar. Thus,

$$\frac{v}{V_w} = \frac{v_a}{V_{wa}} \text{ and } \frac{v}{V_i} = \frac{v_a}{V_{ia}} \quad \text{.. (10·15)}$$

We may further write,

$$\frac{\dfrac{V_w \times v}{g}}{\dfrac{V_i^2}{2g}} = \frac{\dfrac{V_{wa} \times v_a}{g}}{\dfrac{(V_{ia})^2}{2g}}$$

This shows that the ratio of work done to wasted energy is the same under unit head as it is under the working head. Theoretically, therefore, the condition of unchanged efficiency is established, *i.e.*

$$\eta = \frac{V_w v}{gH} = \frac{V_{wa}\ v_a}{g \times 1} \quad \text{..(10·16)}$$

Substituting values from eqn. (10·15)

$$\frac{V_w\ v}{H} = \frac{\left(v_a^2 \times \dfrac{V_w}{v}\right)}{1} = \frac{v_a^2 V_w}{v}$$

$$\left.\begin{array}{l} \therefore \quad v_a^2 = \dfrac{v^2}{H}, \ i.e.\ v_a = \dfrac{v}{\sqrt{H}} \\ \text{and similarly, } V_{ia} = \dfrac{1}{\sqrt{H}} V_i \end{array}\right\} \quad \text{.. (10·17)}$$

In order to reduce the performance of a turbine working under a head H, to conditions of unit head, all velocities must be reduced in ratio of $1/\sqrt{H}$. Or, in otherwords, in order to maintain the same efficiency under different heads, all velocities should be taken proportional to the square root of the given head.

Unit speed : The unit speed, N_u for a particular turbine is the speed with which it runs when running under unit head.

Since, linear speed, $v = \dfrac{\pi DN}{60}$,

$$v \propto N \text{ for given turbine dimensions}$$

$$\text{But, } v \propto \sqrt{H}$$

$$\therefore \quad N \propto \sqrt{H}$$

$$\text{or } N = k_1 \sqrt{H}$$

where, k_1 is a coefficient which will take into account varying conditions of running.

$$\text{When, } H = 1,$$

$$N = k_1 = \text{unit speed, } N_u$$

Hence, unit speed of a turbine, $N_u = k_1 = \dfrac{N}{\sqrt{H}}$ .. (10·18)

This means that unit speed, N_u of a turbine in revolutions per minute is the working speed N (actual rpm.) divided by $\sqrt{H}$.

Unit quantity : This is the discharge of water through the given turbine while operating under the unit head.

$$\text{Discharge,} \quad Q = a \times V$$

$$\therefore \quad Q \propto V \text{ (} a \text{ being constant)}$$

$$\text{But, } V \propto \sqrt{H}$$

$$\therefore \quad Q \propto \sqrt{H}$$

$$\text{or} \quad Q = k_2 \sqrt{H}$$

where, k_2 is a coefficient depending on the condition of running.

When $H = 1$, $Q = k_2$ = quantity, Q_u.

Hence, unit quantity, Q_u of a turbine is

$$Q_u = k_2 = \frac{Q}{\sqrt{H}} \qquad \text{.. (10·19)}$$

This means that the unit quantity, Q_u of a given turbine is the normal quantity in m^3/sec. flowing through the turbine divided by $\sqrt{H}$.

Unit power : The unit power, P_u of a given turbine is the power developed by the turbine when working under unit head.

Power, $P = wQH\eta_o$

$\therefore \quad P \propto QH$ (η_o being kept constant.)

But, $Q \propto \sqrt{H}$

$\therefore \quad P \propto \sqrt{H} \cdot H$

or $P \propto H^{\frac{3}{2}}$

or $P = k_3 H^{\frac{3}{2}}$

where, k_3 is a coefficient which will vary with gate opening and speed.

When $H = 1$,

$P = k_3 =$ unit power, P_u

Hence, unit power, P_u of a turbine is

$$P_u = k_3 = \frac{P}{H^{\frac{3}{2}}} \qquad \text{.. (10-20)}$$

This means that unit power, P_u of a given turbine is the power produced under unit head and is found by dividing the normal output, P by $H^{\frac{3}{2}}$.

Characteristic curves: The performance of a particular turbine under varying conditions of operation is represented by a group of graphs. These graphs (called characteristic curves) help to decide the optimum conditions of working of the turbine. When a turbine is tested under constant head and with fixed gate opening the relations between speed, efficiency, output and discharge are obtained and presented in fig 10-47 and fig. 10-48. These relations are obtained under various heads such as H_1, H_2 and H_3 etc.

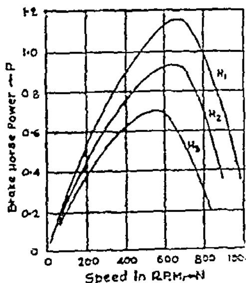


Fig. 10-47 Inward flow turbine characteristics

In order to have a complete picture of turbine performance, such graphs should be plotted with different gate openings. If the quantities such as speed, discharge and output were reduced to unit quantities by reducing the head to unity, it will be seen that under unit conditions, points corresponding to various heads lie fairly well along a single set of curves. In fig. 10-49 is obtained a single curve for each of the quantities, *viz*, power, discharge, and efficiency for a bunch of graphs of figures 10-47 and 10-48. A set of such curves can be drawn for each gate opening.

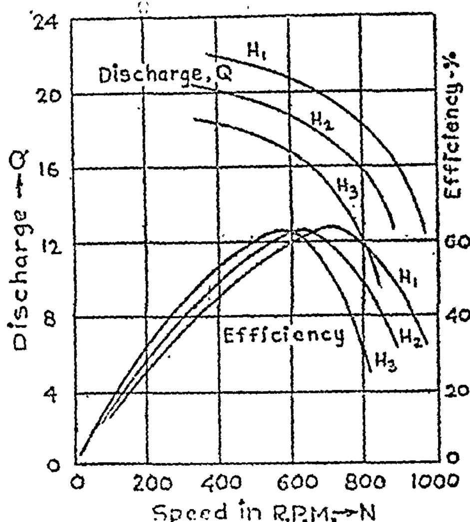


Fig. 10-48 Inward flow turbine characteristics

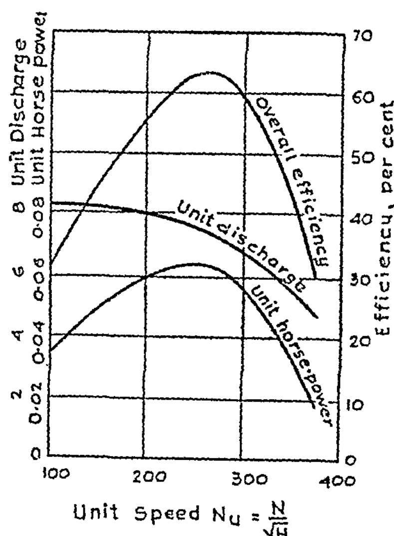


Fig. 10-49 Turbine performance reduced to unit head conditions

Fig. 10-50 shows curves of unit power, P_u on a base of unit speed, N_u for selected gate openings. On this chart, lines of constant efficiency are superimposed. A single point on this chart gives all necessary information. The line corresponding to maxi-

mum efficiency is introduced for various conditions of operation. From this, it can be deduced at a glance, what the speed of the turbine should be at any gate opening to yield maximum efficiency. The condition of operation, i.e. gate opening, speed and corresponding power developed can also be read for optimum efficiency.

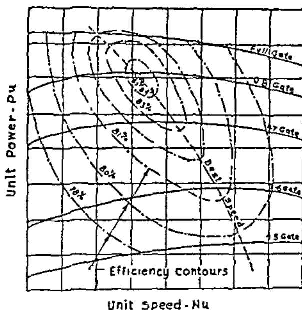


Fig 10-50 Characteristic curves of a turbine

Problem-29 : *The table gives some data relating to a turbine operating at 200 rpm. with full gate opening*

Head, m	*8·3*	*7·5*	*6·8*	*6·3*	*5·8*	*5·3*
H. P.	*357*	*310*	*270*	*236*	*205*	*176*
Efficiency, %	*81·1*	*83·1*	*84·1*	*84·8*	*85 0*	*84·1*

Draw the graphs of unit power and efficiency against unit speed and find how much water is required for power output under a head of 7 m.

From the tabular data the values of unit power, and unit speed are calculated as under.

In order to have a complete picture of turbine performance, such graphs should be plotted with different gate openings. If the quantities such as speed, discharge and output were reduced to unit quantities by reducing the head to unity, it will be seen that under unit conditions, points corresponding to various heads lie fairly well along a single set of curves. In fig. 10-49 is obtained a single curve for each of the quantities, *viz*, power, discharge, and efficiency for a bunch of graphs of figures 10-47 and 10-48. A set of such curves can be drawn for each gate opening.

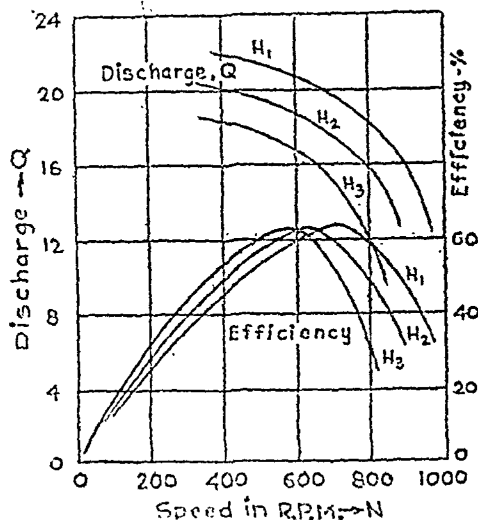


Fig. 10-48 Inward flow turbine characteristics

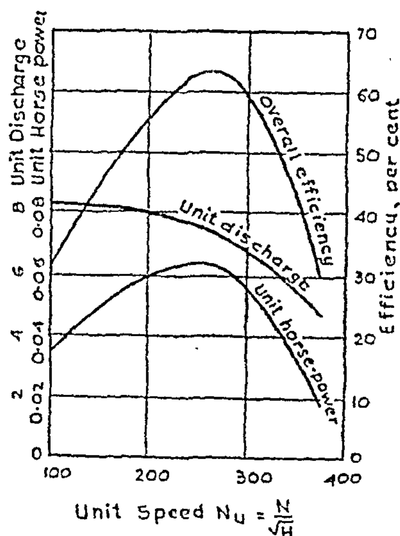


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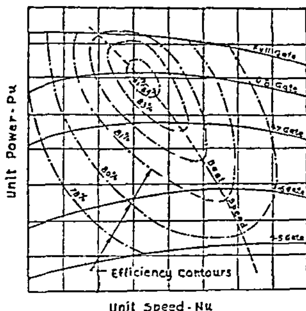


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Draw the graphs of unit power and efficiency against unit speed and find how much water is required for power output under a head of 7 m.

From the tabular data the values of unit power, and unit speed are calculated as under.

Head (H), m	8·3	7·5	6·8	6·3	5·8	5·3
H.P., (P)	357	310	270	236	205	176
$H^{\frac{3}{2}}$	23·92	20·45	17·73	15·81	13·96	12·2
unit power $P_u = \frac{P}{H^{\frac{3}{2}}}$	14·93	15·09	15·23	14·93	14·68	14·42
unit speed $N_u = \frac{N}{H^{\frac{1}{2}}}$	69·4	73	76·68	79·68	83	86·88
Efficiency %	81·1	83·1	84·1	84·8	85	84·1

The graph of unit power against unit speed is plotted in fig.10–51.

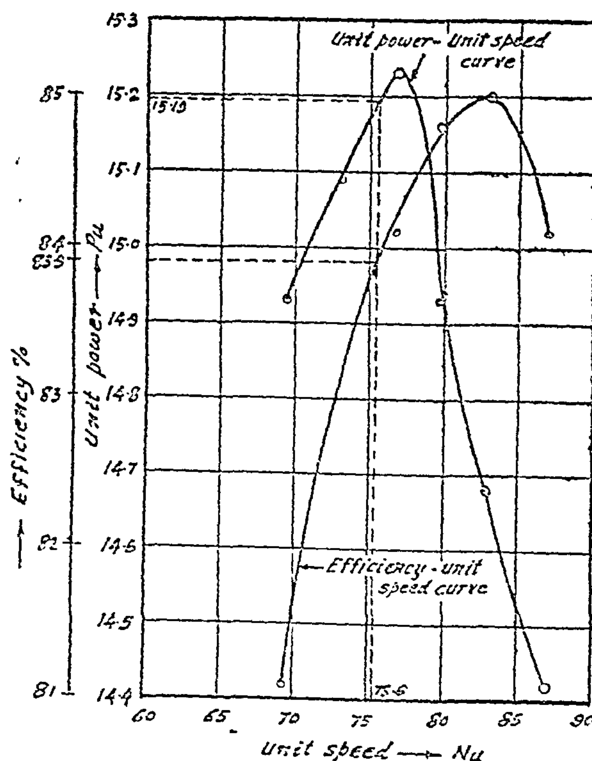


Fig. 10–51

Unit speed corresponding to 7 m head is

$$\frac{200}{\sqrt{7}} = 75.6 \text{ rpm.}$$

The corresponding value of unit power (from graph) is 15·19 H.P. Hence, power developed is,

$$P = P_u \times H^{\frac{3}{2}}$$
$$= 15.19 \times 7^{\frac{3}{2}}$$
$$= 281.5 \text{ H.P.}$$

The efficiency corresponding to power 281·5 H.P. is 83·9%. Hence, quantity of water required, Q is

$$\frac{P \times 75}{wH\eta} = \frac{281.5 \times 75}{1{,}000 \times 7 \times 0.839}$$
$$= 3.6 \text{ m}^3\text{/sec or } 3{,}600 \text{ litres/sec.}$$

Problem-30 : *The nozzle of a Pelton wheel is of 4 cm diameter. The head at the nozzle is 165 m. The coefficient of velocity for the nozzle, $C_v = 0.97$. Wheel diameter is 38 cm The relative velocity at exit is reduced to 0·85 times relative velocity at inlet Water is deflected through 165° in passing over the quckets.*

Calculate the unit power, and unit speed, if the blade speed is 0·46 of the jet speed.

Velocity of jet, $V_1 = C_v \sqrt{2gH}$
$= 0.97\sqrt{2 \times 9.81 \times 165}$
$= 55.2$ m/sec.

Thus, velocity of wheel, $v = 0.46V_1 = 0.46 \times 55.2$
$= 25.4$ m/sec.

Thus, $v = \frac{\pi DN}{60}$

$$\therefore \quad N = \frac{60v}{\pi D} = \frac{60 \times 25.4}{\pi \times 0.38} = 1{,}278 \text{ rpm.}$$

Now, from considerations of velocity triangles,

$V_{w1} = V_1 = 55.2$ m/sec.
$V_{r1} = V_1 - v = 55.2 - 25.4 = 29.8$ m/sec.
$\therefore \quad V_{r2} = 0.85V_{r1} = 0.85 \times 29.8 = 25.3$ m/sec.

$$\text{Now, } V_{w_2} = V_{r_2} \cos (180° - 165°) - v$$
$$= 25{\cdot}3 \cos 15° - 25{\cdot}4 = 25{\cdot}3 \times 0{\cdot}9659 - 25{\cdot}4$$
$$= -1{\cdot}05 \text{ m/sec.}$$

$$\text{Work done /kg} = \frac{(V_{w1} + V_{w2})\, v}{g}$$
$$= \frac{(55{\cdot}2 - 1{\cdot}05)\, 25{\cdot}4}{9{\cdot}81}$$
$$= 140 \text{ kg-m/kg}$$

$$\text{Quantity of water flowing} = \frac{\pi}{4} d^2 \times V_1 \times w$$
$$= \frac{\pi}{4}\left(\frac{4}{100}\right)^2 \times 55{\cdot}2 \times 1{,}000$$
$$= 69{\cdot}4 \text{ kg/sec.}$$

$$\therefore \text{ power developed, } P = \frac{\text{water flowing/sec.} \times \text{work done per kg}}{75}$$
$$= \frac{69{\cdot}4 \times 140}{75}$$
$$= 130 \text{ H.P.}$$

$$\text{Now unit power, } P_u = \frac{P}{H^{\frac{3}{2}}}$$
$$= \frac{130}{(165)^{\frac{3}{2}}} = 0{\cdot}062$$

$$\text{and unit speed, } N_u = \frac{N}{\sqrt{H}}$$
$$= \frac{1{,}278}{\sqrt{165}} = 99{\cdot}5$$

Specific Speed

Having studied how a turbine will behave under unit conditions a further step is necessary before we can arrive at an *equitable* basis for comparing turbines of different types. For this, we should establish the characteristics of an imaginary turbine, identical in shape, having same angles and operating on same gate opening as that for the actual turbine, but reduced to such a *size* that it will develop *one horse power* under *unit head.* This imaginary

turbine is called a *specific turbine* and the speed at which it runs is called the *specific speed* of the actual turbine. The two turbine runners under comparison are shown in fig. 10-52.

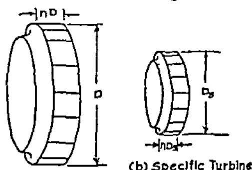


Fig. 10-52 Actual and specific runners

Within certain limits each type of turbine will have its own value of specific speed, thus, if the specific speed is known it is possible to know the type of turbine *i.e.* it serves as a method of classifying the turbines. The alteration in the speed of a given turbine to suit the unit head has been explained earlier. The process is now extended to include the alteration in size of the unit turbine to produce unit power. In altering the size, rotational speed must be altered simultaneously since linear speed depends on the head.

For a geometrically similar turbine with corresponding operating conditions (to given the same efficiency throughout), all the linear speeds will vary as $\sqrt{H}$.

i.e. peripheral speed, $v \propto \sqrt{H}$

$$\text{But, } v = \frac{\pi DN}{60}$$

$$\therefore \quad v \propto DN \propto \sqrt{H} \qquad \qquad \text{(10·21)}$$

Now, discharge, $Q = \pi D \times b \times V_f$

$= \pi D\,(nD)\,V_f$ where, width, $b = nD$

$$\therefore \quad Q \propto D^2 V_f \propto D^2 \sqrt{H} \qquad \qquad \text{(10·22a)}$$

For a Pelton wheel, diameter of jet bears a fixed ratio to the diameter of the wheel (fig. 10–53). Taking,

jet diameter, $d = pD$,

$$\text{discharge, } Q = \frac{\pi}{4} d^2 V \times \text{no. of jets}$$

$$= \frac{\pi}{4} (pD)^2 V \times \text{no. of jets}$$

$$i.e.\ Q \propto D^2 V \propto D^2 \sqrt{H} \qquad (10{\cdot}22b)$$

This is same for other runners also.

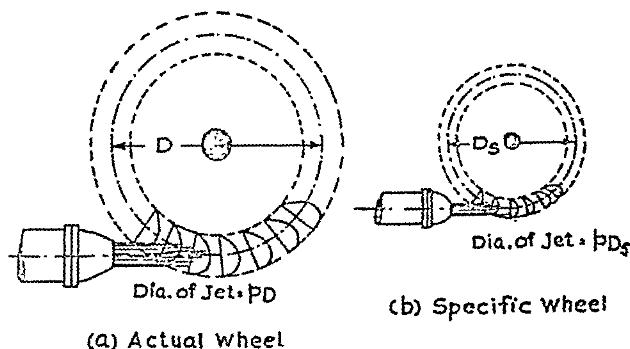


Fig. 10–53 Actual and specific Pelton wheels

Now, power, $P = wQH\eta_h$

$$i.e.\ P \propto QH$$

$$\propto D^2 \sqrt{H} \cdot H$$

$$\propto D^2 H^{\frac{3}{2}} \qquad ..(10{\cdot}23)$$

But, from relation (10·21), $D \propto \dfrac{\sqrt{H}}{N}$

Including this in relation (10·23), we have,

$$P \propto \left(\frac{\sqrt{H}}{N}\right)^2 \cdot (H)^{\frac{3}{2}}$$

$$\propto \frac{H^{\frac{5}{2}}}{N^2}$$

$$i.e.\ N^2 \propto \frac{H^{\frac{5}{2}}}{P}$$

$$\text{or } N \propto \frac{H^{\frac{5}{4}}}{P^{\frac{1}{2}}}$$

$$\therefore \quad N = \frac{k\ (H)^{\frac{5}{4}}}{\sqrt{P}} \qquad \text{.. (10·24)}$$

For a *specific turbine*, $H = 1$ and $P = 1$ and N takes the value of N_s, the specific speed.

$$i.e.\ N_s = k \times 1$$

$$\therefore \quad N = N_s \times \frac{H^{\frac{5}{4}}}{\sqrt{P}}$$

$$\text{or } \quad N_s = \frac{N\sqrt{P}}{H^{\frac{5}{4}}} \qquad \text{.. (10·25)}$$

Since, this expression is independent of diameter or any other linear dimension, specific speed, N_s has the same value for all geometrically similar turbines. It may be noted that $H = 1$ and $P = 1$ are really arbitrary values. A similar comparison of speed of different varieties of turbines may be made for any selected values of H and P.

Specific speed depends upon the flow area. A higher specific speed is obtained for turbines having greater flow area and greater vane inlet angle. Obviously, higher specific speed offers the possibility of generating larger bulk of power and hence cheaper installation.

Here is a general definition : *The specific speed of any turbine is the speed in revolutions per minute of a turbine geometrically similar to the actual turbine, but of such a size that under corresponding conditions it will develop one H.P. under unit head.*

Fig. 10-54 represents the graph of specific speed v_s working head for various types of turbines. Each point on the

graph corresponds to a turbine installation either successfully at work or at the manufacturing stage. It may be observed that it is impossible to find a Pelton wheel, a Francis turbine and a Kaplan turbine all working under the same head. On the other hand, there is a fairly definite range of heads and specific speeds allocated to each type of turbine. The data of the graph is summerized in the tabular form below for purpose of reference.

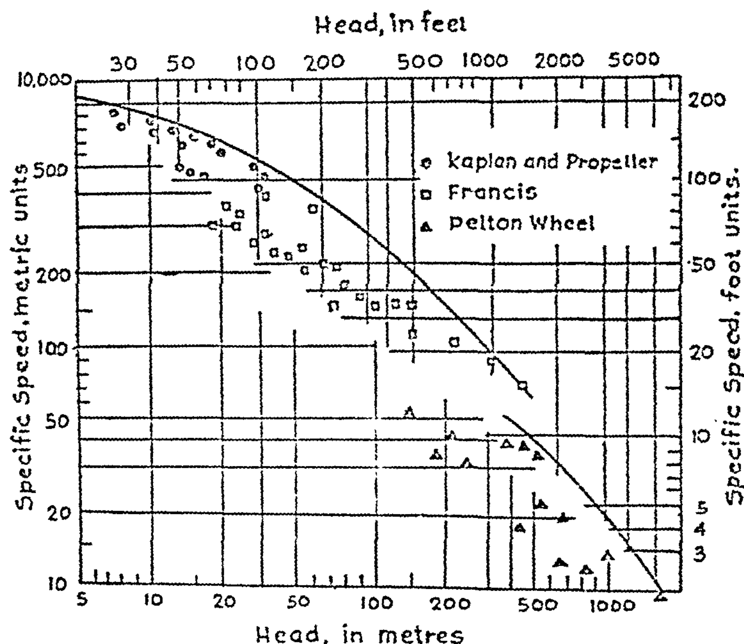


Fig. 10-54 Relation between specific speed and head

Type of turbine	Range of specific speed	Maximum head, m	Maximum output, B.H.P.	Maximum overall efficiency, (%)
Pelton	18-53	1,530	1,00,000	89
Francis	80-400	305	1,50,000	93
Propeller	400-900	30·5	70,000	93

Problem – 31 : *Define the 'specific speed' of a water turbine. Prove that for a single jet Pelton wheel, the specific speed in M.K.S. units is appoximately equal to 250 $\frac{d}{D}\sqrt{\eta}$, where η is the overall efficiency, 'd' is the diameter of the jet, in metre and D is the diameter of the bucket circle in metre, given also that the velocity coefficient for the nozzle is 0·98 and bucket speed is 0·46 times the speed of the jet.*

For a turbine

$$\text{specific speed, } N_s = N\frac{P^{\frac{1}{2}}}{H^{\frac{5}{4}}}$$

The velocity of bucket, $v = \frac{\pi DN}{60}$ m/sec.

$$\text{also, } v = 0{\cdot}46\ V_1 = 0{\cdot}46 \times 0{\cdot}98\sqrt{2gH}$$

$$\text{or } \frac{\pi DN}{60} = 0{\cdot}46 \times 0{\cdot}98\sqrt{2gH}$$

$$\therefore\quad N = \frac{0{\cdot}46 \times 0{\cdot}98 \times \sqrt{2g} \times 60}{\pi} \times \frac{\sqrt{H}}{D}$$

$$= \frac{38H^{\frac{1}{2}}}{D}\text{ rpm. } (g = 9{\cdot}81\text{ m/sec}^2)$$

Further, power output of a turbine is,

$$P = \frac{\frac{\pi}{4}d^2 \times 0{\cdot}98\sqrt{2gH} \times w \times H}{75} \times \eta$$

$$= 45{\cdot}2H^{\frac{3}{2}} \times \eta \times d^2\ (w = 1{,}000\text{ kg/m}^3)$$

$$\text{Hence } N_s = N\frac{P^{\frac{1}{2}}}{H^{\frac{5}{4}}}$$

$$= \frac{38H^{\frac{1}{2}}}{D} \times \frac{(45{\cdot}2H^{\frac{3}{2}} \times \eta \times d^2)^{\frac{1}{2}}}{H^{\frac{5}{4}}}$$

$$= 250\frac{d}{D} \times \sqrt{\eta} \quad \text{(M.K.S. units)}$$

Model Testing-Principle of Similarity

As discussed earlier, geometrically similar turbines, operating under similar conditions, will have identical efficiencies. The conditions which keep efficiency unaltered under charge of head and change of scale are : (i) all velocities must vary as $\sqrt{H}$, and (ii) hydraulic and mechanical losses must vary as H.

The principle of similarity is extended to predict the performance of a future design from the tests on a geometrically similar model. Testing of model also helps in deciding the best shape for blades and draft tubes.

For similar turbines, all the velocities such as V, v, V_r, V_f etc. should be proportional to $\sqrt{H}$.

$$\text{Hence, } v \propto \sqrt{H}$$

$$\text{but } v = \frac{\pi DN}{60}$$

$$\therefore \quad DN \propto \sqrt{H}$$

$$\text{or } D \propto \frac{\sqrt{H}}{N}$$

$$\text{i.e. } D = \frac{k_1 \sqrt{H}}{N} \qquad \text{.. (10·26)}$$

where k_1 is a constant.

Also discharge, $Q = \pi D b V_f$

$$\text{but } b \propto D \text{ and } V_f \propto \sqrt{H}$$

$$\therefore \quad Q \propto D^2 \sqrt{H}$$

$$\text{i.e. } Q = k_2 D^2 \sqrt{H} \qquad \text{.. (10·27)}$$

where k_2 is constant.

Now, power developed, $P = \frac{WH}{75} \times \eta_h$

$$\text{But, } W = wQ$$

$$\therefore \quad P \propto QH \propto D^2\sqrt{H} \cdot H \propto D^2 H^{\frac{3}{2}}$$

$$\text{or } P = k_3 D^2 H^{\frac{3}{2}} \qquad \text{.. (10·28)}$$

where, k_3 is constant.

Further by substituting the value of D from eqn. (10·26) in eqn (10·28),

$$P = k_4 \times \frac{H^{\frac{5}{2}}}{N^2}$$

$$\text{or } N = k_5 \frac{H^{\frac{5}{4}}}{\sqrt{P}} \qquad \text{.. (10·29)}$$

Hence, by measuring values of P, H, Q, N, and D for the model under test, the values of k_1, k_2, k_3, k_4 may be calculated. The values of these constants will also hold good for large (full size) turbine. From these known values of diameter and head under which it is to operate, its power, speed, and discharge can be estimated.

Adopting subscripts $_M$ and $_A$ for model and actual turbines, equations (10·27) to (10·29) can be rewritten as

$$\frac{Q_M}{Q_A} = \left(\frac{D_M}{D_A}\right)^2 \left(\frac{H_M}{H_A}\right)^{\frac{1}{2}} \qquad \text{.. (10·30)}$$

$$\frac{P_M}{P_A} = \left(\frac{D_M}{D_A}\right)^2 \left(\frac{H_M}{H_A}\right)^{\frac{3}{2}} \qquad \text{.. (10·31)}$$

$$\text{and } \frac{N_M}{N_A} = \left(\frac{H_M}{H_A}\right)^{\frac{5}{4}} \left(\frac{P_A}{P_M}\right)^{\frac{1}{2}} \qquad \text{.. (10·32)}$$

Scale effect : In order to maintain hydraulic efficiency the same for the model and the actual turbines, principle of dynamical similarity yields that the respective heads under which they operate should be inversely proportional to the square of their linear dimensions, *i e.* a model $\frac{1}{10}$th of the actual turbine size should be operated under a head hundred times as great. It is usually quite impracticable to test the model under such very high heads. In practice, models are tested under some convenient lower head and test results are then corrected for the *"scale effect"*. The efficiency of the model is estimated from the observed values of power, discharge and head and a suitable correction factor is then applied to the efficiency of the model and gross efficiency of the

actual or full-scale or proto-type turbine is obtained. The correction can be estimated with a sufficient degree of reliability.

The factors which decide the relationship between efficiency of model η_M and efficiency of actual turbine, η_A are : (i) hydraulic losses in model are higher than that in actual turbine as heads used do not confirm to dynamic similarity condition, (ii) the degree of roughness of blades and other passages may not be identical, and (iii) besides the hydraulic losses, losses such as mechanical, disc friction, leakage etc. may not be same.

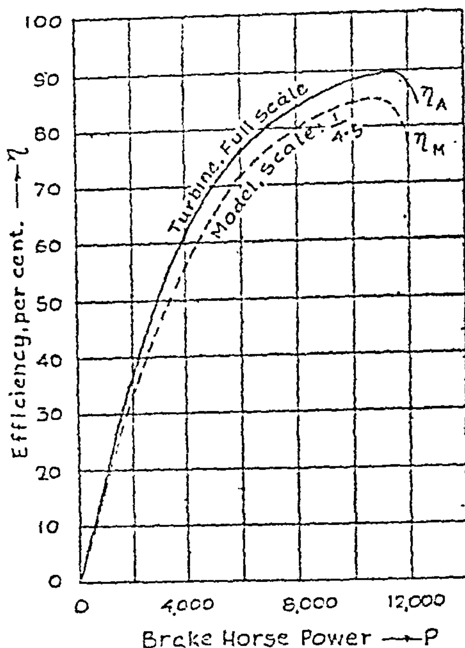


Fig. 10-55 Scale effect

In spite of the complexity of these relations, it was shown by Dr. Moody that η_M and η_A can be inter-related as under :

$$\frac{1-\eta_M}{1-\eta_A} = (n)^{\frac{1}{4}} \quad \text{.. (10·33)}$$

where, n = scale ratio = $\dfrac{\text{diameter of actual runner}}{\text{diameter of model runner}}$

Fig. 10·55 represents "scale effect" on gross efficiency for various loads.

Problem – 32 : *A turbine running at 250 rpm develops 5,000 H.P. under 12 m head. It is proposed to use the same design altered to suitable scale for a turbine giving 3,000 H.P. under 7·5 m head*

Estimate, (a) the scale ratio for the new machine and, (b) the design speed.

Using eqn. (10·26),

$$\frac{N_1}{N_2} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{2}} \times \left(\frac{H_1}{H_2}\right)^{\frac{5}{4}}$$

where, subscripts 1 and 2 represent original and new machine respectively.

$$\therefore \quad \frac{250}{N_2} = \left(\frac{3,000}{5,000}\right)^{\frac{1}{2}} \times \left(\frac{12}{7\cdot5}\right)^{\frac{5}{4}} = 0\cdot775 \times 1\cdot8 = 1\cdot395$$

$$\therefore \quad N_2 = \frac{250}{1\cdot395} = \underline{179\cdot3 \text{ rpm}}$$

Now, using eqn. (10·26),

$$\frac{D_1}{D_2} = \left(\frac{H_1}{H_2}\right)^{\frac{1}{2}} \left(\frac{N_2}{N_1}\right) = \left(\frac{12}{7\cdot5}\right)^{\frac{1}{2}} \left(\frac{179\cdot3}{250}\right) = 0\cdot908$$

$$\therefore \quad \text{scale ratio, } \frac{\text{diameter of original turbine}}{\text{diameter of new turbine}} = \underline{0\cdot908}$$

Problem - 33 : *A model of a turbine, built to a scale of $\frac{1}{6}$, when tested, developed 4·0 H.P. at a speed of 350 rpm under a head of 1·75 m. Estimate the equivalent speed and power of the full size turbine, when working under a head of 6 m. Allow for the scale effect on the gross efficiency. The model efficiency is found to be 0·75.*

Using eqn. (10·26),

$$\frac{D_M}{D_A} = \left(\frac{H_M}{H_A}\right)^{\frac{1}{2}} \left(\frac{N_A}{N_M}\right)$$

$$i.e. \ \frac{1}{6} = \left(\frac{1\cdot75}{6}\right)^{\frac{1}{2}} \left(\frac{N_A}{350}\right)$$

$\therefore$ speed of the full size turbine, $N_A = \frac{1}{6} \times 350 \times 1\cdot85 = \underline{108 \text{ rpm}}$.

Further, using eqn (10·28),

$$\frac{P_M}{P_A} = \left(\frac{D_M}{D_A}\right)^2 \left(\frac{H_M}{H_A}\right)^{\frac{3}{2}}$$

$$i.e. \ \frac{4}{P_A} = \left(\frac{1}{6}\right)^2 \left(\frac{1\cdot75}{6}\right)^{\frac{3}{2}}$$

$\therefore$ H.P. of the full size turbine, $P_A = 915$ H.P.

The above solution may now be modified to take into account the scale effect.

Using eqn. (10·33),

$$\frac{1-\eta_M}{1-\eta_A} = (n)^{\frac{1}{4}}$$

$$i.e. \frac{1-0.75}{1-\eta_A} = (6)^{\frac{1}{4}} = 1.565$$

∴ efficiency of the turbine, $\eta_A = 0.84$, *i.e.* 84%

The true output of the full scale turbine could, therefore, be expected to be about

$$\frac{915 \times 0.84}{0.75} = \underline{1,025 \text{ H.P.}}$$

Comparison of Impulse and Reaction Turbines

After having the foregoing analysis of turbines, the salient features of each type of turbine are worthwhile summarising to obtain a comparative idea about their fields of applications, design aspects and performance characteristics. This may also help to suggest the choice of a turbine for a given set of conditions.

(a) The peripheral velocity of the Pelton wheel for maximum efficiency is about $0.45\sqrt{2gH}$, while in a reaction turbine it varies from $0.65\sqrt{2gH}$ to $1.45\sqrt{2gH}$, depending on the design considerations. Consequently, Pelton wheel is more suitable for adoption for high heads without increasing the rotational speed.

(b) There is an optimum relationship between the diameter of the jet and the diameter of the wheel in the case of Pelton wheel. Moreover, more than two jets cannot be used efficiently on a single wheel. With lower head the diameter of the jet has to be increased in order to develop same power. This increase in jet diameter necessitates an increase in wheel diameter. Furthermore, the lower head is associated with lower jet velocity and hence, the peripheral velocity of the wheel should also be reduced to maintain the optimum ratio of wheel velocity to jet velocity. This necessity of

decrease in wheel velocity and increase in diameter combinedly decreases the *rpm* of the Pelton wheel Thus, if the head falls below 150 m, the Pelton wheel becomes very slow and too big in size. Therefore, the Pelton wheel is not suitable for heads below 150 m and its place is taken by relatively faster running and more compact reaction turbine.

(c) Efficiency of a Pelton wheel is more susceptible to changes of head than that of a reaction turbine. In practice, percentage change of head is greater in low head installations, where a reaction turbine is invariably used. Hence, the adoption of Pelton wheel for high head and of reaction turbine for low head fits in with this consideration too.

(d) In general, the efficiency of a reaction turbine is higher than that of a Pelton wheel. But, it may be observed that, the efficiency of the reaction turbine, with reduction in size, falls greatly. While the efficiency of a Pelton wheel is more or less independent of the size. Thus, other considerations permitting, Pelton wheel is preferable to a reaction turbine for small capacity installations.

(e) That Pelton wheel efficiency is higher than that of reaction turbine at part loads. The Pelton wheel is preferable where fluctuation of load is frequent

(f) Since the exhaust from Pelton wheel is atmospheric, whereas in a reaction turbine the exhaust is under vacuum, by use of a draft tube, the percentage net head available to the turbine is greater for a reaction turbine.

(g) Pelton wheel is simple in construction and parts requiring replacement are more accessible. Hence Pelton wheel is more suitable when the supply is taken from a river carrying silt and dirt. There may also be less wear of buckets because of slower speed.

Selection of a turbine : To sum up the above points, it can be said that, for very high heads, say over 300 m there is nothing

to choose. The Pelton wheel is the only suitable turbine. Between the range of 150 to 300 m, the choice between the Pelton wheel and the reaction turbine of the Francis type is determined by the considerations given below.

When the load is constant, both the turbines are equally good in respect of efficiency. The Francis turbine with its high specific speed and consequent reduced size may work out cheaper. A cheaper generator may be coupled and the building to house the generating set may be smaller. If the load fluctuation is greater, the choice will normally go to the Pelton wheel.

Again, for small size plant, it is preferable to use a Pelton wheel, because the efficiency of a reaction turbine is a function of the capacity. Moreover, there is a possibility of blades being worn out by erosion, if the water carries sand and silt, and the speed is higher. The Pelton wheel with its lower speed demands repair or replacement less frequently. Parts requiring repair or replacement are more easily accessible.

For low head projects, say from 25 to 3 m, propeller or Kaplan turbines are suitable. It is seen previouly, how the Kaplan turbine is preferable to the Francis turbine by virtue of its more compact construction and high speed involving less space, and smaller cost of generator and power house. The superior part load efficiency of the propeller turbine can be readily seen by referring to the efficiency curve. Its uniform efficiency over a wide range of load more than justifies initial high cost specially in an installation where the change of head is very frequent.

Between the propeller turbine and Kaplan turbine, the choice is governed by the head variations. If the head is farily constant, propeller turbine is chosen because it is cheaper. Normally in a low head plant, with a fairly constant head, a number of propeller turbines will be installed to take care of the constant base load. One or two Kaplan turbines will be paralleled to take care of peak loads. Thus a high efficiency is obtained at a reasonable cost.

The maximum head for which propeller or Kaplan turbines are used is limited by the setting in of cavitation. To prevent this, the turbine has to be placed at a lower setting including a further cost of excavation. Hence for heads, greater than 30 m, choice goes to the Francis turbine.

Cavitation in Turbine

Cavitation is a complex physical phenomena arising in a flow through the turbine which leads to the destruction of the rotating elements of the turbine. When obsolute pressure of water inside a pipe or machine falls below 3 m (of water), the air remaining dissolved in water separates out in the form of bubbles in which high pressure is concentrated when the bubbles burst, they exert very heavy force on the metal of the machine which is destroyed due to frequent hamering due to cavitation.

The surface of metal, destroyed by cavitation is shown in fig. 10-56. In water turbines the places getting affected by action of cavitation are : working surface of the moving blades, the vortex chamber of the axial turbines somewhat below axis of blades, the working wheel and the hub of the mixed flow turbine. The occurance of cavitation may lead to total break down of the turbine.

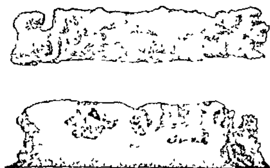


Fig. 10-56 Surface destroyed by cavitation

The surest way of preventing cavitation is to set the turbine sufficiently near the tail race level. If for the constructional reasons, it is necessary to work at a higher suction head than that

desired, then the runner blades may be saved from cavitation effect by choosing specially resistant metal. Stainless steel, nickel steel and bronze have proved satisfactory; or stainless steel armouring may be welded on to the areas most subjected to cavitation.

If may be observed that Pelton wheels are not free from cavitation. The nozzles, needles and buckets are all liable to corrosion.

TUTORIAL-10

Types of turbines

1 Differentiate critically an impulse turbine from a reaction turbine. Name few turbines of each type.

2 What are the different basis of classifying water turbines? Give complete specifications of a turbine you would like to order.

Radial flow impulse turbine

3 The peripheral velocity of the wheel of an inward flow turbine is 22 m/sec. The velocity of whirl of the inflowing water is 16 m/sec. and the radial velocity of flow is 3 m/sec. If the flow is 0·6 m³/sec and the hydraulic efficiency is 80%, find the head on the wheel, the H.P. of the turbine and drawing to scale the triangles of velocities, the inlet angle of the vanes. The outflow is radial.

[44·8 m ; 294 H.P.; 153°26′]

4 In an outward flow impulse turbine the diameters of the wheel at inlet and outlet are 0·5 m and 0·65 m respectively. Jet speed to wheel speed ratio is 0·4. The guide vane angle is 25°. Angle of moving blade at outlet is 20°. The velocity of water in the guide vanes is reduced by 5% of the theoretical velocity under the total head of 25 m. Water is supplied at the rate of 176 litres/sec. Calculate the H.P. developed, hydraulic efficiency, angle of the moving vane tip at inlet, and speed of rotation.

[41·5; 78·7%; 41°, 339 rpm.]

Axial flow impulse turbine

5 Describe the working, and method of governing of an axial flow Girard turbine.

In an axial flow Girard impulse turbine, the angle of the guide blade is 30° and the angle of the rotor vane is 25° at outlet. Find the hydraulic efficiency and the best speed of the turbine when discharge is axial. The available head is 36 m and the mean diameter of the rotor 125 cm.

[93·5%; 221 rpm.]

6 The lead-on angle of the guide vanes in an axial flow impulse turbine is 25°, the wheel vane angle at entrance is 30°, the head is 144 m. If velocity at exit is axial, and if the coefficient of velocity for the guide vanes is 0·98, determine the horse power developed when passing 0·24 m³/sec. and running under maximum efficiency conditions

[319 H P.]

7 The mean blade circle diameter of an axial flow impulse turbine is 1·5 m. The guide vane outlet angle is 25°, radial height of vanes 15 cm and velocity coefficient 0·97 The wheel vane outlet angle is 30° and the relative velocity at outlet is 80% of the relative velocity at inlet. At normal power, the head is 90 m and admission takes place over one tenth of the circumference. Determine the wheel speed and correct inlet vane angle, if outflow is axial. Evaluate the power and efficiency under these conditions, disregarding vane thickness effects

[226 rpm.; 42·1°; 1,070 H P; 74%]

Pelton wheel

8 Sketch the bucket of a Pelton wheel in two views and give its important dimensions in terms of jet diameter Why is the outlet angle of bucket tip not equal to zero ? Explain one of the method of fixing the buckets to the rim of the wheel.

9 Obtain an expression for the theoretical efficiency of a Pelton wheel when the tip of the bucket at exit makes an angle of ϕ

17 In an inward flow reaction turbine operating under a total head of 12 m, the guide vane discharge is inclined at 20° to the wheel tangent. At exit from the runner, pressure is atmospheric, discharge is radial with a speed 3·5 m/sec. and the radial flow velocity through the runner is constant. The ratio of inlet and outlet diameters is 3:2, and the wheel vane inlet angle is 90°. State the correct peripheral speed of the runner and outlet vane angle. Express the fluid friction losses in the machine as a percentage of the total head.

[9·63 m/sec.; 28·6° 16·06 %]

18 An inward flow reaction turbine having an overall efficiency of 75% is required to develop 175 H.P. The head is 9 m. Velocity of periphery of the wheel is $0{\cdot}95\sqrt{2gH}$ and the radial velocity of flow is $0{\cdot}35\sqrt{2gH}$. The wheel is to make 230 rpm. and the hydraulic losses in the turbine are 20% of the available energy. Determine, (a) the angle of the guide vane at inlet, (b) the wheel vane angle at inlet, (c) the diameter of the wheel, (d) the width of wheel at inlet. The turbine discharges radially.

[(a) 39° 32′; (b) 146° 24′; (c) 1·05 m; (d) 12·5 cm]

19 An inward flow reaction turbine has guide vane angle at inlet of 22°. The external diameter of the runner is 60 cm and the internal diameter 40 cm. The velocity of the flow through the runner is constant and the velocity of whirl at exit may be taken at zero. Calculate the angles of the moving vanes at inlet and exit, given that the kinetic head at entrance to the runner is half the total head. If the width of the runner at entrance is 7·5 cm and the effect of vane thickness is to reduce the circumference by 8%, calculate the speed of rotation of the runner and the H.P. developed under a supply head of 22 m. Neglect friction losses.

[100·8°; 29·33°; 467 rpm.; 197 H.P.]

20 An inward flow reaction turbine works under a total head H of 30·5 m. The centre line is 3·05 m above tail water level; the discharge from the wheel vanes is radial and the outlet

diameter is one half the inlet. The rim velocity at inlet is 0 $6V$, the radial velocity of flow at inlet is $0{\cdot}15V$, the velocity at inlet to the draft tube, $0{\cdot}10V$, where $V = \sqrt{2gH}$.

Assume there is no shock loss at inlet; loss in penstock and guide vanes is 4% of H; loss in runner passages is 7% of H, friction loss in the suction tube is 3% of H. Find, (a) the pressure head at inlet to and oulet from the runner, (b) suitable angles for runner and guide vanes, (c) the ratio of rim speed at inlet to water speed at inlet.

[(a) 10·2 m; 2·14 m; (b) 53° 59′, 18° 25′; 11° 57′; (c) 0·828]

21 Define ' degree of reaction ' and ' specific speed ' as applied to turbine.

An inward flow turbine has a degree of reaction of 0·6. The peripheral velocity of the blades at entry is 14 m/sec. and the velocity of flow is constant at 3 m/sec. The rotor diameter at entry is twice that at exit. Find the blade angles at entry and exit, assuming that water leaves the rotor without whirl and that friction losses may be neglected. Sketch the shape and arrangement of the blades.

[137° 5′, 23° 12′]

22 An inward flow Francis turbine having overall efficiency of 75% is required to develop 180 H P. at 120 rpm. The head H is 10 m and velocity of the wheel periphery is $3{\cdot}5\sqrt{H}$. Radial velocity of flow is $1{\cdot}1\sqrt{H}$. The hydraulic losses are 20% of the available energy, the discharge is to be radial and the diameter of the wheel at outlet is to be ⅔rd that at inlet. Determine, (a) the guide vane angle at inlet, (b) the wheel vane angle at inlet and outlet, (c) the diameter of the wheel, (d) the width of the wheel at the inlet and outlet assuming a blade coefficient of 0 09.

[(a) 26° 6′; (b) 143° 30′; 40° 17′;
(c) 1·76 m; (d) 10·25 cm; 8·56 cm;]

23 An inward flow reaction turbine runner has an external diameter of 1 m and its breadth at inlet is 25 cm. If the velocity of flow is uniform at the rate of 2 m/sec, find the

weight of water passing through the turbine per second. If the speed of the runner is 210 rpm. and guide vanes make an angle of 10° to the wheel tangent, find the magnitude and direction of the velocity with which the water enters the turbine rotor at inlet.

If the discharge of the turbine is radial at outlet and the inner diameter of the wheel is half of that at inlet, determine the vane angle at outlet. Find also the H.P. produced, hydraulic efficiency, and the head of water supplied to the turbine wheel.

[(i) 1,570 kg/sec, (ii) 2·03 m/sec, $\theta = 80° 21'$; (iii) 20°, (iv) 266·4 H.P. (v) 97%, (vi) 13·13 m]

Outward flow reaction turbine

24 In an outward flow reaction turbine supplied with 6·8 m^3/sec. and making 200 rpm., the diameters of the wheel are 2 m, and 2·5 m, respectively and the effective width of the wheel-face at inlet and outlet is 30 cm. The head on the wheel is 40 m. and the discharge is free and radial. Neglecting the thickness of the vanes and frictional losses, determine the angles of the vanes at entrance and exit, and sketch a vane showing these angles.

[124·6°; 6·55°]

25 In an outward flow reaction turbine, the rim speed is 304 rpm. The outward diameter is 80 cm and the ratio of the radii is 0·8. The turbine is placed 1 m below the water surface in the tail race and the wheel vane angles are 90° and 20° at inlet and outlet respectively.

The radial velocity of flow at inlet is 3 m/sec. Neglecting frictional losses and taking velocity of outflow as radial, find the guide vane angle, pressure head at inlet to the wheel, speed of flow from guides, total head and hydraulic efficiency.

[132°; 10·015 m of water gauge; 13·1 m/sec; 18·745 m of water; 90%]

26 An outward flow reaction turbine operates with 5,700 litres/sec. water supply. Its rotational speed is 210 rpm. The internal and external diameters of the wheel are 1·75 m and 2·4 m respectively. The width of the wheel at inlet and outlet is same and is 0·225 m. This turbine works under a head of 36·5 m. The turbine discharges radially in the atmosphere. Neglecting vane thickness effect and friction, compute the vane angles at inlet and exit of the moving vanes.

[82°; 7·2°]

27 An outward flow reaction turbine rotating at 250 rpm. has wheel size of 1·8 m and 1·5 m diameters. 32 vanes, 19 mm thick at inlet and 32 mm thick at outlet, are accomodated between discs 250 mm apart. The available head is 42 m above the centre of the wheel and discharge takes place into the atmosphere. The rate of water supply to the turbine is 6 m^3/sec. Determine the vane angles at inlet and outlet for water to leave the wheel radially without shock. Determine the guide vane inlet angle also.

[86·1°; 123°; 16 17°]

Axial flow reaction turbines

28 In a vertical reaction turbine water velocity measured perpendicular to the plane of motion of the wheel is 4 3 m/sec. at inlet and outlet from the runner, the latter velocity being purely axial. The guide vane outlet angle is 15° and the moving vane tips are curved back from the direction of motion of wheel through 118° and 165° at inlet and outlet respectively. The pressure head across the runner is 18 m and the exit edge is 0·6 m below the inlet edge. Construct the velocity diagrams and determine, (i) the work done per kg of water, and (ii) the head lost on friction in the runner.

[(i) 30·2 kg-m; (ii) 1·77 m]

29 In a parallel flow reaction turbine (Jonval type) the shaft axis is vertical and water flow is downward The vertical depth of guide and wheel passages is 23 cm in each case The mean diameter is 1·8 m. The guide vane outlet angle is 30° and vane

thickness coefficient at entrance 0·88. The wheel vane outlet angle is 25° and the thickness coefficient 0·90. The radial length of wheel and guide passages is 30 cm. The loss in the guides is 8% of the velocity head developed therein, and the loss in the wheel is 8% of the total head. The wheel speed is 90 rpm. and the final discharge is axial. Determine the wheel vane inlet angle, the pressure drop across the wheel, head at inlet to guides (if pressure at outlet is atmospheric), power developed, and hydraulic efficiency.

[110° 3′; 3·93 m; 7·44 m; 485 H.P.; 81·5%]

30 What is the function of a draft tube? What are the different types of draft tubes and what are their advantages and disadvantages over one another?

31 Explain why the angle of taper of a draft tube is limited.

A turbine unit is fitted with a conical draft tube. The velocity of water at exit from the runner is 9·8 m/sec. The total loss of head (including friction, eddy and leaving losses from the draft tube) should not exceed 1·5 m of water. The velocity of water at outlet of draft tube is 2·5 m/sec.

If cavitation commences when the absolute pressure of water falls below 2·5 m of water, estimate the maximum height above the tail race at which the turbine can be installed.

[4·42 m]

32 Explain how the suction height of turbine above tail race level is limited by cavitation

The following data refers to an elbow type draft tube:

Area at inlet	25 m^2
Area at outlet	116 m^2
Inlet velocity of water	10 m/sec.
Height of turbine outlet from tail race level	0·6 m
Efficiency of the tube	70 %

Estimate, (a) pressure head at inlet, (b) power wasted in draft tube, and (c) power carried away by the outgoing water.

[4·19 m (vacuum); 4,300 H.P.; 786 H.P.]

Performance characteristics

33 Define the terms :

(a) Unit speed, (b) Unit power, and (c) Unit quantity.

Explain their utility with reference to performance plotting of a turbine.

34 Describe the method of preparation of the characteristic diagram of a turbine. How does this diagram help in giving the results, when operating conditions are changed?

35 Develop the formulae $\frac{P}{H^{\frac{3}{2}}}$ and $\frac{N}{H^{\frac{1}{2}}}$ for unit power and unit speed respectively of a turbine.

Why is unit power plotted against unit speed for a turbine when errected on site, rather than a curve of power against speed?

The table gives some data relating to a turbine operating at 200 rpm. with full gate opening :

Head in m	7·5	6·9	6·3	5·75	5·3	4·9
Horse power	357	310	270	236	205	176
Efficiency	81·1	83·1	84·4	84·8	85·0	84·1

Draw graphs of unit power and efficiency against unit speed and find how much water is required per second for maximum output under a head of 6·4 m

[3,930 litres/sec.]

Specific Speed

36 What do you understand by the term *specific speed* of a water turbine ?

Deduce an expression for the specific speed of a water turbine in terms of the H.P. developed, the operating speed, and the head available.

37 Indicate how the form of a reaction turbine depends upon specific speed.

38 Classify the hydraulic turbines in terms of specific speed, and state how the diameter of runner varies with specific speed.

39 Prove that for a Pelton wheel specific speed is inversely proportional to the ratio of jet diameter to wheel diameter.

40 Determine the H.P. of a machine suitable for a head of 150 m, the quantity available being 150 litres/sec. If the speed is to be 375 rpm., what type of turbine would be used ?

[1,700 H.P. ; $N_s = 29{\cdot}4$; Pelton wheel may be used]

41 Define the "specific speed" of a water turbine. Prove that for a single jet Pelton wheel the specific speed in metric units is approximately equal to 256 $\frac{d}{D}\sqrt{\eta}$, where η is the overall efficiency, d is the diameter of the jet, and D is the diameter of the bucket circle. Given also that the coefficient for the nozzle is 0·98 and the bucket speed is 0·46 time the speed of the jet.

42 A Pelton wheel is working under the following conditions : Diameter of nozzle = 12·5 mm, diameter of the brake wheel = 300 mm, net load on brake = 6 kg ; speed = 538 rpm. ; weight of water used per minute = 150 kg ; coefficient of velocity, $C_v = 0{\cdot}98$.

Determine the specific speed and efficiency of the Pelton wheel.

[9·2 ; 91%]

43 In a Francis turbine, the speed ratio is 0·96 and the flow ratio is 0·3. The breadth of the wheel at outer periphery is $\frac{1}{6}$th of the corresponding diameter. If the efficiency of the turbine is 80%, determine the specific speed in metric units.

[220]

44 Calculate the specific speed of a Francis turbine runner in which the speed ratio is 1·05, the flow ratio is 0·25, and the width of runner at the outer periphery is $\frac{1}{4}$ of the outer diameter. The mechanical efficiency may be taken as 0·80.

45 Calculate the diameter, the speed and the specific speed of a Kaplan turbine runner to develop 8,500 B. H. P. under a head

of 4·9 m, having given that speed ratio based on outer diameter is 2·1 ; flow ratio is 0·65, diameter of boss is 0·35 × external diameter of the runner, mechanical efficiency is 88%.

[5·86 m; 63·6 rpm ; 791]

Model testing

46 Show that in a given turbine the peripheral speed of the runner for maximum efficiency is proportional to $\sqrt{H}$, where H is the available head, and that under these conditions the quantity of water consumed is proportional to $\sqrt{H}$, and the power developed is proportional to $H^{\frac{3}{2}}$.

Hence, show how the performance of a turbine may be predicted from that of a geometrically similar model.

Indicate the importance of model testing.

47 A turbine develops 8,000 H.P. under a head of 25 m at 150 rpm. Calculate the specific speed and its normal speed and output under a head of 20 m.

[240 ; 134 rpm. ; 5,650 H.P.]

48 A model turbine of 0·25 m diameter running at 900 rpm at $\frac{3}{4}$ gate opening has 80% efficiency when working under a head of 20 m. The output of the model was 40 B.H.P. A similar turbine is to be designed to develop 500 B H P at $\frac{3}{4}$th gate opening under a head of 25 m. Calculate its diameter and speed of motion. Determine the specific speed of the model.

[0·745 m ; 337 rpm ; 135]

49 A model of a Francis turbine was tested in a laboratory and it was found that it developed 8 B.H P. at a speed of 360 rpm. under a head of 3 m. A full size turbine having its rotor 6 times the rotor of the model is working under a head of 12 m. Calculate its speed, power and specific speed.

[120 rpm ; 2,304 H.P. ; 258]

50 A model prepared to a scale of $\frac{1}{6}$th of the full size turbine gave the following results when tested under a head of 8 m. Speed 360 rpm., discharge 520 litres/sec., power 40 H.P.

If the actual turbine is to run under a head of 40 m, estimate the speed, discharge and H. P. developed.

[322 rpm. ; 7,300 litres/sec. ; 2,800 H.P.]

51 A turbine has a runner of 0·8 m diameter and it runs under a head of 50 m when developing 250 H.P. Its specific speed is 40. Calculate the speed at which the turbine is running. A model is prepared to a scale of 0·6 to work under a head of 30 m. Determine the H.P., speed and unit speed of the model.

[334 rpm ; 41·84 H.P. ; 432 rpm. ; 79]

52 A turbine running at 250 rpm. gives 5,000 H.P. under 12 m head. It is proposed to use the same design, altered to a suitable scale for a turbine giving 3,000 H.P. under 7·5 m head. Calculate : (a) the scale ratio for the new machine, (b) the design speed. Derive any formula used, explaining carefully the basic assumptions.

[(a) 0·9:1 ; (b) 179 rpm.]

53 A model of a Kaplan turbine is constructed to a scale of 1:10. The actual turbine is to develop 10,000 H.P. when working under a net head of 9·15 m and running at 100 rpm. The model is to be tested under a head of 6·1 m. If the probable efficiency of the turbine and model is 88%, estimate the speed, discharge and H.P. of the model. Calculate also the specific speed of the model.

[816 rpm. ; 0·763 m^3./sec ; 54·2 H.P., 625 rpm.]

54 A propeller turbine is to develop 1,000 H.P. using a head of 2·5 m and its runner is to be geometrically similar to a type of runner 1 m diameter, which develops 25 H.P. when running at 132 rpm. under a head of 1 m. Determine the speed, diameter, and specific speed of the runner required and compare the quantities of water used. If the head were increased to 4 metres, what would be the actual speed of the runner in order to maintain the same efficiency as before ? Neglect the scale effect.

[65·5 rpm. ; 3·18 m ; 660 ; 15·9:1 ; 83 rpm.]

Selection of turbines

55 Explain the factors which decide the choice for a particular turbine for a given project.

56 State the ranges of working conditions most suitable for (a) Pelton wheel, (b) Francis turbine, and (c) Kaplan turbine.

57 Deduce an expression for the specific speed of a reaction turbine.

Under a head of 12 m the maximum specific speed is 460. A particular project requires 20,000 H.P. to be developed under a head of 12 m. If the speed of the turbine is 150 rpm., determine the number of units required.

[5 units]

Governing

58 Describe how the speed of a Pelton wheel can be maintained constant at different loads ? Explain the superiority of speed regulation by deflector over that by spear

59 Describe with neat sketches suitable method of governing a large Pelton wheel.

Show how a sudden increase in pressure in the supply main may be prevented

60 Explain with sketches the governing of a reaction turbine, showing in detail the part played by the floating fulcrum.

61 Sketch the arrangement for adjusting the guide blades in a reaction turbine.

62 Describe with neat sketches an arrangement for the governing of a Kaplan turbine by adjustment of runner blades.

63 Explain the working of pressure relief valves. Where are they used ?

Cavitation

64 What is cavitation ? When does it occur ? and what is its effect ?

65 What are the materials used for Francis turbine runners and why are they chosen ?

66 Why is the upper part of a draft tube lined with metal ?



CENTRIFUGAL PUMPS

Introduction

A pump is a machine which, when driven by power from an external source, lifts water or any other fluid from lower level to higher level or in other words, increases the pressure of the fluid, *i e.* it receives mechanical energy and raises the potential energy of the fluid. Hence, the pump is just the reverse of hydraulic prime-mover which generates mechanical energy by utilizing potential energy of the fluid or water. There are two main types of pumps: *centrifugal* type and *reciprocating* or *positive displacement* types, the former type only is dealt with in this chapter.

Centrifugal pumps are also known as *dynamic pressure generators,* as they are essentially machines for generating dynamic pressure. When they are placed in a pipe line, due to the centrifugal action they impart pressure energy to the fluid and this increase in pressure is responsible for change in pressure gradient and hence increased flow of fluid. The rate of fluid flow depends on the resistance offered to the flow by the pipe line, *i.e.* if the resistance is more, rate of flow is reduced and vice-versa. Thus the discharge from a centrifugal pump is directly influenced by the resistance or pressure against which it works.

Centrifugal pumps are similar in outward appearance, and in respect of construction they are apparently identical with mixed flow inward reaction turbines. They may be regarded indeed, as reversed reaction turbines. The direction of rotation of the shaft may remain the same but, the direction of flow of fluid is reversed.

In a turbine, high pressure water is allowed to enter the casing and mechanical energy is given out at the shaft; while in the pump, mechanical energy is fed into the shaft and pressure energy is increased for the outgoing fluid. Water enters axially in the central opening of the pump and flowing over the moving vanes in the impeller, leaves it radially. The guide vanes of the turbine serve in the pump as diffuser to reduce the exit velocity of fluid. In most pumps, the diffuser is replaced by suitable shaped spiral collecting passages around the impeller.

Construction and Working

The very word 'centrifugal' defines the action of the pump. The motion of water in it is from centre towards the periphery. This motion is caused by the centrifugal force, created in the pump as a result of the revolving motion of the working wheel

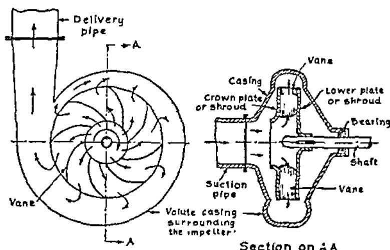


Fig. 11-1 Centrifugal pump

The impeller is made from a strong metal and consists of a number of vanes fixed on usually one side of the disc. Diameter of the wheel is 125 to 900 mm and more. The vanes are held

between two discs and in one disc there is a hole in which the shaft rod is fitted and on the other side there is a wide circular opening. As a result of such construction, passages (canals) are formed between the vanes, which are widening towards the outer rim (fig. 11-1). The impeller is housed in a specially shaped chamber which has the shape of the spiral casing with its area increasing.

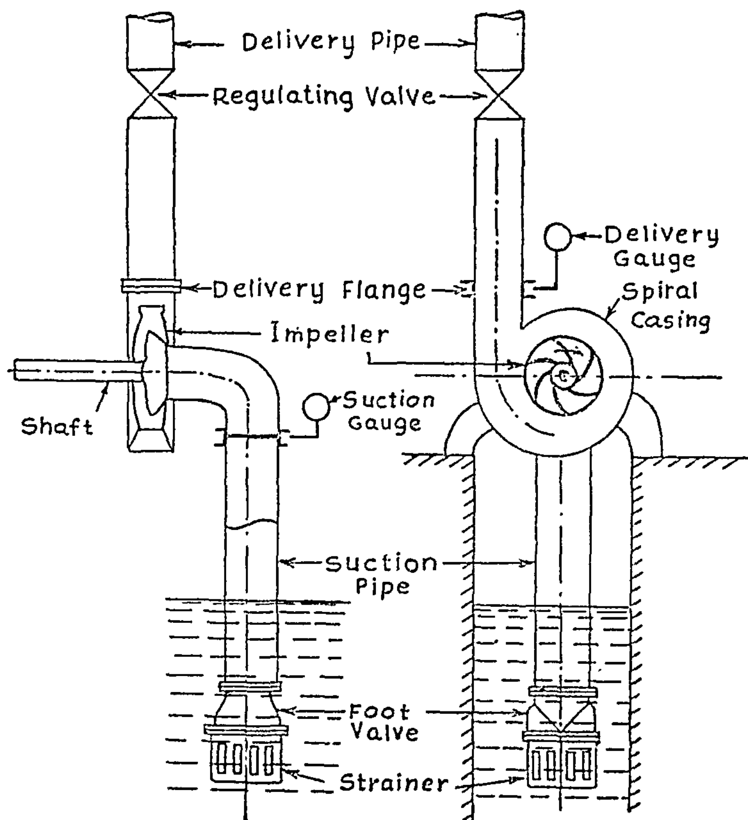


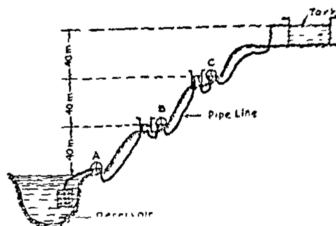
Fig. 11-2 Arrangement of the centrifugal pump

To the suction tube is attached a long vertical pipe at the end of which is fitted a strainer which prevents the solid bodies and debries from entering the pump. These things will cause wear of the impeller, if they are not prevented by the strainer. Above the

strainer a one way valve (foot valve) is fitted which keeps the suction pipe filled with water when the pump is stopped (fig. 11-2). This arrangement effectively simplifies the operation of starting the pump, as the wheel by its rotation cannot create vacuum, which is essential for the fluid to rise. This otherwise needs the priming of the pump by filling it manually with the fluid.

To the delivery end of the pump, the delivery pipe is fitted and very near to the pump a regulating valve is provided on this pipe. Besides this, a non-return valve may also be fitted which may automatically cut out the delivery pipe from the pump in case the pump stops, and may not permit water to flow back to the pump. This valve will be opened by itself once the correct direction of fluid flow is established. This valve is useful also when the pump is to be inspected or repaired. On the suction pipe a vacuum gauge and on the delivery pipe a manometer (pressure gauge) are usually fitted. They serve to indicate the pressures during the working of the pump In the pump which is just now described and which has entry from one side only, the shaft has one bearing with one pedstal

Multi-stage pumps are employed for obtaining very high heads as in feed pumps of boilers. The impellers are mounted on



a common shaft and are enclosed in a common housing. Water successively flows from one to another impeller, gaining higher and higher head developed by each impeller. If the impellers are similar, the total head, $H = nH_1$ where n is number of impellers and H_1 is head developed by one impeller. This is explained in another fashion by diagram of fig. 11–3.

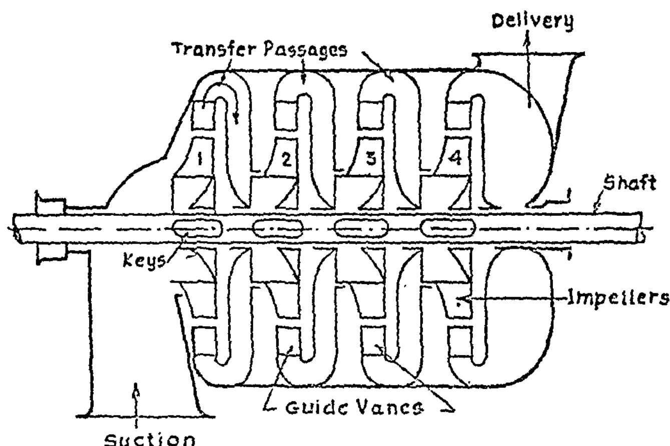


Fig. 11-4 Multi-stage pump

Suppose fluid to be pumped from reservoir to a tank situated at a vertical height of 120 m above the level A. A single impeller pump can raise water through 40 m. If follows that two additional pumps B and C are required to be installed. Thus the three pumps A, B, C will work simultaneously. Each of them will raise water through 40 m. This can, however, be simplified by combining the three units into a single one, by mounting the three impellers on a single shaft and all external energy supplied at point A only. In other words, it is necessary to install a 3-stage pump at A which develops the head of 120m and it will lift water through head of 120 m to the tank.

Multi-stage pumps can develop head upto 700 m with discharge of 8 to 80 litres/sec with 2 to 10 impellers. When it is necessary to increase the discharge of the pump without changing the head, it is advisable to run the pumps in parallel. The total discharge, $Q = nQ_1$ where Q_1 is the discharge of one pump (fig. 11-5).

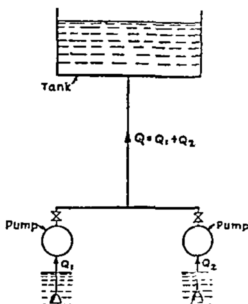


Fig. 11-5 Pumps in parallel operation

The action of the pump can be understood in the following manner : The impeller, having received the rotation from the prime mover throws the liquid towards the periphery (*i.e.* towards the inner surface of the housing), forming in the central parts of the impeller vaccum conditions. As a result, the liquid under the pressure of atmosphere enters, through the suction pipe, to the pump, in which it attains a complex motion. The schematic representation of the liquid can be obtained in this way. In the

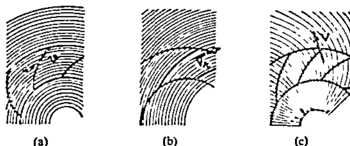


Fig. 11-6 Flow pattern in the impeller

beginning, when water enters the wheel, it moves with velocity v, (fig. 11-6a) parallel to the axis of the shaft. Further, liquid deviates

in the radial direction, and moving along the blade passages, from one side, moves along with the wheel with peripheral velocity, V_r along the blade (fig. 11-6 b). As a result of the combination of these velocities, the absolute movement of the liquid will take place with velocity V (fig. 11-6 c).

Theory or Fundamental Equations of the Pump

As the centrifugal pump is the reversed form of the inward flow turbine, the relationship between the head generated, the impeller speed and blade angles, and the discharge can be worked out in the similar manner. The notations adopted are same as those for turbines. Applying Bernoulli's equation to the fluid at sections where it enters and leaves the impeller we have,

$$\frac{p_1}{w} + Z_1 + \frac{V_1^2}{2g} + \text{W.D./kg} - \text{losses/kg} = \frac{p_2}{w} + Z_2 + \frac{V_2^2}{2g}$$

$$\therefore \quad \text{W.D./kg} = \frac{p_2 - p_1}{w} + Z_2 - Z_1 + \frac{V_2^2 - V_1^2}{2g} + \text{losses} \qquad \ldots(11{\cdot}1)$$

The total energy on the exit is increased by the equivalent energy of mechanical work spent and decreased by the energy lost on friction, etc.

But, work done per kg of fluid pumped is

$$\text{W.D. per unit weight} = \frac{1}{g}\left(V_{w_2}v_2 - V_{w_1}v_1 \right) \qquad ..(11{\cdot}2)$$

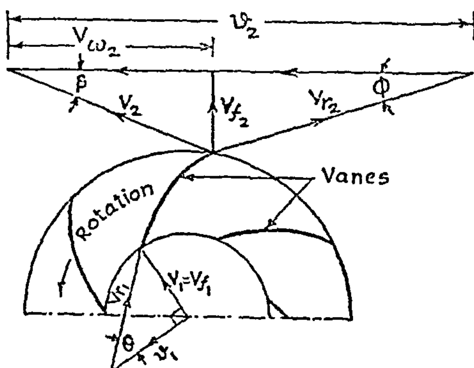


Fig. 11-7 Velocity diagram for the pump

Using velocity triangle relationship (fig. 11-7), we have,

$$V_{r2}^2 = V_{f2}^2 + (v_2 - V_{w_2})^2$$
$$= V_{f2}^2 + v_2^2 + V_{w2}^2 - 2v_2 V_{w_2}$$
$$= V_2^2 + v_2^2 - 2v_2 V_{w_2} \text{ as } V_{f2}^2 + V_w^2 = V_2^2$$

$$\therefore \quad V_{w_2} v_2 = \tfrac{1}{2}\{v_2^2 - V_{r2}^2 + V_2^2\} \qquad ..(11\cdot3)$$

Similarly, from inlet velocity triangle,

$$V_{w_1} v_1 = \tfrac{1}{2}\{v_1^2 - V_{r1}^2 + V_1^2\} \qquad ..(11\cdot4)$$

$\therefore$ substituting from eqns. (11·3) and (11·4) in eqn. (11·2), we have,

$$\text{W.D./kg} = \left(\frac{v_2^2 - v_1^2}{2g}\right) - \left(\frac{V_{r2}^2 - V_{r1}^2}{2g}\right) + \frac{V_2^2 - V_1^2}{2g} \qquad ..(11\cdot5)$$

In this expression, the bracketed term is the pressure head rise in the impeller and the last term is the increase in absolute kinetic energy head (the mechanical energy is converted into equivalent pressure energy head and K.E. head)

From eqns. (11·1) and (11·5) together, we have,

$$\frac{p_2 - p_1}{w} = \frac{v_2^2 - v_1^2}{2g} - \frac{V_{r2}^2 - V_{r1}^2}{2g} - (h_2 - h_1) - \text{losses} \qquad ..(11\cdot6)$$

where $h_2 = Z_2$ and $h_1 = Z_1$

This is the turbine equation proper multiplied by (– 1). In eqn. (11·6) the term $\frac{v_2^2 - v_1^2}{2g}$ is termed the vortex head, term $\frac{V_{r2}^2 - V_{r1}^2}{2g}$ the relative velocity head, and term $(h_2 - h_1)$ is the gravity head which, being small, is neglected.

Neglecting losses and gravity head eqn. (11·6) can be written

$$\text{as } \frac{p_2 - p_1}{w} = \frac{v_2^2 - v_1^2}{2g} - \frac{V_{r2}^2 - V_{r1}^2}{2g} \qquad ..(11\cdot6\,a)$$

The pump efficiency depends, to a great extent, on the conversion of the kinetic energy back to the pressure head in the volute chamber or the diffuser. It is usually assumed in calculations that V_1 is radial, *i.e.* water enters the impeller in the radial direction. Any deviation from the radial direction is due to the impeller and must be credited to it. It follows that,

$$V_{w_1} = 0$$

in the radial direction, and moving along the blade passages, from one side, moves along with the wheel with peripheral velocity, V_r along the blade (fig. 11-6 b). As a result of the combination of these velocities, the absolute movement of the liquid will take place with velocity V (fig. 11-6 c).

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But, work done per kg of fluid pumped is

$$\text{W.D. per unit weight} = \frac{1}{g}\left(V_{w_2} v_2 - V_{w_1} v_1 \right) \qquad ..(11 \cdot 2)$$

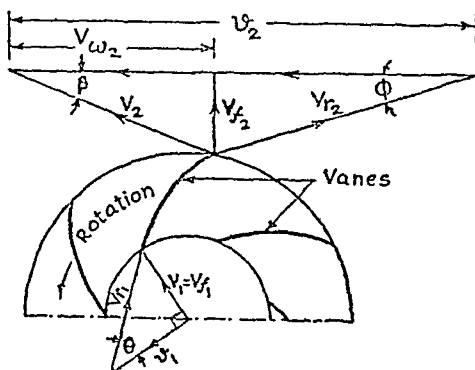


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$$= V_2^2 + v_2^2 - 2v_2 V_{w2} \text{ as } V_{f2}^2 + V_{w}^2 = V_2^2$$

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$$\text{W.D./kg} = \left(\frac{v_2^2 - v_1^2}{2g} - \frac{V_{r2}^2 - V_{r1}^2}{2g}\right) + \frac{V_2^2 - V_1^2}{2g} \qquad ..(11\text{·}5)$$

In this expression, the bracketed term is the pressure head rise in the impeller and the last term is the increase in absolute kinetic energy head (the mechanical energy is converted into equivalent pressure energy head and K E. head)

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$$\frac{p_2 - p_1}{w} = \frac{v_2^2 - v_1^2}{2g} - \frac{V_{r2}^2 - V_{r1}^2}{2g} - (h_2 - h_1) - \text{losses} \qquad ..(11\text{·}6)$$

where $h_2 = Z_2$ and $h_1 = Z_1$

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The pump efficiency depends, to a great extent, on the conversion of the kinetic energy back to the pressure head in the volute chamber or the diffuser. It is usually assumed in calculations that V_1 is radial, *i e.* water enters the impeller in the radial direction. Any deviation from the radial direction is due to the impeller and must be credited to it. It follows that,

$$V_{w1} = 0$$

$$\text{and W.D./unit weight} = \frac{1}{g} V_{w_2} v_2$$

If the pump runs with zero discharge,

$$\frac{p_2 - p_1}{w} = \frac{v_2^2 - v_1^2}{2g} - \text{losses} \quad \ldots \ldots \ldots \ldots (11{\cdot}7)$$

This means that if the pump is to start delivering the fluid, the pressure developed by it must exceed the static lift, H of the pump. Thus, the least theoretical speed for flow to commence will be when

$$H + \text{losses} = \frac{v_2^2 - v_1^2}{2g}$$

$$\text{or} \quad \frac{H}{\eta} = \frac{v_2^2 - v_1^2}{2g} \qquad \text{where } \eta \text{ is efficiency,}$$

$$\text{or} \left(\frac{\pi D_2 N}{60}\right)^2 - \left(\frac{\pi D_1 N}{60}\right)^2 = \frac{2gH}{\eta}$$

$$\therefore \quad N = \frac{60}{\pi}\sqrt{\frac{2gH}{(D_2^2 - D_1^2)\eta}} \quad \ldots \ldots \ldots \ldots (11{\cdot}8)$$

This equation may be used to determine the pump speed at which delivery will just commence.

Apparatus for Converting Kinetic Energy into Pressure Energy

In order to increase the efficiency of the pump, major part of the excessive kinetic energy generated by the impeller should be converted into pressure energy before water enters the delivery pipe. The velocity of water, V_d in the delivery pipe is generally limited to 1–2 m/sec. The various devices adopted for converting kinetic energy into pressure energy of water once it leaves the impeller are :

(i) Volute casing (chamber) or spiral casing gradually increasing in cross sectional area,

(ii) Vortex or Whirlpool chamber,

(iii) Guide vanes, and

(iv) Conical diffuser (connecting the pump outlet to the delivery pipe).

In practice, the pumps are fitted with one or more of the above stated devices depending upon the operational conditions of the pump.

According to the type of diffuser fitted to the pump, the pumps are classified as *turbine pump* or *spiral* (*volute*) *pump*. In the turbine pump, before liquid enters the spiral casing, or passes on to the successive impellers, it flows through the stationary guide vanes *similar to those in the turbine*. In the spiral pump, the liquid leaving the impeller directly enters the spiral casing and then flows in the delivery pipe.

Volute casing : The impeller is surrounded by a spiral casing of increasing area known as a volute casing or volute chamber (fig. 11-8). Water leaving the impeller flows inside this chamber circumferentially, the velocity gradually decreasing. The velocity of water when it reaches the inlet of delivery pipe reduces to a very small value, V_d and this is accompanied by corresponding increase in pressure head. It has been observed from tests that this type of chamber is very inefficient and only slightly increases the efficiency of the pump. This is due to considerable eddy losses taking place as increasing quantity of water flows through the chamber.

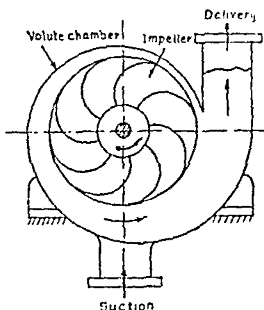


Fig. 11-8 Pump with volute casing

Vortex or Whirlpool chamber : Prof. Thomson improved on volute chamber by combining a circular chamber with a spiral chamber (fig. 11-9). This circular casing is known as a vortex or whirlpool chamber. This increases the conversion efficiency.

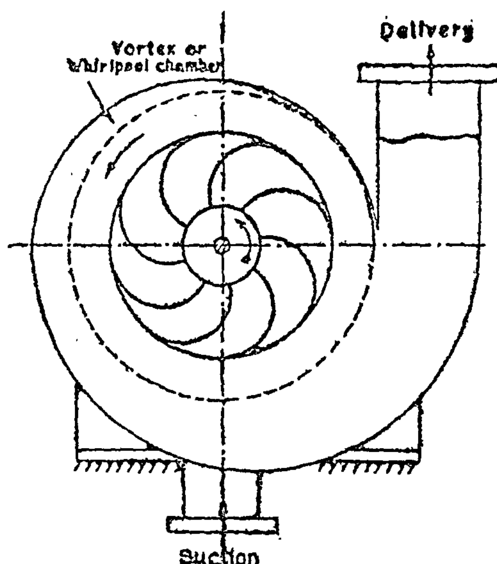


Fig. 11-9 Pump with vortex or whirlpool chamber

Guide blades : The pump casing is modified and is provided with guide vanes which form passages of increasing area as shown in fig. 11-10. Water leaving the impeller with velocity V_2 is given a change to flow through this increasing area which gradually reduces its velocity to V_0 with a consequent gain in pressure head. The pressure head, h_g generated in the guide vanes is

$$h_g = \left\{ \frac{V_2^2 - V_0^2}{2g} \right\} \times \eta_d$$

where, η_d is the efficiency of conversion of the diverging passages. For a good design, value of η_d is about 70%.

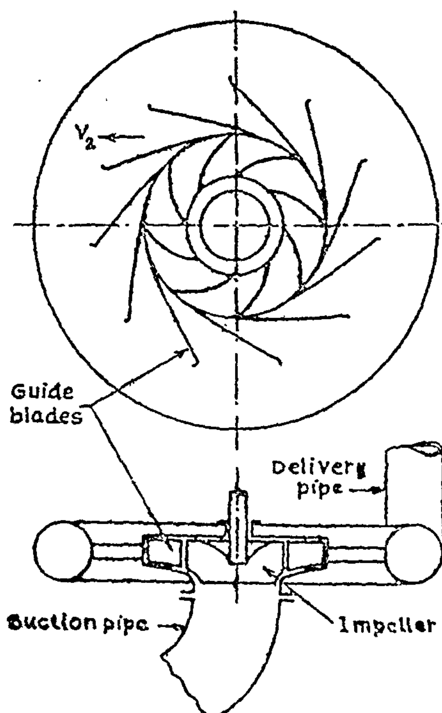


Fig. 11-10 Pump fitted with guide vanes

The ring of guide vanes is called a *diffuser*. A pump fitted with such vanes is known as a turbine pump, and is similar in principle to a reversed inward flow turbine.

Conical diffuser: By adding to the volute or whirlpool chamber, a conical tapered outlet passage (fig. 11-11), it is easy to reduce the water velocity to still lower outlet velocity with a consequent additional gain in presssure energy.

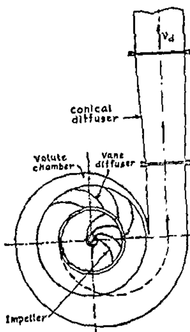


Fig. 11-11 Pump fitted with conical diffuser

Hydraulic Efficiency of Centrifugal Pump

Before defining the efficiency of the pump, it is necessary to list various heads imparted to water during its passage through the pump. The gross head imparted to water is termed the *manometric head* H_m. It is the sum of the static lift, h_s, the velocity head, $\frac{V^2}{2g}$, and the friction and other losses in the suction and delivery pipes h_{fs} and h_{fd}. The manometric head, H_m also seems to be the sum of the vacuum gauge, fixed on the suction pipe just before the pump inlet, and of the pressure gauge, on the delivery pipe, fitted just after the outlet of the pump. Thus,

$$H_m = H_{ms} + H_{md}$$

$$\text{where, } H_{ms} = h_s + h_{fs} + \frac{V^2_s}{2g}$$

$$\text{and } H_{md} = h_d + h_{fd} + \left(\frac{V^2_d}{2g} - \frac{V^2_s}{2g}\right)$$

If the suction and delivery pipes are of the same size, then the velocity of water in both the pipes will be same and then,

$$\left.\begin{aligned} H_m &= h_s + h_{fs} + \frac{V_s^2}{2g} + h_d + h_{fd} \\ &= (h_s + h_d) + (h_{fs} + h_{fd}) + \frac{V_s^2}{2g} \\ &= \begin{matrix}\text{static}\\ \text{lift, } H\end{matrix} + \begin{matrix}\text{friction}\\ \text{head, } h_f\end{matrix} + \begin{matrix}\text{velocity}\\ \text{head}\end{matrix} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots (11 \cdot 19)$$

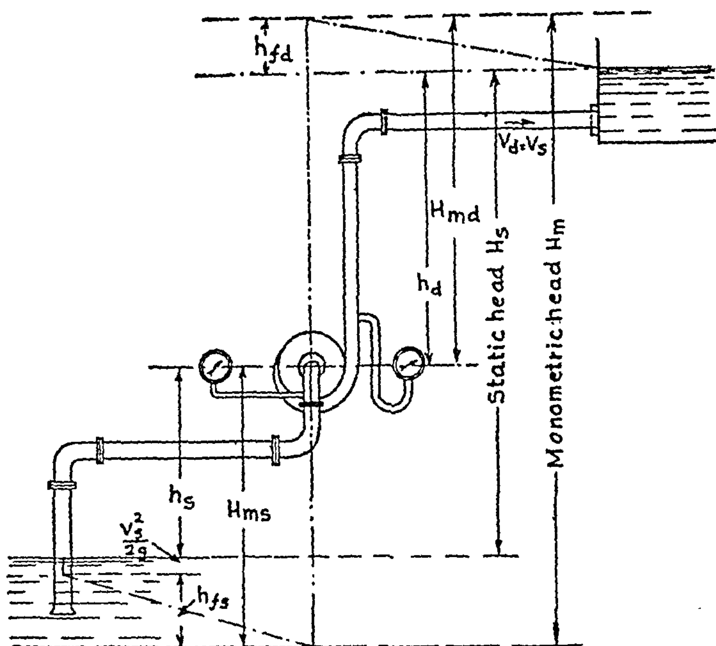


Fig. 11-12 Hydraulic heads in the pump system

In general, the manometric head, H_m shows the total increase in energy of water during its passage through the pump. This manometric head is sometimes referred to as *dynamic head.*

Manometric efficiency : The work supplied per kg water to the impeller is $\frac{V_{w_2}v_2 - V_{w_1}v_1}{g}$ which generates head H_m. The difference

between the two is on account of the loss of head due to shocks at entry and exit, friction losses in vane passages and eddy losses in the casing. These losses together are given a single name of *hydraulic losses in the pump.* Hence, the manometric or hydraulic efficiency of the pump can be defined as

$$\eta_m = \frac{H_m}{\dfrac{V_{w_2}v_2 - V_{w_1}v_1}{g}} \quad \ldots \quad \ldots \quad \ldots (11\cdot10)$$

For radial entry, which is the usual case for most of the C F. pumps, $V_{w_1} = 0$. Hence the expression for η_m reduces to

$$\eta_m = \frac{H_m}{\dfrac{V_{w_2}v_2}{g}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (11\cdot11)$$

Overall efficiency : The overall or gross efficiency of a pump is defined as the ratio of output water horse-power (W.H.P) to *the output power at the pump shaft (S.H.P.) viz.*

$$\text{overall efficiency, } \eta_o = \frac{\text{W H P.}}{\text{S H.P.}} \quad \ldots \quad \ldots (11\cdot12)$$

The energy of the prime mover (S.H P.) which runs the pump is not wholly spent on suction and delivery of the pump, but a part *of it is spent on overcoming various harmful resistances* which arise from imperfectness of the pump as a mechanism as a result of which the resistance is caused when water flows through it. The losses in the pump *are estimated by an overall efficiency which is* determined by taking a product of a number of efficiencies.

$$\eta_o = \eta_m \times \eta_v \times \eta_{mech} \quad \ldots \quad \ldots \quad \ldots \quad \ldots (11\cdot13)$$

where, η_m – manometric (hydraulic) efficiency, accounting for hydraulic loss of head when water flows through the pump (*e g friction* and eddy loss when water passes through the impeller and guide passage).

η_v – volumetric efficiency accounting for power required to pump through the impeller additional weight of water which *slips or leaks* through clearances, bad packing etc.

η_{mech} - mechanical efficiency accounting for surface or disc friction on the external faces or shrouds of the impeller and mechanical friction of the main bearings and glands.

The average value of the overall efficiency, η_o for C. F. pumps fluctuates widely between limits of 50% and 90% and depends on type, size, workmanship and liquid pumped. With increase in size and specific speed of the pump, the overall efficiency increases. It may be noted that while calculating water horse-power (W.H.P.) of the pump, manometric head, H_m should be used and not static head, H_s.

Problem-1 : *A centrifugal pump has impeller of 25 cm diameter at inlet and 50 cm diameter at outlet and runs at 1,000 rpm. The vanes are set back at an angle of 30° to the outer rim. If velocity of flow through the impeller is constant at 2 m/sec and the entry to the pump is radial, calculate the vane angle at inlet and the work done on the wheel per kg of water.*

$$v_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times \frac{25}{100} \times 1{,}000}{60} = 13{\cdot}1 \text{ m/sec.}$$

$$\text{and } v_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times \frac{50}{100} \times 1{,}000}{60} \doteq 26{\cdot}2 \text{ m/sec.}$$

Since, $V_{f_1} = V_{f_2} = 2$ m/sec. and angle $\phi = 30°$,

$$V_{w_2} = v_2 - \frac{V_{f_2}}{\tan\phi} = 26{\cdot}2 - \frac{2}{0{\cdot}5774} = 22{\cdot}73 \text{ m/sec}$$

$$\text{Now, W. D. per kg of water} = \frac{V_{w_2} v_2}{g} = \frac{22{\cdot}73 \times 26{\cdot}2}{9{\cdot}81}$$

$$= \underline{60{\cdot}5 \text{ kg-m}}$$

$$\text{Further, } \tan\theta = \frac{V_{f_1}}{v_1} = \frac{2}{13{\cdot}1} = 0{\cdot}153$$

$$\therefore \quad \theta = \underline{8°42'}$$

Problem-2 : *A centrifugal pump having a wheel of 30 cm outside diameter rotates at 1,000 rpm. The vanes are radial at exit and are 75 mm wide. The velocity of radial flow through the wheel is 3 m/sec. The velocities in suction and delivery pipes are 2·5 and 1·5 m/sec. respectively. Neglecting frictional losses, determine, (i) the height through which the pump lifts water, and (ii) the H.P. required to drive the pump.*

The wheel velocity at exit, $v_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0{\cdot}3 \times 1{,}000}{60}$

$= 15{\cdot}71$ m/sec.

As the vanes are radial at exit,

angle $\phi = 90°$ and hence $V_{w_2} = v_2 = 15{\cdot}71$ m/sec.

and $V_{f_2} = 3$ m/sec.

Neglecting friction losses, we have

$$\frac{V_{w_2} v_2}{g} = H_s + \frac{V_d^2}{2g}$$

$$\therefore \quad H_s = \frac{V_{w_2} v_2}{g} - \frac{V_d^2}{2g}$$

$$= \frac{15{\cdot}71 \times 15{\cdot}71}{9{\cdot}81} - \frac{(1{\cdot}5)^2}{2 \times 9{\cdot}81} = \underline{25{\cdot}086 \text{ m}}$$

Now, discharge $Q = \pi D_2 b_2 V_{f_2} = \pi \times 0{\cdot}3 \times 0\,075 \times 3$

$= 0{\cdot}212$ m³/sec.

$$\therefore \quad \text{H.P. required} = \frac{W \times \frac{V_{w_2} v_2}{g}}{75}$$

$$= \frac{1{,}000 \times 0{\cdot}212}{9{\cdot}81} \times \frac{15{\cdot}71 \times 15{\cdot}71}{75} = \underline{71{\cdot}2}$$

Problem-3 : *The inlet and outlet effective radial flow areas of a turbine pump impeller are respectively 16 cm² and 14 cm² and water enters with an absolute velocity of 5 m/sec. in the radial direction. The vanes at outlet are set back at 30° to the tangent.*

η_{mech} – mechanical efficiency accounting for surface or disc friction on the external faces or shrouds of the impeller and mechanical friction of the main bearings and glands.

The average value of the overall efficiency, η_o for C. F. pumps fluctuates widely between limits of 50% and 90% and depends on type, size, workmanship and liquid pumped. With increase in size and specific speed of the pump, the overall efficiency increases. It may be noted that while calculating water horse-power (W.H.P.) of the pump, manometric head, H_m should be used and not static head, H_s.

Problem–1 : *A centrifugal pump has impeller of 25 cm diameter at inlet and 50 cm diameter at outlet and runs at 1,000 rpm. The vanes are set back at an angle of 30° to the outer rim. If velocity of flow through the impeller is constant at 2 m/sec and the entry to the pump is radial, calculate the vane angle at inlet and the work done on the wheel per kg of water.*

$$v_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times \frac{25}{100} \times 1{,}000}{60} = 13{\cdot}1 \text{ m/sec.}$$

$$\text{and} \quad v_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times \frac{50}{100} \times 1{,}000}{60} = 26{\cdot}2 \text{ m/sec.}$$

Since, $V_{f_1} = V_{f_2} = 2$m/sec. and angle $\phi = 30°$,

$$v_{w_2} = v_2 - \frac{V_{f_2}}{\tan\phi} = 26{\cdot}2 - \frac{2}{0{\cdot}5774} = 22{\cdot}73 \text{ m/sec}$$

$$\text{Now, W. D. per kg of water} = \frac{V_{w_2} v_2}{g} = \frac{22{\cdot}73 \times 26{\cdot}2}{9{\cdot}81}$$

$$= \underline{60{\cdot}5 \text{ kg–m}}$$

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Problem-2 : *A centrifugal pump having a wheel of 30 cm outside diameter rotates at 1,000 rpm. The vanes are radial at exit and are 75 mm wide. The velocity of radial flow through the wheel is 3 m/sec. The velocities in suction and delivery pipes are 2·5 and 1·5 m/sec. respectively. Neglecting frictional losses, determine, (i) the height through which the pump lifts water, and (ii) the H.P. required to drive the pump.*

The wheel velocity at exit, $v_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.3 \times 1{,}000}{60}$

$= 15{\cdot}71$ m/sec.

As the vanes are radial at exit,

angle $\phi = 90°$ and hence $V_{w_2} = v_2 = 15{\cdot}71$ m/sec.

and $V_{f_2} = 3$ m/sec.

Neglecting friction losses, we have

$$\frac{V_{w_2} v_2}{g} = H_s + \frac{V_d^2}{2g}$$

$$\therefore \quad H_s = \frac{V_{w_2} v_2}{g} - \frac{V_d^2}{2g}$$

$$= \frac{15{\cdot}71 \times 15{\cdot}71}{9{\cdot}81} - \frac{(1{\cdot}5)^2}{2 \times 9{\cdot}81} = \underline{25{\cdot}086 \text{ m}}$$

Now, discharge $Q = \pi D_2 b_2 V_{f_2} = \pi \times 0{\cdot}3 \times 0{\cdot}075 \times 3$

$= 0{\cdot}212$ m³/sec.

$$\therefore \quad \text{H.P. required} = \frac{W \times \frac{V_{w_2} v_2}{g}}{75}$$

$$= \frac{1{,}000 \times 0{\cdot}212}{9{\cdot}81} \times \frac{15{\cdot}71 \times 15{\cdot}71}{75} = \underline{71{\cdot}2}$$

Problem-3 : *The inlet and outlet effective radial flow areas of a turbine pump impeller are respectively 16 cm² and 14 cm² and water enters with an absolute velocity of 5 m/sec. in the radial direction. The vanes at outlet are set back at 30° to the tangent.*

The outer peripheral speed is 28 m/sec. and the inner one is 14 m/sec. The manometric efficiency is 70%.

Neglecting losses in the impeller estimate the percentage of the outlet velocity head recovered in the diffuser passage and determine the inlet angle for the diffuser vanes.

As the entry is radial, $V_{f1} = V_1 = 5$ m/sec. and $V_{w1} = 0$

$$\therefore \quad V_{f2} = V_{f1} \times \frac{A_1}{A_2} = 5 \times \frac{16}{14} = 5{\cdot}7 \text{ m/sec.}$$

$$\text{Now, } V_{w2} = v_2 - \frac{V_{f2}}{\tan\phi} = 28 - \frac{5{\cdot}7}{\tan 30^\circ} = 28 - 9{\cdot}9 = 18{\cdot}1 \text{ m/sec.}$$

$$\text{Work done/kg} = \frac{V_{w2} v_2}{g} = \frac{18{\cdot}1 \times 28}{9{\cdot}81} = 51{\cdot}8 \text{ m-kg}$$

$$\text{Manometric efficiency, } \eta_m = \frac{\text{manometric head}}{\text{work done/kg}}$$

$$i.e.\ 0{\cdot}70 = \frac{H_m}{51{\cdot}8}$$

$$\therefore \quad H_m = 36{\cdot}26 \text{ m}$$

Neglecting losses in the impeller,

losses in the diffuser = W.D./kg–H_m

$= 51{\cdot}8 - 36{\cdot}26 = 15{\cdot}54$ m-kg/kg

Now, velocity at outlet is given by

$$V_2^2 = V_{w2}^2 + V_{f2}^2 = 18{\cdot}1^2 + 5{\cdot}7^2 = 360{\cdot}4$$

$$\therefore \quad \frac{V_2^2}{2g} = \frac{360{\cdot}4}{19{\cdot}62} = 18{\cdot}4 \text{ m of water}$$

$$\therefore \quad \left.\begin{array}{l}\text{velocity head recovered}\\ \text{in the diffuser}\end{array}\right\} = \frac{\frac{V_2^2}{2g} - \text{losses}}{\frac{V_2^2}{2g}} = \frac{18{\cdot}4 - 15{\cdot}54}{18{\cdot}4}$$

$$= 0{\cdot}155 \ i.e.\ 15{\cdot}5\%$$

Now, inlet angle to the diffuser vane is given by

$$\tan \beta = \frac{V_{f_2}}{V_{w_2}} = \frac{5{\cdot}7}{18{\cdot}1} = 0{\cdot}3143$$

$$\therefore \quad \underline{\beta = 17°27'}$$

Problem-4 : *A centrifugal pump of 1·4 m exit diameter runs at 200 rpm. and pumps 1,900 litres/sec. of water, the static lift being 9 m. The angle which the vanes make at exit with the tangent to the impeller is 155°, and the radial velocity of flow is 2·5 m/sec. Determine the useful H.P. and the efficiency. Find also the lowest speed to start pumping against a head of 9 m, the inner diameter of the impeller being 0·55 m*

Velocity of the rim of the impeller, $v_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1{\cdot}4 \times 200}{60}$

$$= 14{\cdot}8 \text{ m/sec.}$$

Now, $V_{w_2} = v_2 - \frac{V_{f_2}}{\tan\phi} = 14{\cdot}8 - \frac{2{\cdot}5}{\tan 25°} = 14{\cdot}8 - \frac{2{\cdot}5}{0{\cdot}4663} = 9{\cdot}42$ m/sec.

$$\left.\begin{array}{l}\text{H. P. of} \\ \text{the impeller}\end{array}\right\} = \frac{V_{w_2} v_2}{g} \times \frac{W}{75} = \frac{9{\cdot}42 \times 14{\cdot}8}{9{\cdot}81} \times \frac{1900}{75} = 360$$

$$\text{Useful H. P.} = \frac{WH_s}{75} = \frac{1{,}900 \times 9}{75} = 288$$

$$\text{and efficiency, } \eta = \frac{\text{useful H P.}}{\text{H. P. supplied to impeller}} = \frac{228}{360} \times 100 = \underline{63{\cdot}3\%}$$

The lowest speed for delivery to commence is given by

$$H = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\therefore \quad 2gH = v_2^2 - v_1^2 = \left\{\frac{\pi D_2 N}{60}\right\}^2 - \left\{\frac{\pi D_1 N}{60}\right\}^2 = \left(\frac{\pi N}{60}\right)^2 \{ D_2^2 - D_1^2 \}$$

$$\therefore \quad \frac{2 \times 9{\cdot}81 \times 9 \times 3{,}600}{\pi^2} = (1{\cdot}4^2 - 0{\cdot}55^2)\, N^2$$

$$\therefore \quad N^2 = 39{,}000$$

$$\therefore \quad N = \underline{197 \text{ rpm.}}$$

Problem – 5 : *A centrifugal pump running at 400 rpm discharges 113 litre/sec. The impeller is 25 cm diameter at inlet and 52 cm at outlet. The inlet width is 12 cm and the outlet width is 8 cm. Neglecting friction losses and the effect of thickness of the vanes in reducing flow area, determine the head pumped against, if the vanes at outlet are curved back to an angle of 30° the wheel tangent.*

$$\text{Now } v_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0{\cdot}25 \times 400}{60} = 5{\cdot}24 \text{ m/sec.}$$

$$\text{and } v_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0{\cdot}52 \times 400}{60} = 10{\cdot}9 \text{ m/sec.}$$

Now, discharge, $Q = \pi D_2 b_2 V_{f_2}$

$$\therefore \quad \frac{113}{1{,}000} = \pi \times 0{\cdot}52 \times 0{\cdot}08 \times V_{f_2}$$

$$\therefore \quad V_{f_2} = \frac{113}{1{,}000 \times \pi \times 0{\cdot}52 \times 0{\cdot}08} = 0{\cdot}867 \text{ m/sec.}$$

$$\text{and } V_{w_2} = v_2 - \frac{V_{f_2}}{\tan\phi} = 10{\cdot}9 - \frac{0{\cdot}867}{\tan 30^\circ} = 9{\cdot}4 \text{ m/sec.}$$

As data regarding frictional losses and pipe velocity is not given

$$H_s = \frac{V_{w_2} v_2}{g} = \frac{9{\cdot}4 \times 10{\cdot}9}{9{\cdot}81} = \underline{10{\cdot}4 \text{ m}}$$

Problem – 6 : *Show that the pressure rise in the impeller of a centrifugal pump is given by the expression* $\dfrac{V_{f_1}^2 + v_2^2 - V_{f_2}^2 \operatorname{cosec}^2\phi}{2g}$ *provided the frictional and other losses in the impeller are neglected.*

Applying Bernoulli's equation between inlet and outlet edges of the impeller for points 1 and 2 respectively and as frictional losses and other losses are neglected,

Total energy at inlet = total energy at outlet – work done

$$\frac{p_1}{w} + Z_1 + \frac{V_1^2}{2g} = \frac{p_2}{g} + Z_2 + \frac{V_2^2}{2g} - \frac{V_{w_2} v_2}{g}$$

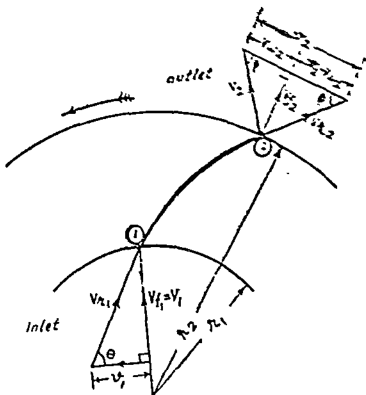


Fig. 11-13

Now neglecting gravitational head $(Z_2 - Z_1)$ and as $V_1 = Vf_1$ the pressure rise is

$$\frac{p_2 - p_1}{w} = \frac{Vf_1^2}{2g} - \frac{V_2^2}{2g} + \frac{V_{w_2} v_2}{g}$$

From outlet triangle of velocities

$$V_2^2 = Vf_2^2 + V_{w_2}^2$$

and $V_{w_2} = v_2 - vf_2 \cot\phi$

$$\therefore \quad V_2^2 = Vf_2^2 + (v_2 - Vf_2 \cot\phi)^2 = Vf_2^2\,(1 + \cot^2\phi) + v_2^2 - 2v_2\, Vf_2 \cot\phi$$

Hence,

$$\frac{p_2 - p_1}{w} = \frac{Vf_1^2}{2g} - \frac{Vf_2^2\,(1 + \cot^2\phi)}{2g} - \frac{v_2^2}{2g} + \frac{2v_2 Vf_2 \cot\phi}{2g} + \frac{v_2\,(v_2 - Vf_2 \cot\phi)}{g}$$

But $1 + \cot^2\phi = \text{cosec}^2\phi$

$$\therefore \quad \frac{p_2 - p_1}{w} = \frac{Vf_1^2 - Vf_2^2 \text{cosec}^2\phi - v_2^2 + 2v_2 Vf_2 \cot\phi + 2v_2^2 - 2v_2 Vf_2 \cot\phi}{2g}$$

$$= \frac{Vf_1^2 + v_2^2 - Vf_2^2 \text{cosec}^2\phi}{2g} \quad ..(11{\cdot}14)$$

$$\therefore \quad \text{pressure rise} = \frac{1}{2g}(Vf_1^2 + v_2^2 - Vf_2^2 \text{cosec}^2\phi)$$

Problem-7 : *A 30 cm external diameter impeller rotates at 800 rpm. The vanes are curved backward at 35° to the tangent at outlet. The thickness of the vanes occupies 15% of the periphery. The clear flow width at the exit from the impeller is 5 cm. Calculate the pressure rise in the impeller when flow is 65 litres/sec. What percentage of total work done is converted into kinetic energy? Velocity of flow is constant from inlet to outlet.*

$$\text{Now, } v_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0{\cdot}3 \times 800}{60} = 12{\cdot}55 \text{ m/sec.}$$

$$Q = \frac{65}{1{,}000} = 0{\cdot}065 \text{ m}^3\text{/sec.}$$

But, $Q = k\pi D_2 b_2 Vf_2$

$$\therefore \quad Vf_2 = \frac{Q}{k\pi D_2 b_2} = \frac{0{\cdot}065}{0{\cdot}85 \times \pi \times 0{\cdot}3 \times 0{\cdot}05} = 1{\cdot}73 \text{ m/sec.}$$

$$\therefore \quad Vf_1 = Vf_2 = 1{\cdot}73 \text{ m/sec.}$$

$$\text{Now, } V_{w_2} = v_2 - \frac{Vf_2}{\tan\phi} = 12{\cdot}55 - \frac{1{\cdot}73}{\tan 35°} = 12{\cdot}55 - \frac{1{\cdot}73}{0{\cdot}7002}$$

$$= 10{\cdot}09 \text{ m/sec.}$$

$$\text{and } V_2^2 = Vf_2^2 + Vw_2^2 = (1{\cdot}73)^2 + (10{\cdot}09)^2 = 105$$

$$\therefore \quad V_2 = 10{\cdot}25 \text{ m/sec.}$$

Using eqn. (11·14),

$$\text{pressure rise in the impeller} = \frac{Vf_1^2 + v_2^2 - Vf_2^2 \text{cosec}^2\phi}{2g}$$

$$= \frac{(1{\cdot}73)^2 + (12{\cdot}55)^2 - (1{\cdot}73)^2 \text{cosec}^2 35°}{2 \times 9{\cdot}81}$$

$$= \frac{3 + 157 - 3/0.33}{19.62} = 7.7 \text{ m}$$

K.E. at outlet per kg of water $= \frac{V_2^2}{2g} = \frac{10.25^2}{2 \times 9.81} = 5.2$ m

W D. per kg of water $= \frac{V_{w_2} v_2}{g} = \frac{10.09 \times 12.55}{9.81} = 12.9$ kg-m

Percentage of total work done converted into K.E. at outlet is

$$= \frac{5.2}{12.9} \times 100 = 40.3\%$$

Problem-8 : *A centrifugal pump has an impeller of internal diameter 12 cm and external diameter 24 cm, which rotates at 1,200 rpm. The absolute velocity of water at inlet is radial and vanes are curved back at an angle of 25° to the tangent at outlet. The widths of the impeller at inlet and outlet are 16 mm and 8 mm respectively. Determine the gain of (pressure) head as the water passes through the impeller, neglecting losses. The pump discharges water at the rate of 500 litres per minute.*

$$v_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.12 \times 1,200}{60} = 7.54 \text{ m/sec.}$$

$$v_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.24 \times 1,200}{60} = 15.08 \text{ m/sec.}$$

$$Q = \pi d_2 b_2 V_{f_2} = \pi d_1 b_1 V_{f_1}$$

$$\text{But, } Q = \frac{500}{60 \times 1,000} = 0.00834 \text{ m}^3/\text{sec.}$$

$$\therefore \quad V_{f_2} = \frac{Q}{\pi d_2 b_2} \text{ and } V_{f_1} = \frac{Q}{\pi d_1 b_1}$$

$$\therefore \quad V_{f_2} = \frac{0.00834}{\pi \times 0.24 \times 0.8/100} = 1.38 \text{ m/sec.}$$

$$\text{and } V_{f_1} = \frac{0.00834}{\pi \times 0.12 \times \frac{1.6}{100}} = 1.38 \text{ m/sec.}$$

Now, gain of pressure head is

$$\frac{p_2 - p_1}{w} = \frac{Vf_1^2 = v_2^2 - v_{t2}^2 \operatorname{cosec}^2\phi}{2g}$$

$$= \frac{(1{\cdot}38)^2 + (15{\cdot}08)^2 - (1{\cdot}38)^2 \times \dfrac{1}{(\cos 25°)^2}}{19{\cdot}62}$$

$$= \frac{1{\cdot}9 + 266 - 2{\cdot}33}{19{\cdot}62} = 11{\cdot}5 \text{ m of water}$$

Problem – 9 : *A centrifugal pump with a manometric efficiency of 0·8 operates against a manometric head of 32 m. Assume radial velocity of flow to be constant and that the increase in pressure through the impeller is 65% of the total head generated by the pump. Determine: (a) the operating speed in rpm., (b) the discharge in litres per minute. The impeller outlet diameter is 15 cm and witdh 1·5 cm. The blades at outlet are set back at 60° to the wheel tangent. Impeller losses may be neglected.*

Manometric efficiency, $\eta_m = \dfrac{H_m}{\dfrac{V_{w2} v_2}{g}}$

$$i.e. \quad 0{\cdot}8 = \frac{32}{\dfrac{V_{w2} \times v_2}{g}}$$

$$\therefore \quad \frac{V_{w2} \times v_2}{g} = \frac{32}{0{\cdot}8} = 40 \text{ m-kg/kg}$$

$$\therefore \quad V_{w2} v_2 = 40 \times 9{\cdot}81 = 392{\cdot}4$$

Now, pressure head across the impeller is

$$\frac{p_2 - p_1}{w} = 0{\cdot}65 \times \frac{V_{w2} \times v_2}{w} = 0{\cdot}65 \times 40 = 26 \text{ m of water}$$

Again, neglecting losses in the impeller,

$$\frac{p_2 - p_1}{g} = \frac{v_2^2 - v_1^2}{2g} - \frac{V_{r2}^2 - V_{r1}^2}{2g}$$

$$i.e.\ 26 = \frac{1}{19{\cdot}62}\{v_2^2 - v_1^2 - V_{r2}^2 + V_{r1}^2\}$$

Since entry is radial,

$$V_{r1}^2 = v_1^2 + V_1^2$$

i.e. $V_{r1}^2 - v_1^2 = V_1^2 = V_{f1}^2$

$$\therefore\ 26 \times 19{\cdot}62 = v_2^2 - V_{r2}^2 + V_{f1}^2$$

$$510 = v_2^2 - V_{r2}^2 + V_{f2}^2 \quad (\because V_{f1} = V_{f2})$$

$$= v_2^2 - (V_{r2}^2 - V_{f2}^2)$$

$$= v_2^2 - (v_2 - V_{w2})^2 \text{ (from outlet triangle)}$$

$$= v_2^2 + 2v_2V_{w2} - v_2^2 - V_{w2}^2$$

$$= 2v_2\, V_{w2} - V_{w2}^2 = 2 \times 40 \times 9{\cdot}81 - V_{w2}^2$$

$$\therefore\ V_{w2}^2 = 274{\cdot}8$$

$$\therefore\ V_{w2} = 16{\cdot}5 \text{ m/sec.}$$

Again, $\dfrac{V_{w2}v_2}{g} = 40$ m-kg

$$\therefore\ v_2 = \frac{40 \times 9{\cdot}81}{15{\cdot}5} = 23\ 8 \text{ m/sec.}$$

But, $v_2 = \dfrac{\pi D_2 N}{60}$

$$\therefore\ N = \frac{23{\cdot}8 \times 60}{\pi \times 0{\cdot}15} = \underline{3{,}020 \text{ rpm.}}$$

Now, $v_{f2} = (v_2 - V_{w2}) \tan\phi$

$$= (23{\cdot}8 - 16{\cdot}5) \tan 60^\circ = 12{\cdot}4 \text{ m/sec.}$$

Hence, discharge, $Q = \pi D_2 b_2 V_{f2}$

$$= \pi \times 0{\cdot}15 \times \frac{1{\cdot}5}{100} \times 12{\cdot}4$$

$$= \underline{0\ 0878\ \text{m}^3\text{/sec.}}$$

$$= 0{\cdot}0878 \times 1{,}000 \times 60$$

$$= \underline{5{,}408 \text{ litres per min.}}$$

Problem - 10 : *A centrifugal pump is required to work against a head of 25 m. Assume that the blade outlet angle is 40°, velocity of flow is 2·5 m/sec , the pump runs at 1,000 rpm. and that 40% of head due to whirl is converted into pressure energy. Calculate the diameters and widths at inlet and outlet of the impeller of the pump if it discharges 125 litres/sec. of water. Assume the diameter at outlet to be twice that at inlet.*

When the flow is just to commence, centrifugal head impressed by the rotating impeller is

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{.. (i)}$$

Let k be the fraction of the velocity head due to the whirl velocity which is converted into pressure head.

When the flow is just to commence, $V_d = 0$ hence $V_{f_2} = 0$

Thus, $V_{w_2} = v_2 - \frac{V_{f_2}}{\tan\theta} = v_2$

Also $H_s = h + k\frac{V_{w_2}^2}{2g}$

$$\therefore \quad h = H_s - k\frac{V_{w_2}^2}{2g} \quad \text{.. (ii)}$$

From eqns. (i) and (ii), we have,

$$H_s - k\frac{V_{w_2}^2}{2g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$\therefore \quad H_s = \frac{kv_2^2}{2g} + \frac{v_2^2}{2g} - \frac{v_2^2}{4\times 2g} \text{ (as } V_{w_2} = v_2 \text{ and } v_2 = 2v_1\text{)}$$

$$= \frac{v_2^2}{2g}\left\{k + 1 - \tfrac{1}{4}\right\}$$

$$\therefore \quad v_2^2 = \frac{2gH_s}{k + 0{\cdot}75}$$

$$\therefore \quad v_2 = \sqrt{\frac{2 \times 9{\cdot}81 \times 25}{0{\cdot}4 + 0{\cdot}75}} = 20{\cdot}6 \text{ m/sec}$$

But, $v_2 = \frac{\pi D_2 N}{60}$

$$\therefore \quad D_2 = \frac{60 v_2}{\pi N} = \frac{60 \times 20{\cdot}6}{\pi \times 1{,}000} = \underline{0{\cdot}394 \text{ m or } 39{\cdot}4 \text{ cm}}$$

and $D_1 = \frac{D_2}{2} = 0{\cdot}197$ m or $\underline{19{\cdot}7 \text{ cm}}$

Now, discharge, $Q = \pi D_2 b_2 V_{f_2} = \pi D_1 b_1 V_{f_1}$

$$\therefore \quad b_1 = \frac{Q}{\pi D_1 V_{f_1}} = \frac{0{\cdot}125}{\pi \times 0{\cdot}197 \times 2{\cdot}5}$$

$$= \underline{0{\cdot}0808 \text{ m or } 8{\cdot}08 \text{ cm}}$$

$$\text{and} \quad b_2 = \frac{b_1}{2} = \frac{0{\cdot}0808}{2} = \underline{0{\cdot}0404 \text{ m or } 4{\cdot}04 \text{ cm}}$$

Problem-11 : *A test carried out on a centrifugal pump in a hydraulic laboratory gave the following readings :*

Vacuum gauge reading on suction side	..	*250 mm of mercury*
Pressure gauge reading on delivery side	..	*1·5 kg/cm²*
The vertical distance between the gauges	..	*50 cm*
The output of the electric motor	..	*22 H.P.*
Diameter of the delivery pipe	..	*15 cm*
Diameter of the suction pipe	..	*20 cm*
Length of the sump	..	*9 m*
Breadth of the sump	..	*6 m*
Depth of the sump	..	*2 m*
Time required to empty the sump	..	*30 minutes.*

Determine the overall efficiency of the pump.

$$\text{The rate of discharge} = \frac{\text{Volume of the sump}}{\text{time taken to empty the sump}}$$

$$\therefore \quad Q = \frac{9 \times 6 \times 2}{1{,}800} = \frac{108}{1{,}800} = 0{\cdot}06 \text{ m}^3/\text{sec.}$$

Velocity of water in the delivery pipe,

$$V_d = \frac{Q}{A_d} = \frac{0{\cdot}06}{\frac{\pi}{4}(0{\cdot}15)^2} = 3{\cdot}4 \text{ m/sec}$$

Velocity of water in the suction pipe,

$$V_s = \frac{Q}{A_s} = \frac{0{\cdot}06}{\frac{\pi}{4}(0{\cdot}2)^2} = 1{\cdot}91 \text{ m/sec}$$

Pressure head at outlet, $\frac{p_2}{w} = \frac{1.5 \times 10,000}{1,000} = 15$ m of water

Pressure head at inlet, $\frac{p_1}{w} = -25$ cm of mercury $= -0.25$ m of mercury

$= -0.25 \times 13.6 = -3.4$ m of water

$$H_m = \left(\frac{p_2}{w} + \frac{V_2^2}{2g} + z_2\right) - \left(\frac{p_1}{w} + \frac{V_1^2}{2g} + z_1\right) \quad \text{neglecting losses.}$$

Considering $V_2 = V_d$ and $V_1 = V_s$,

$$H_m = \left\{(15 + \frac{(3.4)^2}{19.62} + 0.5)\right\} - \left\{(-3.4 + \frac{(1.91)^2}{19.62} + 0)\right\}$$

$$= 15 + 0.59 + 0.5 + 3.4 - 0.186 = 19.304$$

Overall efficiency, $\eta_o = \frac{\text{W H.P.}}{\text{S.H P. of motor}} = \frac{wQ \times \frac{H_m}{75}}{\text{S.H.P.}}$

$$= \frac{1,000 \times 0.06 \times 19.304}{75 \times 22} = 0.704 \text{ i.e. } 70.4\%$$

Problem-12: *A centrifugal pump is to discharge 0·12 m³/sec, at a speed of 1,500 rpm. against a head of 32 m. The impeller diameter is 30 cm, its width is 5 cm and the manometric efficiency is 80%. Determine the vane angle at the outer periphery of the impeller.*

$$v_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.3 \times 1,500}{60} = 23.6 \text{ m/sec}$$

Discharge, $Q = \pi d_2 b_2 V_{f2}$

$$\therefore \quad V_{f2} = \frac{Q}{\pi d_2 b_2} = \frac{0.12}{\pi \times 0.3 \times 0.05} = 2.55 \text{ m/sec}$$

Manometric efficiency, $\eta_m = \frac{H_m}{\frac{V_{w2} v_2}{g}}$

$$\therefore \quad \frac{V_{w2} v_2}{g} = \frac{H_m}{\eta_m} = \frac{32}{0.8} = 40$$

$$\therefore \quad V_{w2} v_2 = 9.81 \times 40 = 392.4$$

$$\therefore \quad V_{w2} = \frac{392.4}{23.6} = 16.6 \text{ m/sec}$$

$$\text{Now, } \tan\phi = \frac{V_{f_2}}{v_2 - V_{w2}} = \frac{2.55}{23.6 - 16.6} = \frac{2.55}{7} = 0.364$$

$$\therefore \quad \phi = \tan^{-1}(0.364) = \underline{20^\circ}$$

Problem-13 : *A centrifugal pump having an overall efficiency of 75% delivers 1,800 litres of water per minute through a pipe 15 cm diameter and 125 m long. Calculate the H. P. required to drive the pump if it lifts water to a height of 25 m The coefficient of friction for the pipe may be taken as 0.01.*

Discharge of water, $Q = \dfrac{1,800}{1,000 \times 60} = 0.03 \text{ m}^3/\text{sec}$

Velocity of water in the delivery pipe, $V_d = \dfrac{Q}{A_d}$

$$\therefore \quad V_d = \frac{0.03}{\frac{\pi}{4}(0.15)^2} = 1.7 \text{ m/sec}$$

As no specification of suction pipe is given we consider the given length as delivery pipe and assume the suction pipe to be small and hence frictional loss in it is negligible.

$\therefore$ head lost in friction in the delivery pipe is

$$H_{fd} = \frac{4flV_d^2}{D \cdot 2g} = \frac{4 \times 0.01 \times 125}{0.15} \times \frac{(1.7)^2}{19.62} = 4.93 \text{ m of water}$$

$$\text{Now, } H_m = (H_s + H_d) + H_{fd} + \frac{V_d^2}{2g}$$

$$= 25 + 4.93 + \frac{(1.7)^2}{19.62}$$

$$= 25 + 4.93 + 0.148 = 33.078 \text{ m of water}$$

$$\text{H.P. required to drive the pump} = \frac{wQH_m}{75\eta_o}$$

$$= \frac{1,000 \times 0.03 \times 30.078}{75 \times 0.75} = \underline{16}$$

Problem - 14: *At the designed speed of 1,200 rpm. a centrifugal pump is to deliver water against a net head of 20 m. The vanes are curved back to an angle of 30° with the tangent to the wheel. The impeller has an outside diameter 30 cm and the inner diameter of 15 cm. The velocity of flow is constant from inlet to outlet. The width of the impeller wheel at outlet is 5 cm and the hydraulic (manometric) efficiency is 90%. Determine the discharge of the pump in litres per minute, the width of the impeller at inlet, and the angle of the vane tip at inlet.*

$$\text{Now, } \eta m = \frac{Hm}{Vw_2 \, v_2/g}$$

$$\therefore \quad Vw_2 v_2 = \frac{g \times Hm}{\eta m} = \frac{9.81 \times 20}{0.9} = 218$$

$$\text{But } Vw_2 = v_2 - \frac{Vf_2}{\tan\phi}$$

$$\therefore \quad \left(v_2 - \frac{Vf_2}{\tan\phi}\right) v_2 = 218$$

$$\text{But, } v_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.3 \times 1{,}200}{60} = 18.8 \text{ m/sec}$$

$$\therefore \quad \left(18.8 - \frac{Vf_2}{\tan 30°}\right) 18.8 = 218$$

$$\therefore \quad \frac{Vf_2}{\tan 30°} = 18.8 - \frac{218}{18.8} = 18.8 - 11.6 = 7.2$$

$$\therefore \quad Vf_2 = 7.2 \tan 30° = 7.2 \times 0.5774 = 4.15 \text{ m/sec}$$

$$\therefore \quad Vf_1 = Vf_2 = 4.15 \text{ m/sec}$$

Now, discharge $Q = \pi d_1 b_1 Vf_1 = \pi d_2 b_2 Vf_2$

Hence, the width of the impeller at inlet is;

$$b_1 = b_2 \times \frac{d_2}{d_1} = 5 \times \frac{30}{15} = \underline{10 \text{ cm}}$$

Velocity of the periphery at inlet,

$$v_1 = v_2 \times \frac{d_1}{d_2} = 18.8 \times \frac{15}{30} = 9.4 \text{ m/sec}$$

$$\text{Hence, } \tan\theta = \frac{V_{f2}}{v_2} = \frac{4 \cdot 15}{9 \cdot 4} = 0.442$$

$$\therefore \quad \theta = \underline{23° 51'}$$

$$Q = \pi d_2 b_2 V_{f2} = \pi \times 0.3 \times \frac{5}{100} \times 4.15 = 0.196 \text{ m}^3/\text{sec.}$$

$$\therefore \quad W = wQ \times 60 = 1{,}000 \times 0.196 \times 60 = 11{,}800 \text{ kg/min.}$$

$$i.e. \ \underline{11{,}800 \text{ litres/min.}}$$

Problem – 15 : *A centrifugal pump has an impeller diameter of 30 cm. The vanes are set back at an angle of 20° with the tangent to the wheel at outlet. The diameters of the suction and delivery pipes are 20 cm and 15 cm respectively. The impeller has discharge area of 0·093 m². Pressure gauges at points on the suction and delivery pipes close to the pump, and each gauge 2 m above the water level in the sump, read pressure heads of 4 m below and 28 m above atmospheric pressure respectively when the rate of discharge of water was 0·11 m³/sec. It requires 80 H. P. to drive the pump when running at 1,500 rpm.*

Calculate, (a) the overall efficiency, (b) manometric efficiency assuming that water enters the impeller without whirl and (c) the loss of head in the suction pipe.

Velocity of water in the suction pipe,

$$V_s = \frac{Q}{A_s} = \frac{0.11}{\frac{\pi}{4} \times (0.2)^2} = 3.5 \text{ m/sec.}$$

Velocity of water in the delivery pipe,

$$V_d = \frac{Q}{A_d} = \frac{0.11}{\frac{\pi}{4}(0.15)^2} = 6.22 \text{ m/sec.}$$

$$\text{Total suction head at inlet} = H_s + H_{fs} + \frac{V_s^2}{2g}$$

$$\therefore \quad 4 = 2 + H_{fs} + \frac{(3.5)^2}{19.62} = 2 + H_{fs} + 0.625$$

$$\therefore \quad H_{fs} = 4 - 2.625 = 1.375 \text{ m of water}$$

Velocity of the periphery of wheel,

$$v_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.3 \times 1,500}{60} = 23.6 \text{ m/sec}$$

Velocity of flow at outlet,

$$V_{f2} = \frac{Q}{A_1} = \frac{0.11}{0.093} = 1.18 \text{ m/sec}$$

Velocity of whirl at outlet,

$$V_{w2} = v_2 - \frac{V_{f2}}{\tan\phi} = 23.6 - \frac{1.18}{0.364} = 20.35 \text{ m/sec.}$$

$$\text{W.D. per kg of water/sec.} = \frac{V_{w2}\, v_2}{g} = \frac{20.35 \times 23.6}{9.81} = 49 \text{ m-kg}$$

$$\text{Manometric head, } H_m = \begin{Bmatrix} \text{total energy at} \\ \text{outlet} \end{Bmatrix} - \begin{Bmatrix} \text{total energy} \\ \text{at inlet} \end{Bmatrix}$$

$$\therefore \quad H_m = \left(\frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2\right) - \left(\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1\right)$$

But, $V_2 \doteq V_d = 6.22$ m/sec; $V_1 \doteq V_s = 3.5$ m/sec; $Z_2 = Z_1 = 2$ m

$$\frac{p_2}{w} = 28 \text{ m of water and } \frac{p_1}{w} = -4 \text{ m of water}$$

$$\therefore \quad H_m = \left\{28 + \frac{(6.22)^2}{19.62} + 2\right\} - \left\{-4 + \frac{(3.5)^2}{19.62} + 2\right\}$$

$$= 32 + 1.98 - 0.625 = 33.335$$

$$\text{Manometric efficiency, } \eta_m = \frac{H_m}{V_{w2}\, v_2/g}$$

$$= \frac{33.335}{49} = \underline{0.675 \text{ i.e. } 67.5\%}$$

$$\text{Now overall efficiency, } \eta_o = \frac{wQH_m/75}{\text{S.H.P. of the motor}}$$

$$= \frac{1,000 \times 0.11 \times 33.335}{75 \times 80}$$

$$= \underline{0.61 \text{ i.e. } 61\%}$$

Problem-16 : *A centrifugal pump is required to discharge water at the rate of 0·566 m³/sec and to develop a head of 16 m when the impeller rotates at 800 rpm. Water enters the impeller wheel radially and the velocity of flow is constant from inlet to outlet equal to 3 m/sec. The manometric efficiency is to be 80% and the loss of head in the impeller due to friction being assumed to be* $0{\cdot}025V_2^2$ *m of water where* V_2 *is the absolute velocity of water with which it leaves the impeller.*

Calculate : (a) the vane angle at outlet edge of the impeller, (b) the impeller diameter and area of flow at outlet, and (c) the vane angle at inlet, if the inlet diameter is half the outlet diameter.

Manometric efficiency, $\eta_m = \dfrac{H_m}{\dfrac{V_{w_2} v_2}{g}}$

$$\therefore \quad \frac{V_{w_2} v_2}{g} = \frac{H_m}{\eta_m} = \frac{16}{0{\cdot}8} = 20 \text{ m}$$

$\therefore$ hydraulic losses in the impeller $= \dfrac{V_{w_2} v_2}{g} - H_m$

$$= 20 - 16 = 4 \text{ m of water}$$

$$\therefore \quad 0{\cdot}025 V_2^2 = 4$$

$$V_2 = \sqrt{\frac{4}{0{\cdot}025}} = 12{\cdot}6 \text{ m/sec}$$

$$\therefore \quad V_{w_2} = \sqrt{V_2^2 - V_{f_2}^2}$$

$$= \sqrt{(12{\cdot}6)^2 - (3)^2} = 12{\cdot}25 \text{ m/sec}$$

Now, $V_{w_2} v_2 = 9{\cdot}81 \times 20$

$$\therefore \quad v_2 = \frac{9{\cdot}81 \times 20}{12{\cdot}25} = 16{\cdot}1 \text{ m/sec}$$

But, $v_2 = \dfrac{\pi d_2 N}{60}$

$$\therefore \quad d_2 = \frac{60\, v_2}{\pi N} = \frac{60 \times 16{\cdot}1}{\pi \times 800} = 0{\cdot}384 \text{ m } i.e.\ 38{\cdot}4 \text{ cm}$$

Discharge, Q = velocity of flow × area of flow $= V_f \times A_f$

$$\therefore \quad A_f = \frac{Q}{V_f} = \frac{0{\cdot}566}{3} = 0{\cdot}189 \text{ m}^2$$

Now, $\tan\phi = \frac{V_{f2}}{v_2 - V_{w2}} = \frac{3}{16{\cdot}1 - 12{\cdot}25} = \frac{3}{3{\cdot}85} = 0{\cdot}78$

$\therefore \quad \phi = \underline{38°57'}$

As the inlet diameter is half the outlet diameter,

$$d_1 = \frac{d_2}{2} = \frac{0{\cdot}384}{2} = 0{\cdot}192 \text{ m and, } v_1 = \frac{v_2}{2} = \frac{16{\cdot}1}{2} = 8{\cdot}05 \text{ m/sec,}$$

$$\tan\theta = \frac{V_{f1}}{v_1} = \frac{3}{8{\cdot}05} = 0{\cdot}373$$

$\therefore \quad \theta = \underline{20°\ 27'}$

Performance Characteristics

By characteristics of a centrifugal pump, we mean the relationship between the output (discharge), Q and the total head developed, H under constant rpm., N, represented in a graphical manner of the head-discharge or H–Q curve. It can be obtained theoretically from the basic equation of the centrifugal pump (ref. eqn. 11·10) or in a practical manner by conducting a test on it at the time of its manufacture in the factory. The latter one is carried out like this : The pump is started and run while the delivery valve is kept closed and hence $Q = 0$. Then the valve is gradually opened. For each gate opening, Q and H are noted (from readings of manometer and vacuum gauge) for the given rpm., N. With the chosen scales, on x-axis the discharge Q and on y-axis the head H are plotted. By joining the points, the mean curve H-Q is obtained.

Simultaneously the brake test is conducted on the shaft of the pump for different Q, so as to obtain a power-discharge or P–Q curve for the same rpm., N. Then, using the formula W.H.P. $= \frac{wQH}{75}$ the useful power is determined and knowing S.H.P. (shaft horse power) supplied to the pump, the efficiency for all points on the H – Q curve is determined from formula,

$$\eta_h = \frac{\text{W.H.P.}}{\text{S.H.P.}}$$

The $\eta - Q$ curve is, thus, obtained.

Such H-Q, P-Q and η-Q characteristic curves for a typical *centrifugal pump* are presented *in fig. 11-14*. The head-discharge

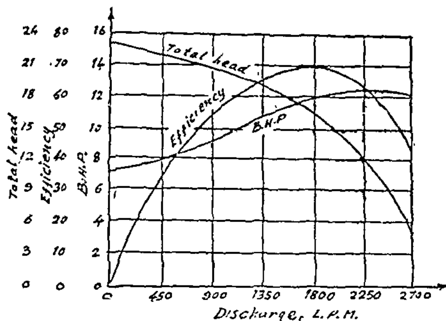


Fig. 11-14 Performance characteristics of a C F. pump

characteristic, after indicating a maximum head near the zero discharge, descends steadily; the head generated, when the efficiency is maximum is only 75% of the head corresponding to zero discharge. The power discharge characteristic rises and gives maximum horse power at the discharge corresponding to maximum efficiency.

The set of curves H-Q, P-Q and η-Q carry the name, *working characteristics*, moreover the $H - Q$ characteristic called the *main one* and a point on the η-Q curve having highest efficiency is called the *working point*, as it represents the optimum working conditions for the pump. On the basis of these curves it is possible to obtain a full presentation of the working of the pump.

Geometrically Similar Pumps and Model Testing

As in the case of turbines, the conception of geometrical similarity can be extended to centrifugal pumps. Two geometrically similar pumps, operating under different hydraulic conditions, will have same manometric efficiency. For such pumps, their velocity triangles will be similar, and magnitude of all the velocities will be proportional to square root of their manometric heads. On the basis of this, the values of various constants for geometrically similar pumps can be derived as follows :

As, $v = \frac{\pi DN}{60}$ and $v \propto \sqrt{H_m}$

$$DN \propto \sqrt{H_m}$$

$$\therefore \quad D \propto \frac{\sqrt{H_m}}{N}$$

$$\text{or} \quad \frac{\sqrt{H_m}}{DN} = \text{constant} \qquad ..(11{\cdot}15)$$

Now considering discharge, Q of the pump

$$Q = k\pi DbV_f$$

$$\therefore \quad Q \propto DbV_f$$

$$\propto D^2\sqrt{H_m} \text{ as } b \propto D \text{ and } V_f \propto \sqrt{H_m}$$

$$\propto \frac{H_m}{N^2}\sqrt{H_m} \text{ as } D \propto \frac{\sqrt{H_m}}{N}$$

$$\propto \frac{H_m^{\frac{3}{2}}}{N^2}$$

$$\therefore \quad \frac{\sqrt{Q}\cdot N}{H^{\frac{3}{4}}} = \text{constant} \qquad ..(11{\cdot}16)$$

Considering power to be supplied to pump,

$$\text{S.H.P., } P = \frac{\text{W.H.P.}}{\eta_m} = \frac{WH_m}{k\eta_m} = \frac{wQH_m}{k\eta_m}$$

$$\therefore \quad P \propto QH_m$$

$$\propto \frac{H_m^{\frac{3}{2}}}{N^2} \times H_m \quad \text{as } Q \propto \frac{H_m^{\frac{3}{2}}}{N^2}$$

$$\propto \frac{H_m^{\frac{5}{2}}}{N^2}$$

$$\therefore \quad \frac{\sqrt{P}N}{H_m^{\frac{5}{4}}} = \text{constant} \qquad ..(11\cdot17)$$

Rewriting eqn. (11·16) and (11·17) in terms of D, we have,

$$\frac{\sqrt{Q}N}{(ND)^{\frac{3}{2}}} = \frac{\sqrt{Q}}{N^{\frac{1}{2}}D^{\frac{3}{2}}} = \text{constant}$$

$$\text{or} \quad \frac{Q}{ND^3} = \text{constant} \qquad ..(11\cdot18)$$

$$\text{and} \quad \frac{\sqrt{P}N}{(DN)^{\frac{5}{2}}} = \frac{\sqrt{P}}{D^{\frac{5}{2}}N^{\frac{3}{2}}} = \text{constant}$$

$$\therefore \quad \frac{P}{D^5N^3} = \text{constant} \qquad ..(11\cdot19)$$

From eqns. (11·18), (11·15), (11·19) it can be seen that for a given pump,

$$\left.\begin{array}{ll} \text{discharge, } Q & \propto \text{speed, } N \\ \textit{head, } H_m & \propto (\textit{speed, } N)^2 \\ \text{and power } P & \propto (\text{speed, } N)^3 \end{array}\right\} \qquad ..(11\cdot20)$$

Thus, from the given operating conditions for a pump, running at one speed, the new operating conditions, namely Q, H_m and P for another speed can be deduced from the above relations This set of relationships is very useful in manipulating the operating conditions of a pump by simply changing the speed of its rotation.

Further, considering two similar pumps, following relations can be easily established from eqns. (11·15) to (11·19).

$$\frac{\sqrt{H_{m1}}}{D_1N_1} = \frac{\sqrt{H_{m2}}}{D_2N_2} \qquad ..(11\cdot21)$$

$$\frac{Q_1}{D_1^3N_1} = \frac{Q_2}{D_2^3N_2} \qquad ..(11\cdot22)$$

$$\frac{P_1}{D_1^5N_1^3} = \frac{P_2}{D_2^5N_2^3} \qquad ..(11\cdot23)$$

$$\frac{N_1\sqrt{Q_1}}{(H_{m1})^{\frac{3}{4}}} = \frac{N_2\sqrt{Q_2}}{(H_{m2})^{\frac{3}{4}}} \qquad ..(11\cdot24)$$

The above relations can be used in envisaging the performance of a proto-type pump from the laboratory test results of a

geometrically similar model. This technique of model testing is extremely useful in the design practice of pumps. Before a newly designed pump is manufactured, a geometrically similar model is constructed and tested in the laboratory. Concluding from the test results of the model pump, necessary modifications are effected on it till satisfactory performance is secured. Then only the actual pump is constructed. This helps in minimizing the cost of designing and testing of the pump.

Specific Speed

Centrifugal pumps are available in different varieties, handling different quantities of water at different heads. In order to be able to compare one class of pumps with another, the concept of specific pump is developed. Under it, is thought of such a pump which being geometrically similar to the given pump and having identical efficiency, imparts to the liquid an energy of 1 H. P. while it is being raised through head, $H = 1$ m and discharging 75 litres/sec.

The understanding of the concept of specific pump has great practical value. In fact, specific pumps have small dimensions and heads. This permits to study them in the laboratory through experiments the aim of which is to perfect the design which is to be adopted for manufacturing a series of that type of pumps.

The specific speed, N_s of a pump is that speed in rpm. at which the specific pump performs for head, $H = 1$ m, power $P = 1$ H.P. and discharge $Q = 75$ litres/sec. For actual values of the quantities discharge Q, head H, and rpm. N, the specific speed is

$$N_s = \frac{3{\cdot}65\, N\sqrt{Q}}{H^{\frac{3}{4}}} \quad \ldots (11{\cdot}25)$$

It may be noted that, the speed, N_s of the specific pump itself constitutes a characteristic not only for itself, but, for all pumps which are geometrically similar to it. The specific speed permits the following classification of centrifugal pumps according to the range of speed.

(a) Slow speed pumps : For them $50 < N_s < 100$; these pumps have lower discharge, but they develop very high head.

(b) Normal or medium-speed pumps : For them $100 < N_s < 200$; they are characterized by the fact that with increase in discharge, the head decreases.

(c) High-speed pumps : For them $200 < N_s < 350$; they develop not very high heads, for a very large discharge.

Problem-17 : *A propeller pump 20 cm in diameter was found to be most efficient when delivering 5,600 litres/sec. of water at 1,200 rpm against a head of 6 m. A similar pump is required to deliver 1,400 litres/sec. at 700 rpm. Calculate diameter of second pump, and the head developed by it.*

Using eqn. (11·24)

$$\frac{Q_1 N_1^2}{(H_{m_1})^{\frac{3}{2}}} = \frac{Q_2 N_2^2}{(H_{m_2})^{\frac{3}{2}}}$$

or
$$\frac{5{,}600 \times (1{,}200)^2}{(6)^{\frac{3}{2}}} = \frac{1{,}400 \times (700)^2}{(H_{m_2})^{\frac{3}{2}}}$$

$$\therefore \quad (H_{m_2})^{\frac{3}{2}} = \frac{1{,}400}{5{,}600} \times \left(\frac{700}{1{,}200}\right)^2 \times (6)^{\frac{3}{2}} = 1{\cdot}25$$

$$H_{m_2} = (1{\cdot}25)^{\frac{2}{3}} = \underline{1{\cdot}161 \text{ m of water}}$$

Again, using eqn. (11·21),

$$\frac{N_1 D_1}{\sqrt{H_{m_1}}} = \frac{N_2 D_2}{\sqrt{H_{m_2}}}$$

$$\therefore \quad D_2 = D_1 \times \frac{N_1}{N_2} \times \sqrt{\frac{H_{m_2}}{H_{m_1}}}$$

$$= 20 \times \frac{1{,}200}{700} \sqrt{\frac{1{\cdot}161}{6}}$$

$$= \underline{15{\cdot}1 \text{ cm}}$$

Problem-18 : *It is required to predict the performance of a large centrifugal pump from that of a scale model of one fourth the diameter. The model absorbs 25 H P when pumping under the test head of 7·58 m at its best speed of 360 rpm. The large pump is required to pump against 21 m head. What will be its working speed, the H.P. required to drive it and what will be the ratio of quantities discharged by the larger pump and the model ?*

Using eqn. (11·24),

$$\frac{N_1 D_1}{\sqrt{Hm_1}} = \frac{N_2 D_2}{\sqrt{Hm_2}}$$

$$\therefore \quad N_2 = N_1 \times \frac{D_1}{D_2} \times \sqrt{\frac{Hm_2}{Hm_1}}$$

$$= 360 \times \frac{1}{4} \times \sqrt{\frac{21}{7 \cdot 5}} = 150 \text{ rpm.}$$

Again, using eqn. (11·24),

$$\frac{N_1 \sqrt{Q_1}}{(Hm_1)^{\frac{3}{4}}} = \frac{N_2 \sqrt{Q_2}}{(Hm_2)^{\frac{3}{4}}}$$

$$\therefore \quad \sqrt{\frac{Q_2}{Q_1}} = \frac{N_1}{N_2} \times \left\{\frac{Hm_2}{Hm_1}\right\}^{\frac{3}{4}}$$

$$\therefore \quad \frac{Q_2}{Q_1} = \left\{\frac{N_1}{N_2}\right\}^2 \times \left\{\frac{Hm_2}{Hm_1}\right\}^{\frac{3}{2}}$$

$$= \left(\frac{360}{150}\right)^2 \times \left(\frac{21}{7 \cdot 5}\right)^{\frac{3}{2}} = 27$$

Hence, discharge of the proto-type is 27 times the discharge of the model.

Now, using eqn. (11·23),

$$\frac{P_1}{D_1^5 N_1^3} = \frac{P_2}{D_2^5 N_2^3}$$

$$\therefore \quad P_2 = P_1 \left\{\frac{D_2}{D_1}\right\}^5 \times \left\{\frac{N_2}{N_1}\right\}^3$$

$$= 25 \times (4)^5 \left(\frac{150}{360}\right)^3$$

$$= 1{,}870 \text{ H.P.}$$

Problem-23 : *A 15 cm pump running at 1,800 rpm. delivers 4,000 litres per minute against a head of 25 m and requires 20 H.P. to drive it. Determine the discharge, head developed and power required for the pump when running at 2,000 rpm.*

Using eqn. (11·20),

Discharge, $Q \propto$ speed, N

$$\therefore \quad \frac{Q_2}{Q_1} = \frac{N_2}{N_1}$$

i.e. $Q_2 = 4{,}000 \times \frac{2{,}000}{1{,}800} = \underline{4{,}430 \text{ litres/min.}}$

Head, $H_m \propto$ (speed, N)^2

$\therefore \quad H_{m_2} = H_{m_1} \times \left(\frac{N_2}{N_1}\right)^2 = 25 \times \left(\frac{2{,}000}{1{,}800}\right)^2 = \underline{30{\cdot}8 \text{ m}}$

and power, $P \propto$ (speed, N)^3

$\therefore \quad P_2 = P_1 \left\{\frac{N_2}{N_1}\right\}^3 = 20 \times \left\{\frac{2{,}000}{1{,}800}\right\}^3 = \underline{27{\cdot}28 \text{ H.P.}}$

Problem-24: *A single stage centrifugal pump which has an impeller diameter of 30 cm, rotates at 2,000 rpm., and lifts 2,600 litres of water per minute to a height of 32 m with an efficiency of 80%. Obtain the number of stages and the diameter of each impeller of a similar multi-stage pump to lift 4,000 litres per minute to a height 200 m when rotating 1,600 rpm.*

Total theoretical head $= \frac{H_s}{\eta} = \frac{32}{0{\cdot}8} = 40$ m

Now, constant for the pump (specific speed) $= \frac{N\sqrt{Q}}{(40)^{\frac{3}{4}}}$

$= \frac{2{,}000\sqrt{2{,}000}}{(40)^{\frac{3}{4}}}$

$= 5{,}620$

For a single stage of the multi-stage pump, let the theoretical head be x metres.

$\therefore \quad 5{,}620 = \frac{N\sqrt{Q}}{(x)^{\frac{3}{4}}}$

$= \frac{1{,}600\sqrt{4{,}000}}{(x)^{\frac{3}{4}}}$

$\therefore \quad (x)^{\frac{3}{4}} = \frac{1{,}600 \times 63{\cdot}2}{5{,}620} = 18{\cdot}0$

$\therefore \quad x = 47{\cdot}5$ m

Actual lift $= 0{\cdot}8 \times 47{\cdot}5 = 37{\cdot}6$ m

$\therefore$ no. of stages required $= \frac{\text{total lift}}{\text{lift per stage}} = \frac{200}{37{\cdot}6}$

$= 5{\cdot}31$ say $\underline{6 \text{ stages}}$

Actual head per stage = $H = \frac{200}{6} = 33{\cdot}3$ m

$$\text{Now,} \quad \frac{N_1 D_1}{\sqrt{H_{m1}}} = \frac{N_2 D_2}{\sqrt{H_{m2}}}$$

$$\therefore \quad D_2 = D_1 \frac{N_1}{N_2} \times \sqrt{\frac{H_{m2}}{H_{m1}}} = 30 \times \frac{2{,}100}{1{,}600} \times \sqrt{\frac{33{\cdot}3}{32}}$$

$$= \underline{38{\cdot}1 \text{ cm}}$$

i.e. diameter of impeller should be 38·1 cm.

Limits of Suction Lift

Suction of the liquid forms one of the important processes of the pump. The proper functioning of the pump depends upon correct suction. Suction or lifting is due to the atmospheric pressure. It follows that under normal pressure, under complete vacuum (airless space) in the pump, and under absence of any loss of head, liquid can be raised to 10 m, above the level of water in sump from which it is derived.

Here, $Z_0 = 0$, $V_0 = 0$; $\frac{p_0}{w}$ = atmospheric pressure head, H_{atm}; V_1 = velocity of liquid in suction pipe, V_s; Z_1 = static suction lift, h_s (*i e.* height of the centre line of the pump above the liquid level in the sump); p_1 consists of pressures of vapour and air; and losses, h_l consist of frictional resistances in foot valve and suction pipe. Thus, the above equation is rewritten as

$$h_s = H_{atm} - \frac{p_{vapour}}{w} - \frac{V_s^2}{2g} - h_l \qquad ..(11\text{-}26)$$

The usual value of the vapour pressure head is 1 to 2 m of water column, and since the velocity in the suction pipe does not exceed 1 to 2 m/sec. the velocity head is not big and may be neglected. Hence, h_s is usually limited to 6 to 7 m of water.

Priming

The pressure generated by the impeller of the centrifugal pump is proportional to the density of the fluid filling the vane passages. If the pump is at rest for some time, water from the casing and the suction pipe leaks out to the sump and thus casing and pipe remain filled with air. The pump, when started under this condition, will produce only negligible pressure difference across the impeller which is insufficient to create proper vacuum to enable water to rise along the suction pipe to reach the impeller. Thus, it is necessary to first fill the space in the casing and suction pipe with the liquid to be pumped. This operation is known as priming.

For small pumps, priming is done by pouring liquid directly in the casing through a funnel The air vent cock provided above the casing is kept open. When all the air has been displaced from the suction pipe and the pump casing and the system is throughout filled with liquid the cocks are closed and the pump started. Larger pumps are primed by evacuating the casing and suction pipe with the help of a vacuum pump or a steam ejector. Specially designed self-priming pumps are also available. In these pumps, priming is done by providing a special reservoir in the suction pipe. It becomes possible for the pump to prime the casing and suction pipe automatically.

Here, $Z_0 = 0$, $V_0 = 0$; $\frac{p_0}{w}$ = atmospheric pressure head, H_{atm}; V_1 = velocity of liquid in suction pipe, V_s; Z_1 = static suction lift, h_s (*i.e.* height of the centre line of the pump above the liquid level in the sump); p_1 consists of pressures of vapour and air; and losses, h_l consist of frictional resistances in foot valve and suction pipe. Thus, the above equation is rewritten as

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Bore-hole Pump

For water supply and minor irrigation schemes bores are drilled even up to 500 m depth. The pumps used are multi-stage pumps with vertical spindles. The pump is suspended on one end of the shaft and is allowed to be well under water. The prime-mover (*i.e.* engine or electric motor) is installed at the ground level. The shaft is co-axial with the rising main or the delivery pipe and is supported by a number of bearings which are water-lubricated. A representative bore-hole pumping installation is outlined in fig. 11-15 The overall efficiency of such a pump is about 70%. Such pumps yield discharge varying from 1,350 to 11,300 litres/min.

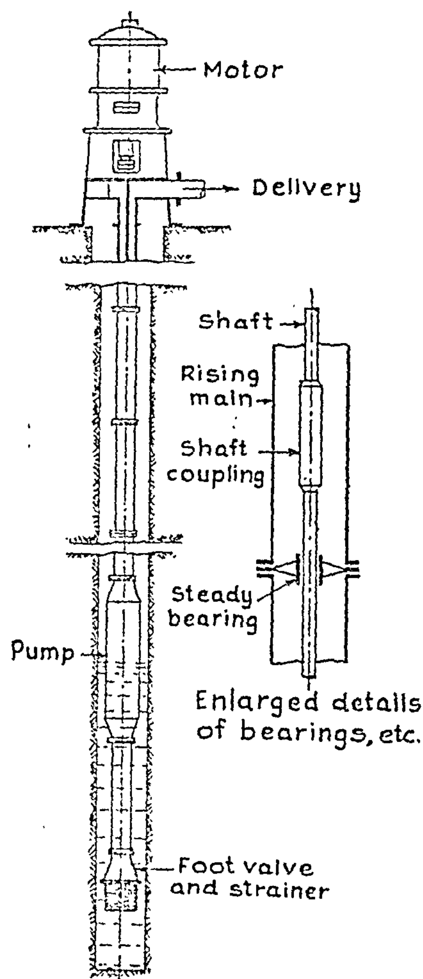


Fig. 11-15 Outline of bore-hole pump installation

The disadvantages of long shaft, connecting the motor, and pump with numerous bearings, may be overcome by adopting special motors designed to work under water. A complete unit, consisting of submergible motor directly coupled to the pump is lowered down in the bore-hole or well. This type of pump is known as submersible pump.

TUTORIAL - 11

Principle and working

1 Explain the general features of construction and working of a centrifugal pump. Why it is termed as dynamic pressure generator ?

2 Explain the conditions in which two pumps may be operated in series or in parallel.

3 What is a multi-stage pump and what are its advantages ?

Theory (fundamental equation and efficiencies)

4 Prove that the minimum speed of the pump for the delivery to commerce is given by

$$N = \frac{60}{\pi}\sqrt{\frac{2gH}{(D_2^2 - D_1^2)\,\eta}}$$

where, H, η, D_1, D_2, are the static lift, efficiency, inlet and outlet impeller diameters respectively.

5 Prove that the pressure head developed across the impeller is given by

$$\frac{V_{f1}^2 + v_2^2 - V_{f2}^2 \operatorname{cosec}^2\phi}{2g}$$

where V_{f1}, v_2, V_{f2} and ϕ are flow velocity at inlet, peripheral velocity, flow velocity at outlet, and blade angle at outlet of the impeller respectively. Assume radial entry.

6 Give a short account of the various methods which have been adopted to increase the efficiency of a centrifugal pump (by altering the shape of the casing or chamber surrounding the impeller).

7 Write short notes on the following, giving neat sketches wherever possible.

(a) Volute chamber, (b) Diffusers, and (c) Whirl pool chamber.

8 Explain the terms, hydraulic efficiency, volumetric efficiency, mechanical efficiency, and overall efficiency in case of a hydrodynamic pump.

9 Define the following terms :

(a) Static head, (b) Friction head, (c) Manometric head in connection with a centrifugal pump.

10 The impeller of a centrifugal pump is 0·75 m in diameter and rotates at 1,000 rpm.; the velocity of flow is constant from inlet to outlet and is equal to 6 m/sec. The vanes are curved back at an angle of 35° to the tangent at outlet tip. The inlet diameter of the impeller is 0·4 m and water enters the impeller radially. Determine the work done per kg of water and angle of vane at inlet.

Find also the theoretical maximum lift to which water can be raised if the pump is provided with whirlpool chamber which reduces the velocity of water by 50%.

[123 kg-m ; 16°; 110·5 m]

11 The following data were obtained from a test on a centrifugal pump :

Reading of vacuum gauge on suction side	.. 3 m of water.
Reading of pressure gauge on delivery side	.. 20 m of water,
Difference of levels between the gauges on the suction and delivery side	.. 0·5 m
Discharge of the pump	.. 13 litres/sec.
Power required to drive the pump ..	.. 6 H. P.

Find the efficiency of the pump.

[67·8%]

12 A centrifugal pump impeller has 500 mm external diameter and 250 mm internal diameter. It runs at 950 rpm. The vanes are set back at an angle of 35° to the outer rim. If the radial velocity of water through the wheel is 2 m/sec. from inlet to outlet, find the angle of the vane at inlet, the velocity and direction of water at outlet and the work done per kg/sec.

[9·15°; 22·2 m/sec, 5·2°; 56 kg-m]

13 A centrifugal pump having wheel of 0·3 m diameter pumps water to a height of 25 m. The velocity of flow is constant through the wheel and is equal to 3 m/sec. The width of the

impeller at exit is 75 mm The vanes are radial at exit. The velocity in the suction and delivery pipes are 2·5 m/sec. and 1·5 m/sec. respectively. Neglecting frictional losses, determine, (i) the speed of rotation of the pump, (ii) H.P. of the pump, (iii) angle of the vane at inlet if the inlet diameter is half that at outlet.

[(i) 1,000 rpm ; (ii) 70·7 H P. (iii) 20·8°]

14 A centrifugal pump wheel has 50 cm external and 25 cm internal diameters It runs at 1,000 rpm. The vanes are set back at an angle of 35° to the outer rim If the radial velocity of water through the wheel be maintained constant at 1·8 m/sec, find the angle of the vanes at inlet; the velocity and direction of water at outlet and work done by the wheel per kg of water.

[7 8°, 23 8 m/sec ; 4 35°; 63·5 kg.m]

15 A centrifugal pump, designed to discharge 120 litres/sec when running at 400 rpm, has the impeller 20 cm in diameter at inlet and 50 cm in diameter at outlet The width of the impeller at inlet is 12 cm and at outlet is 8 cm The impeller vanes at the outlet are curved back to give a discharge angle of 28°. Neglecting friction and other losses in the impeller, calculate the head against which the pump should work at the designed discharge. What would be the inlet angle of the impeller vanes, if the entry to the pump is radial ?

[9 25 m, 13·72°]

16 A centrifugal pump runner has an external diameter of 36 cm an internal diameter of 18 cm, and runs at a speed of 1,500 rpm. The vanes are set back at an angle of 30° to the outer rim. Assuming a constant radial velocity of water through the runner as 2·5 m/sec, find, (a) the angle of vanes at inlet, (b) velocity and direction of water at outlet, and (c) the work done per kg of water.

[(a) 10 05°, (b) 24·13 m/sec, 5·94°, (c) 69 kg-m.]

17 A centrifugal pump discharges 8,000 L P M, when running at 240 rpm. The outlet vane angle is 30° Determine the rise in pressure as water passes through the impeller, given that the

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(a) Static head, (b) Friction head, (c) Manometric head in connection with a centrifugal pump.

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[123 kg-m ; 16°; 110·5 m]

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Discharge of the pump	13 litres/sec.
Power required to drive the pump	6 H. P.

Find the efficiency of the pump.

[67·8%]

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[9·15°; 22·2 m/sec, 5·2°; 56 kg-m]

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impeller at exit is 75 mm The vanes are radial at exit. The velocity in the suction and delivery pipes are 2·5 m/sec. and 1·5 m/sec. respectively Neglecting frictional losses, determine, (i) the speed of rotation of the pump, (ii) H.P. of the pump, (iii) angle of the vane at inlet if the inlet diameter is half that at outlet

[(i) 1,000 rpm ; (ii) 70·7 H P. (iii) 20·8°]

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[7 8°, 23 8 m/sec , 4 35°, 63·5 kg m]

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[(a) 10·05°, (b) 24 13 m/sec, 5·94°, (c) 69 kg-m.]

17 A centrifugal pump discharges 8,000 L P M. when running at 240 rpm. The outlet vane angle is 30° Determine the rise in pressure as water passes through the impeller, given that the

impeller is 30 cm in diameter at inlet and 60 cm. in diameter at outlet, 15 cm broad at inlet and 10 cm broad at outlet.

[2·84 m of water]

18 A centrifugal pump of 1·25 m diameter pumps 2 m^3/sec. through a height of 16 m when running at a speed of 300 rpm. The vanes are curved back at exit to an angle of 30° with the tangent at outlet. The radial velocity of flow is 2·5 m/sec. and is constant from inlet to outlet. Determine useful H.P., and the efficiency. Find also the lowest speed to start pumping against a head of 16 m, the inner diameter of the impeller being 0·5 m.

[815; 52·7%; 295 rpm.]

19 A centrifugal pump has to discharge 13,000 litres of water per minute and develop a head of 25 m when the impeller rotates at 1,500 rpm. Determine, (a) the diameter of the impeller, and (b) blade angle at the outlet edge of the impeller.

Assume the following :

Manometric efficiency is 75%; the absolute velocity of water at outlet is 16 m/sec.; the area of the impeller at outlet is 1·2 D_2^2 m^3 where D_2 is the diameter of the impeller in m; water enters the impeller without shock.

[(a) 26·4 cm; (b) 27·65°]

20 A centrifugal pump is placed with the centre of the impeller at a height of 3·5 m above water in the suction well. The suction pipe is 12·5 cm in diameter and the discharge is 1,250 litres/sec. The total head through which water is lifted is 25 m. The vanes of the impeller at exit are set back and make an angle of 150° with the tangent to the wheel. The radial velocity at exit from the wheel is 3 m/sec and the efficiency of the pump is 70%. Determine, (a) the velocity of the rim of the wheel, (b) the pressure at inlet and outlet of the wheel on the assumption that the whole loss of head takes place after water leaves the wheel.

[(a) 21·5 m/sec; (b) 6·713 & 28·56 m of water ab.]

21 A centrifugal pump is required to deliver 450 litres of water per second against a head of 6 m at a speed of 600 rpm. Assuming that all the velocity head is lost and that the actual head is 75% of the theoretical head, find the diameter and breadth of the impeller at outlet. The velocity of flow, taken as constant, is 3 m/sec and the blades are curved back at 30° to the tangent at outlet Also determine the inlet blade angles, if the inlet diameter is made half the outlet diameter.

[0·377 m; 0·127 m; 26° 51′]

22 A centrifugal pump is employed to pump water from a river into a canal. The pump is fixed with its centre at a height of 6 m above the level of water in the river, and the mouth of the delivery pipe is 9 m above the level of water in the river. The velocity of flow through the delivery pipe is 1·5 m/sec. If the angle made by the tangent to the blade with the tangent to the wheel at the discharge edge is 125°, and the radial velocity of flow through the wheel is 1·5 m/sec, determine, (i) the pressure head at the inlet circumference of the wheel, (ii) the pressure at outlet circumference of wheel.

[(i) 4·25 & (ii) 9·21 m of water abs.]

23 *The following data refer to a centrifugal pump :*

Static head	35 m
Suction height	4 m
Diameter of the suction pipe	15 cm
Diameter of the delivery pipe	*15 cm*
Loss of head in suction pipe	2 m
Loss of head in delivery pipe	7 m
Impeller diameter at outlet	40 cm
Impeller width at outlet	2·5 cm
Speed	1,200 rpm
Manometric efficiency	80%
Overall efficiency	70%

The vanes are set back at an angle of 30° with the tangent at outlet.

Find the H. P. required to drive the pump and the discharge in litres/sec. What pressure will be indicated by gauges mounted on suction and delivery sides ?

[44·7 H.P.; 67·1 litres/sec.
3·265 m & 35·3 m of water ab.]

24 It is proposed to build a three-stage centrifugal pump to handle 60 litres of water per second at a speed of 1,000 rpm. under manometric head of 72 m. The vanes are to be radial at inlet and to be curved backwards at exit at an angle of 30° to the tangent at outer rim. Assuming manometric efficiency of 75% and overall efficiency of 65% and considering that vane thickness accounts for 8% of circumferential area, determine the probable external diameter and width of each impeller and the H. P. input necessary. The velocity of flow may be assumed constant at 2·5 m/sec. (Assume, the stages are in series.)

[37·4 cm; 2·04 cm; 29·6]

25 A centrifugal pump with a manometric efficiency of 0·8 operates against a manometric head of 25 m. Assume the radial velocity of flow constant and that the increase in pressure through the impeller is 65% of the total head generated by the pump. Determine, (a) the operating speed in rpm., and (b) the discharge in litres/sec. The impeller outlet diameter is 20 cm and width 1·5 cm. The blades at outlet are set back at 60° with the tangent. Impeller losses may be neglected.

[(a) 2,000; (b) 10·38]

26 A centrifugal pump with a spiral volute has impeller outlet and inlet diameters 35 cm and 17·5 cm respectively. It is proportioned to give constant radial velocity and vane at outlet makes an angle of 155° with the direction of peripheral velocity. The static lift is 17 m and when dealing with 110 litres/sec., there is an additional 5 m head due to friction, discharge velocity, etc. For a speed of 1,150 rpm ; and manometric efficiency 70%, determine, (a) the impeller width at outlet, (b) the vane

12
RECIPROCATING PUMPS

Introduction

A reciprocating pump is known as a *positive displacement* pump because if the liquid reaches the cylinder and a delivery or return stroke follows, the liquid is bound to get out through the delivery valve unlike as in a centrifugal pump in which the impeller may be rotating, but there may not be delivery of water. Positive displacement pumps invariably consist of one or more chambers which are alternately filled with liquid to be pumped and then *emptied again, their rate of discharge consequently depends almost* wholly on the speed of rotation and hardly upon the working pressure; whereas, as discussed in chapter 11, the discharge from the centrifugal pump is directly influenced by the pressure or resistance against which it works. Thus, behaviour of positive pumps is much different from that of the centrifugal pumps.

Working of a Reciprocating Pump

The elements of a reciprocating pump are a *piston* or *plunger* moving to and fro in a *cylinder* or *barrel*. Thus, pump is driven *through the crank* and *connecting rod* mechanism. Water is sucked through the suction valve and is sent out through delivery valve as shown in fig. 12-1.

During the suction stroke, the piston moves to the right, creating vacuum in the cylinder. The atmospheric pressure acting

Model testing

31 Describe the assumptions made in deriving the similarity conditions for hydrodynamic machines.

32 Define speed and show that the specific speed for model and actual machine must be same for similarity considerations.

33 Bring out the importance of model testing during stages of pump manufacture.

34 Prove that for a given pump:

Discharge $\propto$ speed,

Head $\propto$ $(\text{speed})^2$,

Power $\propto$ $(\text{speed})^3$.

35 A centrifugal pump having a 25 cm diameter impeller develops a head of 12 m when running at 1,000 rpm. and discharging 100 litres/sec. Determine the impeller diameter and rpm. for a geometrically similar pump discharging 1,250 litres/sec. against a head of 6 m when running under dynamically similar conditions. Prove any formula you use.

[105 cm; 169 rpm.]

Suction lift

36 What are the factors which limit the suction lift of the pump and what is the maximum value of suction lift in practice ?

37 Oil of specific gravity 0·9 is lifted by a centrifugal pump from an oil tank which is open to atmosphere. The loss of head in the suction pipe and foot–valve amounts to 1·5 m. The velocity in the suction pipe is 3 m/sec. If the oil starts vaporising at a pressure of 0·5 kg/cm², estimate the maximum permissible suction lift. The barometer reading is 760 mm of mercury.

[3·98 m]

Priming

38 What is priming ? Why it is necessary ? What are the various methods of priming ?

Bore-hole pump

39 Describe the construction and working of a bore-hole pump. Which are the special fields of its applications ?

on the surface of water in the sump forces water into the suction pipe and in turn water opens the suction valve and enters the cylinder. On the return stroke of the piston, the pressure in the cylinder is increased which closes the suction valve and opens the delivery valve. Thus, water is forced through the delivery pipe to the required height. The cycle is repeated.

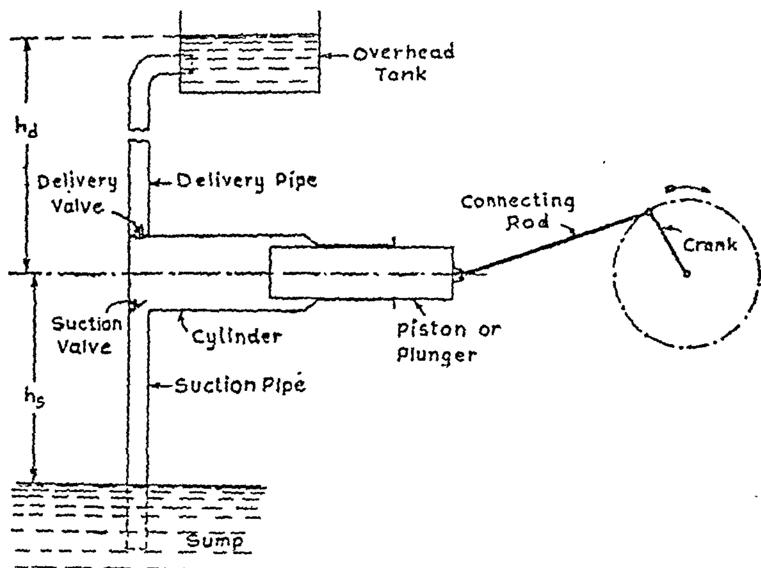


Fig. 12-1 Schematic diagram of a reciprocating pump

If water acts on one side of the piston, the pump is said to be *single-acting*. In this case water is sucked in during outward stroke and forced out during inward stroke, *i.e.* there is one suction and one delivery taking place in one revolution. If water is pumped by both sides of the piston, both suction and delivery take place in every stroke, *i.e.* water is sucked on one side of piston and simultaneously water is delivered from the other side during the same stroke and these two phenomenas reverse during the other stroke; such a pump is said to be *double-acting*.

Pumps which raise water by suction only are known as *suction* or *lift pumps*. Such pumps are suitable only for low lift, as the maximum height to which water can be lifted is theoretically equal to the barometric height and is actually less than about 8 m. Pumps which lift water with the help of pressure generated by the piston or plunger are known as *force pumps*.

In order to obtain better balance and reduce inertia head in the pipes, a pump set may consist of more than one cylinders. A two cylinder pump is known as *duplex pump* and a three cylinder pump is known as *triplex pump*. These pumps are discussed later on.

Theory of the Pump

Referring to fig. 12.1,

let, $A = \frac{\pi}{4} D^2$, area of piston,

$A_s = \frac{\pi}{4} d_s^2$, area of cross section of suction pipe,

$A_d = \frac{\pi}{4} d_d^2$, area of cross-section of delivery pipe,

r = radius of crank,

l = length of connecting rod,

$L = 2r$ = length of stroke,

h_s = height of centre line of cylinder above water surface in the sump,

h_d = height to which water is pumped above the centre line of cylinder,

h_{atm} = atmospheric pressure head (10 m of water)

l_s = total length of suction pipe,

l_d = total length of delivery pipe,

a = acceleration of the piston,

$\omega = \frac{2\pi N}{60}$ = angular velocity of crank (assumed uniform).

N – revolutions per minute of the crankshaft,

v – velocity of piston,

V_s – velocity of water in suction pipe,

V_d – velocity of water in delivery pipe,

f – coefficient of friction in pipes,

Q – volume of water pumped per second,

W – weight of water actually lifted per second,

w – density of water, and

p – pressure of water.

Then, total height through which water is lifted is $h_s + h_d$.

Velocity head of water leaving delivery pipe $= \dfrac{V_d^2}{2g}$

Neglecting friction losses in the pipe,

total head generated, $H = h_s + h_d + \dfrac{V_d^2}{2g}$

As V_d is usually small and varies during the stroke, it may be neglected unless the total head, $(h_s + h_d)$ is small.

Then, work done $= W \times H$

$$= W(h_s + h_d) \quad \ldots \quad \ldots \quad \ldots (12\cdot1)$$

Theoretical horse power required $= W(h_s + h_d)\,K \quad \ldots (12\cdot2)$

The actual horse power required would be greater than this on account of friction losses of water and mechanical parts and of leakage. K is a conversion factor for H.P. $\left(\dfrac{1}{75}\text{ for m.k.s. units.}\right)$

Efficiency of the pump : The performance of a pump is expressed as gross or overall efficiency. This is defined as :

$$\text{gross or overall efficiency, } \eta_o = \frac{\text{hydraulic energy output}}{\text{mechanical energy input}}$$

$$= \frac{\text{water horse power (W.H.P.)}}{\text{shaft horse power (S.H.P.)}} \ldots (12\cdot3)$$

It may be noted that this expression is the inverse of the expression for turbine efficiency. Further, the effective head, H_m will be greater than the static lift, $h_s + h_d$, because of friction and secondary losses

The water horse power, W.H.P. under different operating data can be calculated as shown below.

$$\text{W.H.P.} = \frac{WH_m}{75} \text{ (}W\text{ in kg/sec., } H_m \text{ in metres)}$$

$$= \frac{Q \times p}{7.5} \text{ (}Q\text{ in litres/sec., } p \text{ in kg/cm}^2\text{)}$$

Coefficient of discharge and slip: In practice, the volume of water discharged by the pump is different from the volume swept through by piston or plunger. This may be on account of the following:

(i) Leakage of water past the plunger, valves and through the glands–This will cause reduction in the discharge.

(ii) Inability of water to follow closely the piston–This will cause reduction in discharge.

(iii) Lag in the valve closure–This also will cause reduction in discharge.

(iv) Opening of suction valve during delivery stroke or of delivery valve during suction stroke–This will cause increase in discharge. This is discussed in detail later in this chapter.

The ratio of the actual volume discharged, Q to swept volume, Q_s of the piston per unit time is termed as *coefficient of discharge* or *volumetric efficiency*.

$$\text{The, coefficient of discharge, } C_d = \frac{\text{actual volume discharged}}{\text{swept volume}}$$

$$= \frac{Q_a}{Q_s}$$

N – revolutions per minute of the crankshaft,

v – velocity of piston,

V_s – velocity of water in suction pipe,

V_d – velocity of water in delivery pipe,

f – coefficient of friction in pipes,

Q – volume of water pumped per second,

W – weight of water actually lifted per second,

w – density of water, and

p – pressure of water.

Then, total height through which water is lifted is $h_s + h_d$.

Velocity head of water leaving delivery pipe $= \frac{V_d^2}{2g}$

Neglecting friction losses in the pipe,

total head generated, $H = h_s + h_d + \frac{V_d^2}{2g}$

As V_d is usually small and varies during the stroke, it may be neglected unless the total head, $(h_s + h_d)$ is small.

Then, work done $= W \times H$

$$= W(h_s + h_d) \quad \ldots \quad (12{\cdot}1)$$

Theoretical horse power required $= W(h_s + h_d)K$ .. (12·2)

The actual horse power required would be greater than this on account of friction losses of water and mechanical parts and of leakage. K is a conversion factor for H.P. $\left(\frac{1}{75} \text{ for m.k.s. units.}\right)$

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(i) Leakage of water past the plunger, valves and through the glands–This will cause reduction in the discharge.

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(iv) Opening of suction valve during delivery stroke or of delivery valve during suction stroke–This will cause increase in discharge. This is discussed in detail later in this chapter

The ratio of the actual volume discharged, Q to swept volume, Q_s of the piston per unit time is termed as *coefficient of discharge* or *volumetric efficiency*.

$$\left.\begin{aligned} \text{The, coefficient of discharge, } C_d &= \frac{\text{actual volume discharged}}{\text{swept volume}} \\ &= \frac{Q_a}{Q_s} \end{aligned}\right\} \quad \ldots(12\cdot4)$$

Now, swept volume per sec.,

$$\left.\begin{aligned} Q_s &= \frac{ALN}{60} \text{ for single-acting pump} \\ &= \frac{2ALN}{60} \text{ for double-acting pump} \end{aligned}\right\} \quad ..(12\cdot5)$$

The difference between the volume swept through by the plunger and actual discharge is called the *slip*.

$$\textit{i.e.} \quad \text{slip} = \begin{pmatrix}\text{volume swept by} \\ \text{plunger per sec.,}\end{pmatrix} - \begin{pmatrix}\text{actual discharge} \\ \text{per sec.}\end{pmatrix}$$

$$= Q_s - Q_a \quad ..(12\cdot6)$$

and percentage slip, $S = \dfrac{Q_s - Q_a}{Q_s} \times 100$

$$\left.\begin{aligned} &= \left(1 - \frac{Q_a}{Q_s}\right) \times 100 \\ &= (1 - C_d) \times 100 \end{aligned}\right\} \quad ..(12\cdot7)$$

In well maintained pumps, the percentage slip is of the order of 2% or even less. But, for delivery pressures above 700 m, due to compressibility effect, the slip increases, especially if the clearance volume of the cylinder is excessive.

Simple indicator diagram : The principle of indicator diagram of a reciprocating pump is similar to the indicator diagram of heat engines. The diagram shows the pressure on one side of the plunger or piston during a cycle of operation. Pressure is plotted on vertical ordinate while horizontal ordinate represents stroke length. Fig. 12-2 represents the indicator diagram for a simple reciprocating pump in which acceleration and friction in pipes

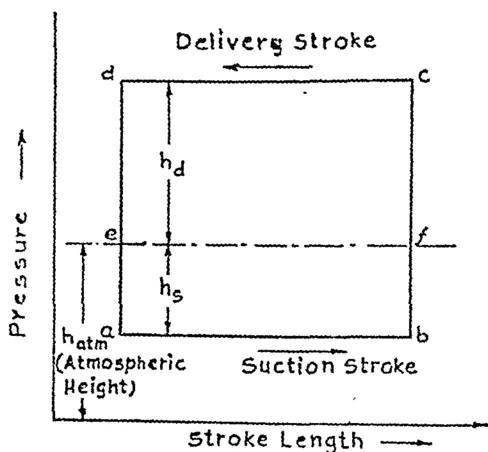


Fig. 12-2 Simple indicator diagram

are neglected. The horizontal line *ef* represents atmospheric pressure. The line *ab* represents the pressure in the cylinder during the suction stroke, it being below the atmospheric line by amount h_s. The line *cd* represents the pressure in the cylinder during delivery stroke, it being above the atmospheric line by amount h_d. The width of the diagram which represents stroke length can also represent weight of water sucked in or delivered out per cycle to some other scale. Thus, the area of the indicator diagram represents work done. Hence, area *abfe* represents work done during suction stroke, while area *fcde* represents work done during delivery stroke. The total work done per cycle is represented by the area *abcd*. This will also be the work done per revolution of the crank for single-acting pumps. If the pump is double-acting, the work done is approximately twice this amount, since on the crank end the useful volume of water sucked is reduced by the piston rod volume.

As in the case of heat engines, such diagrams can be obtained with the help of an indicator placed on the cylinder. An actual indicator diagram taken by an indicator would be similar to fig. 12-2 if the pump were a low speed one.

Effect of acceleration head Owing to the to and fro motion of the plunger or piston and its coming to standstill positions at ends of strokes, its instantaneous velocity and acceleration will vary from point to point. Since water closely follows the piston in its movement, the velocity and acceleration of the piston will be transmitted to water in suction pipe during the suction stroke and to water in delivery pipe during the delivery stroke. The piston will have no velocity, but, it will have maximum acceleration at the beginning of the stroke. The acceleration of the piston goes on reducing while the velocity increases till about the middle of the stroke, when acceleration is zero and velocity is maximum. The acceleration thereafter becomes negative (retardation) and velocity decreases till the end of the stroke when the retardation is maximum and velocity is zero. It is

obvious that if the suction and delivery pipes are long, the inertia of water columns in these pipes will cause a variation of pressure in the cylinder. Acceleration will cause reduction in pressure while retardation will cause increase in pressure.

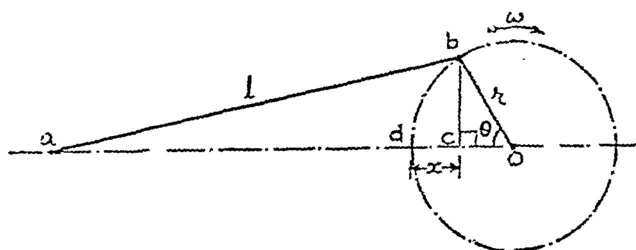


Fig. 12-3 Crank mechanism

In order to simplify the problem, it is usual to assume that the piston moves with simple harmonic motion. This would be the case if connecting rod were very long compared with the crank. Referring to fig. 12-3, the crank is rotating with uniform angular velocity, ω. Noting time from the instant when the crank is at the inner dead centre, *i.e.* piston being at the commencement of the stroke, at time of seconds later, the crank would have rotated through an angle $\theta = \omega t$, after some time, t and this would cause the piston to move forward by an amount,

$$x = dc = do - co = r - r\cos\theta$$

$$= r(1 - \cos\omega t)$$

Then, velocity of piston, $v = \dfrac{dx}{dt} = \omega r \sin\omega t$ .. (12·8)

and, acceleration of piston, $a = \dfrac{dv}{dt} = \omega^2 r \cos\omega t$ .. (12·9)

Considering the suction stroke first, and knowing that volume of water flowing from pipe per second equals volume of water flowing into the cylinder per second,

velocity in suction pipe, V_s = velocity of piston × $\frac{\text{area of piston}}{\text{area of pipe}}$

$$= \omega r \sin\omega t \times \frac{A}{A_s}$$

$$\left.\begin{aligned} &= \frac{A}{A_s}\,\omega r \sin\theta \\ &= \left(\frac{D}{d_s}\right)^2 \omega r \sin\theta \end{aligned}\right\} \quad \ldots (12\cdot10)$$

and, acceleration in suction pipe, $\quad a_s = \frac{A}{A_s}\,\omega^2 r \cos\omega t$

$$\left.\begin{aligned} &= \frac{A}{A_s}\,\omega^2 r \cos\theta \\ &= \left(\frac{D}{d_s}\right)^2 \omega^2 r \cos\theta \end{aligned}\right\} \quad \ldots (12\cdot11)$$

Now, force would be required to accelerate water in the suction pipe and this would be considerable if the suction pipe is long. According to the second law of Newton,

force = mass × acceleration.

Let, p_{as} = intensity of pressure to accelerate water in suction pipe,

$$\text{Then, } p_{as} \times A_s = \frac{wA_s l_s}{g} \times \frac{A}{A_s}\,\omega^2 r \cos\theta$$

$$\text{or} \quad p_{as} = \frac{wl_s}{g} \times \frac{A}{A_s}\,\omega^2 r \cos\theta$$

Let, h_{as} = acceleration head in metres of water $= \frac{p_{as}}{w}$

$$\text{Then,} \quad \left.\begin{aligned} h_{as} &= \frac{l_s}{g} \times \frac{A}{A_s}\,\omega^2 r \cos\theta \\ &= \frac{l_s}{g}\left(\frac{D}{d_s}\right)^2 \omega^2 r \cos\theta \end{aligned}\right\} \quad \ldots \quad \ldots (12\cdot12)$$

The pressure head due to acceleration of the piston will, therefore, vary with the angle θ.

Similarly, acceleration head, h_{ad} in the delivery pipe can be obtained by substituting the subscript $_d$ in place of $_s$ in equation (12·12).

$$\left.\begin{aligned} i.e.\ h_{ad} &= \frac{l_d}{g} \times \frac{A}{A_d}\, \omega^2 r \cos\theta \\ &= \frac{l_d}{g}\left(\frac{D}{d_d}\right)^2 \omega^2 r \cos\theta \end{aligned}\right\} \qquad \ldots \quad \ldots (12{\cdot}13)$$

The values of acceleration head at salient points can be worked out by using eqns. (12·12) and (12·13) as under :

Acceleration head	Beginning of stroke	Middle of stroke	End of stroke
	$\theta = 0°$,	$\theta = 90°$	$\theta = 180°$
h_{as}	$h_{as} = \frac{l_s}{g} \times \frac{A}{A_s}\, \omega^2 r$	$h_{as} = 0$	$h_{as} = -\frac{l_s}{g} \times \frac{A}{A_s}\, \omega^2 r$
	$\theta = 180°$	$\theta = 270°$	$\theta = 360°$
h_{ag}	$h_{ad} = -\frac{l_d}{g} \times \frac{A}{A_d}\, \omega^2 r$	$h_{ad} = 0$	$h_{ad} = \frac{l_d}{g} \times \frac{A}{A_s}\, \omega^2 r$

The simple indicator diagram of fig. 12–2 must now be modified to take in account the acceleration head. The modified diagram is shown in fig. 12–4.

At the beginning of the suction stroke, the acceleration head is $h_{as} = \frac{l_s}{g} \times \frac{A}{A_s}\, \omega^2 r$, which is positive and should be added to the suction Head, h_s as water is to be accelerated and vacuum is to be increased. This is done by setting up $ap = h_{as}$. Equation (12·10) gives a straight sloping line, the acceleration head being zero at the centre of the stroke. At the end of the stroke, water is retarded, and hence, causes positive pressure on the piston, which reduces

the vaccum in the cylinder by the amount h_{as}. This is allowed by setting $bq = h_{as}$. Thus pq represents pressure variation during

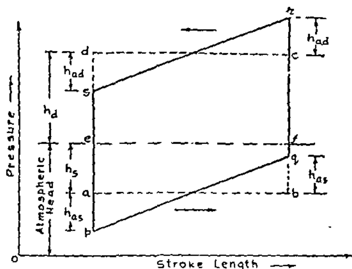


Fig. 12-4 Indicator diagram incorporating acceleration head

suction stroke. The work done during the suction stroke is now represented by area *pqfe*; but as this equals the area *abfe*, the net work done remains same as before. Thus, the inertia of water does not affect the net work done, but only causes a variation of pressure in the cylinder.

Similarly, allowance for accelerating head in delivery pipe can be made and this is represented by line *rs*. At the beginning of the delivery stroke, water is to be accelerated and it builds up positive pressure. At the end of the stroke, water is to be retarded and it causes reduction in pressure. Here also work done is not affected by the acceleration of water.

If simple harmonic motion is not assumed, the acceleration heads can be written more correctly as under :

$$\left.\begin{aligned} h_{as} &= \frac{l_s}{g} \times \frac{A}{A_s}\, \omega^2 r \left(\cos\theta + \frac{r}{l}\cos 2\theta\right) \\ &= \frac{l_s}{g}\left(\frac{D}{d_s}\right)^2 \omega^2 r \left(\cos\theta + \frac{r}{l}\cos 2\theta\right) \end{aligned}\right\} \quad \text{.. (12-12)}$$

$$\left.\begin{aligned} \text{and } h_{ad} &= \frac{l_d}{g} \times \frac{A}{A_d}\,\omega^2 r\left(\cos\theta + \frac{r}{l}\cos 2\theta\right) \\ &= \frac{l_d}{g}\left(\frac{D}{d_d}\right)^2 \omega^2 r\left(\cos\theta + \frac{r}{l}\cos 2\theta\right) \end{aligned}\right\} \quad ..(12\cdot13a)$$

These expressions should be used when the connecting rod is *not very long*. In this case, straight lines *pq* and *rs* of indicator diagram of fig. 12-4 will be slightly curved and acceleration heads at middle of the strokes will not be zero. Further, h_{as} at the beginning of suction stroke and h_{ad} at the end of delivery stroke are slightly more and h_{as} at the end of suction stroke and h_{ad} at the beginning of delivery stroke are slightly less, compared with those obtained by assuming simple harmonic motion for piston. These changes are represented in fig. 12-5.

Effect of friction in pipes : There will be frictional resistance to the flow in the suction and delivery pipes, which follows the ordinary friction laws for pipes. The head lost in pipes due to friction is given by

$$h_f = \frac{4fl}{d} \times \frac{V^2}{2g}$$

Considering simple harmonic motion, for suction pipe,

$$\left.\begin{aligned} h_{fs} &= \frac{4fl_s}{d_s} \times \frac{V_s^2}{2g} \\ &= \frac{4fl_s}{d_s 2g}\left[\frac{A}{A_s}\,\omega r\sin\theta\right]^2 \\ &= \frac{4fl_s}{d_s 2g}\left[\left(\frac{D}{d_s}\right)^2 \omega r\sin\theta\right]^2 \end{aligned}\right\} \quad ..(12\cdot14)$$

and similarly for delivery pipe,

$$\left.\begin{aligned} h_{fd} &= \frac{4fl_d}{d_d} \times \frac{V_d^2}{2g} \\ &= \frac{4fl_d}{d_d 2g}\left[\frac{A}{A_d}\,\omega r\sin\theta\right]^2 \\ &= \frac{4fl_d}{d_d 2g}\left[\left(\frac{D}{d_d}\right)^2 \omega r\sin\theta\right]^2 \end{aligned}\right\} \quad ..(12\cdot15)$$

At the ends of the strokes, $\sin\theta = 0$; therefore, the velocity of water in the pipe is zero, and there will be no loss of head due to friction. Friction head has its maximum values at middle of strokes when $\theta = 90°$ or $270°$. The equations (12·14) and (12·15) are parabolic ones. If friction heads are added to the indicator diagram of fig. 12–4, a more correct diagram is obtained as shown in fig. 12–5. The area *pmqp* represents work done against friction in the suction pipe, and area *rnsr* that in delivery pipe.

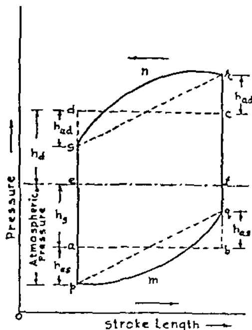


Fig. 12-5 Indicator diagram with acceleration and friction effect accounted

It may be noted that the mean ordinate of a parabola is equal to two-thirds of the maximum ordinate. Therefore, mean head lost in friction in suction pipe is

$$\tfrac{2}{3} h_{fs} = \tfrac{2}{3} \times \frac{4 f l_s}{d_s 2g}\left(\frac{A}{A_s}\,\omega r\right)^2 \qquad ..(12{\cdot}14a)$$

Total work done during suction stroke = area *pmqfep*

= area *pqfe* + area *pmqp*

= area *abfe* + area *pmqp*

$= W\left(h_s + \tfrac{2}{3} h_{fs\,max}\right)$

where W is the weight of water pumped per sec.

Total work done during delivery stroke = area *efrnse*

= area *efrs* + area *rnsr*

= area *efcd* + area *rnsr*

$= W\left(h_d + \tfrac{2}{3} h_{fd\,max}\right)$

$\therefore$ total work done per sec, $= W\left(h_s + h_d + \tfrac{2}{3} h_{fs} + \tfrac{2}{3} h_{fd}\right)$ (12·16)

The *vacuum pressure* in the cylinder during the suction stroke for any angle θ of the crank is

$$h_s + h_{a_s} + h_{f_s}$$
$$= h_s + \frac{l_s}{g} \times \frac{A}{A_s}\,\omega^2 r \cos\theta + \frac{4fl_s}{d_s 2g}\left(\frac{A}{A_s}\,\omega r \sin\theta\right)^2 \quad .. \, (12\cdot17)$$

The *pressure* (above atmospheric) in the cylinder, during the delivery stroke is

$$h_d + \frac{l_d}{g} \times \frac{A}{A_d}\,\omega^2 r \cos\theta + \frac{4fl_d}{d_d 2g}\left(\frac{A}{A_d}\,\omega r \sin\theta\right)^2 \quad .. \, (12\cdot18)$$

It may be observed from eqns. (12·17) and (12·18) that the acceleration heads are maximum at the ends of the strokes and zero at the middle, whilst the frictional heads are zero at the ends and maximum at the middle of the strokes.

Problem-1 : *Water is raised to a height of 25 m by a single-acting single-cylinder reciprocating pump. The pump runs at 30 rpm. The diameter of the cylinder of the pump is 15 cm, and the stroke is 22·5 cm. Find, theoretical discharge in L. P. M.; ideal H. P.; percentage slip; and coefficient of discharge, if the actual discharge is 115 L.P.M.*

Theoretical discharge, $Q_t = \frac{\pi}{4}\, D^2 L N$ m³/min.

$$= 1{,}000 \times \frac{\pi}{4}\, D^2 L N \quad \text{L.P.M.}$$

$$= 1{,}000 \times \frac{\pi}{4} \times \left(\frac{15}{100}\right)^2 \times \frac{22\cdot5}{100} \times 30$$

$$= \underline{119\cdot5 \text{ L.P.M.}}$$

$$\text{Ideal H.P.} = \frac{WH}{75} = \frac{119\cdot5 \times 25}{60 \times 75} = 0\cdot665$$

$$\text{Percentage slip} = \frac{\text{theoretical discharge-actual discharge}}{\text{theoretical discharge}} \times 100$$

$$= \frac{119\cdot5 - 115}{119\cdot5} \times 100 = 3\cdot76$$

$$\text{Coefficient of discharge} = \frac{\text{actual discharge}}{\text{theoretical discharge}}$$

$$= \frac{115}{119.5} = 0.962$$

Problem-2: *Water is lifted to a total height of 30 m by a single acting reciprocating pump having bore of 225 mm and stroke of 370 mm. If the speed of the pump is 30 rpm, and the actual quantity of water lifted is 430 litres/min., find, percentage slip, coefficient of discharge, and ideal H.P.*

$$\text{Theoretical discharge} = wALN = 1{,}000 \times \frac{\pi}{4}\left(\frac{225}{1{,}000}\right)^2 \times \frac{370}{1{,}000} \times 30$$

$$= 442 \text{ kg/min.}$$

$$= 442 \text{ litres/min.}$$

$$\text{Percent slip} = \frac{442 - 430}{442} \times 100 = 2.72$$

$$\text{coefficient of discharge} = \frac{430}{442} = 0.972$$

$$\text{Ideal H.P.} = \frac{WH}{75} = \frac{442 \times 30}{60 \times 75} = 2.95$$

Problem-3: *A single-acting, reciprocating pump has bore of 15 cm and stroke of 30 cm. The length of the suction as well as delivery pipe is 7.5 cm. The length of the suction and delivery pipes are 10 m and 50 m respectively. The head lost in friction in the delivery pipe at the middle of the delivery stroke is 5 m of water. If coefficient of friction, f = 0.01, determine, (a) the speed of the pump, and (b) the maximum inertia and friction heads in the suction pipe.*

Maximum friction head in the delivery pipe is

$$h_{fd} = \frac{4f\,l_d}{2gd_d}\left\{\frac{A}{A_d}\,\omega r\right\}^2$$

$$\text{i.e.}\quad 5 = \frac{4 \times 0.01 \times 50}{19.62 \times 0.075}\left\{\left(\frac{15}{7.5}\right)^2 \times \omega \times \frac{15}{100}\right\}^2$$

$$\therefore \quad \omega = 3.17 \text{ rad./sec.}$$

$$\therefore \quad N = \frac{60\omega}{2\pi} = \frac{60 \times 3{\cdot}17}{2\pi} = \underline{30{\cdot}3 \text{ rpm.}}$$

Now maximum friction head occurs at the middle of the stroke. The friction head in the suction pipe is

$$h_{fs} = \frac{4fl_s}{2gd_s}\left\{\frac{A}{A_s}\,\omega r\right\}^2$$

$$= h_{fd} \times \frac{l_s}{l_d}\left\{\frac{A_d}{A_s}\right\}^2$$

$$= 5 \times \frac{10}{50} \times \left\{\frac{7{\cdot}5}{7{\cdot}5}\right\}^4$$

$$= \underline{1 \text{ m of water}}$$

Maximum inertia head in the suction pipe occurs at the beginning or at the end of the suction stroke :

$$i.e. \quad h_{as} = \frac{l_s}{g} \cdot \frac{A}{A_s}\,\omega^2 r$$

$$= \frac{10}{9{\cdot}81} \times \left\{\frac{15}{7{\cdot}5}\right\}^2 \times 3{\cdot}17^2 \times \frac{15}{100}$$

$$= \underline{6{\cdot}2 \text{ m of water}}$$

Problem - 4 : *In a pumping installation, of a force pump is required to deliver 5,00,000 litres of water per hour at a static height of 40 m, measured vertically between the water levels in the sump and the delivery tank. The delivery pipe has 450 mm diameter and a total length of 5,500 m. Find the H.P. required to drive the pump if the overall efficiency is 70%. Take coefficient of friction $f = 0{\cdot}01$. Friction in the suction pipe may be neglected.*

$$\text{Discharge,} \quad Q = \frac{5{,}00{,}000 \times 1{,}000}{3{,}600 \times 100^3} = 0{\cdot}1386 \text{ m}^3/\text{sec.}$$

Average velocity of water in the delivery pipe is

$$V_d = \frac{Q}{\frac{\pi}{4}\,d_d^2} = \frac{0{\cdot}1386}{\frac{\pi}{4}\left(\frac{450}{1{,}000}\right)^2} = 0{\cdot}875 \text{ m/sec.}$$

Now, head lost in the delivery pipe is

$$h_{fd} = \frac{4fl_d}{d_d} \times \frac{V_d^2}{2g}$$

$$= \frac{4 \times 0.01 \times 5,500}{\frac{450}{1,000}} \times \frac{(0.875)^2}{2 \times 9.81} = 19.1 \text{ m}$$

Hence, total head to be developed, $H_m = (h_s + h_d) + h_{fd}$

$$= 40 + 19.1 = 59.1 \text{ m}$$

$$\therefore \quad \text{W.H P.} = \frac{wQH_m}{75} = \frac{1,000 \times 0.1386 \times 59.1}{75} = 109.1$$

$$\therefore \quad \text{S.H.P.} = \frac{\text{W.H.P.}}{\eta_0} = \frac{109.1}{0.7} = 155.5$$

Problem – 5 : *The plunger of a single-acting, reciprocating pump moves with S.H M. Its diameter is 15 cm and stroke 22·5 cm. The suction pipe is 7·5 cm in diameter and 5 m long If the pump runs at a speed of 30 rpm, determine the velocity and acceleration of water in the suction pipe at the beginning, middle and end of the suction stroke. Also determine the inertia head at the beginning, middle, and end of the suction stroke.*

Velocity of water in the suction pipe during suction stroke is

$$V_s = \frac{A}{A_s}\,\omega r \sin\theta = \left(\frac{15}{7.5}\right)^2 \times \left(\frac{2\pi \times 30}{60}\right) \times \frac{22.5}{200} \sin\theta$$

$$= 1.41 \sin\theta \text{ m/sec.} \qquad \text{.. (i)}$$

and acceleration of water in the suction pipe during suction stroke is

$$a_s = \frac{A}{A_s}\,\omega^2 r \cos\theta = \left(\frac{15}{7.5}\right)^2 \times \left(\frac{2\pi \times 30}{60}\right)^2 \times \frac{22.5}{200} \times \cos\theta$$

$$= 4.45 \cos\theta \text{ m/sec}^2 \qquad \text{.. (ii)}$$

and acceleration head in suction pipe is

$$h_{as} = \frac{l_s}{g} \times a_s = \frac{5}{9.81} \times 4.45 \cos\theta$$

$$= 2.27 \cos\theta \text{ m of water} \qquad \text{.. (iii)}$$

Using eqns. (i), (ii), (iii), velocity, acceleration, and accelerating head of water in the suction pipe can be calculated at the beginning, middle, and end of the suction stroke as shown in the table below.

Point on the suction stroke	Beginning	Middle	End
Rotation of the crank from i.d.c., θ (degrees)	0	90	180
Velocity of water, V_s (m/sec.)	0	1·41	0
Acceleration of water a_s (m/sec².)	4·45	0	– 4·45
Accelerating head, h_{as} (m of water)	2·27	0	– 2·27

Problem – 6 : *A double-acting reciprocating pump running at 30 rpm. supplies water through a main of 40 cm diameter and 800 m long. The bore and stroke of the pump are 60 cm and 90 cm respectively. The pump supplies water through a static head of 15 m. Mechanical efficiency of the pump is 80%. If the slip of the pump is positive and is equal to 3%, determine the H.P. required to drive the pump and the quantity of water delivered in litre/sec. Take coefficient of friction in the pipes as 0·005.*

Theoretical discharge, $Q_t = 2 \times \frac{\pi}{4} D^2 LN$

$$= 2 \times \frac{\pi}{4} \left(\frac{60}{100}\right)^2 \times \frac{90}{100} \times 30$$

$$= 15{\cdot}28 \text{ m}^3/\text{min.}$$

Actual discharge, $Q_a = Q_t \,(1 - \text{slip})$

$$= 15{\cdot}28\,(1 - 0{\cdot}03) = 14{\cdot}8 \text{ m}^3/\text{min.}$$

or $\frac{14{\cdot}8 \times 1{,}000}{60} = 247$ litres/sec. or $0{\cdot}247$ m³/sec.

Now, velocity in the mains, $V_d = \dfrac{Q_a}{\frac{\pi}{4} d^2_d} = \dfrac{0{\cdot}247}{\frac{\pi}{4}\left(\frac{40}{100}\right)^2} = 1{\cdot}97$ m/sec.

Head lost in friction, $h_{fd} = \frac{4fl_d}{d_d}\left(\frac{V_d^2}{2g}\right)$

$$= \frac{4 \times 0.005 \times 800}{0.4} \times \left(\frac{1.97^2}{2 \times 9.81}\right)$$

$$= 7.92 \text{ m}$$

$$\therefore \quad \text{W.H.P.} = \frac{WH_a}{75} = \frac{247 \times (15 + 7.92)}{75}$$

$$= 75.5$$

$$\text{S.H.P} = \frac{\text{W.H.P.}}{\eta_o} = \frac{75.5}{0.8} = 94.5$$

Problem – 7 : *A double-acting reciprocating pump has piston diameter of 15 cm and stroke of 22·5 cm. The pump makes 120 strokes per minute. The centre line of the pump is 2·5 m above the water level in the sump. The length of the suction pipe which is 7·5 cm diameter is 3 m. Determine the pressure in kg/cm² absolute inside the cylinder, at the (a) beginning, (b) middle, and (c) end of the suction stroke. Take $f = 0.01$ for pipes. Neglect the velocity head in the suction pipe.*

As the piston makes 120 strokes/min , the speed is $\frac{120}{2} = 60$ rpm. of the crank.

$$\therefore \quad \omega = \frac{2\pi N}{60} = \frac{2\pi \times 60}{60} = 2\pi \text{ rad/sec}$$

Now, $h_{as} = \frac{l_s}{g}\,\frac{A}{A_s}\,\omega^2 r \cos\theta$

$$= \frac{3}{9.81} \times \left(\frac{15}{7.5}\right)^2 \times (2\pi)^2 \times \frac{22.5}{2 \times 100} \cos\theta$$

$$= 5.5 \cos\theta \qquad \text{.. (i)}$$

and $\quad h_{fs} = \frac{4fl_s}{2gd_s}\left\{\frac{A}{A_s}\,\omega\, r \sin\theta\right\}^2$

$$= \frac{4 \times 0.01 \times 3}{2 \times 9.81 \times 7.5/100}\left\{\left(\frac{15}{7.5}\right)^2 \times 2\pi \times \frac{22.5}{2 \times 100}\right\}^2 \sin^2\theta$$

$$= 0.653 \sin^2\theta \qquad \text{.. (ii)}$$

With the help of eqns. (i) and (ii) the following table is computed.

Head, m of water	Beginning of stroke, $\theta = 0°$	Middle of stroke, $\theta = 90°$	End of stroke $\theta = 180°$
Suction lift, h_s	2·5	2·5	2·5
Acceleration head h_{as}	5·5	0·0	− 5·5
Friction head, h_{fs}	0·0	0.653	0·0
Total vacuum head, $h_s + h_{as} + h_{fs}$	8·0	3·153	− 3·0
Absolute pressure, H	10 − 8 = 2·0	10−3·153 = 6·847	10−(−3·0) = 13·3
Absolute pressure in kg/cm², $H \times \frac{1,000}{10,000}$	0·20	0·6847	1·30

Separation and cavitation

As discussed in chap. 10 (suction lift) when absolute pressure of water falls below 3 m (*i.e.* vacuum below 8 m) of water, water gets vapourized. This phenomenon is known as separation or cavitation. In case of reciprocating pumps, the possibility of such high vacuum pressure during the suction stroke is more. Once separation occurs, water will no longer follow the piston, but will get vaporized and air remaining dissolved in water starts separating out from water. Water vapour and air separated from water will form layers or cavities of gaseous fluid in water, inside the cylinder. Thus, separation finally results in cavitation; hence cavitation and separation go together. The pressure in the cylinder remains constant at vapour pressure of water till the void formed due to separation is completely filled. The phenomenon is diagramatically represented in fig. 12-6.

In the diagram below, $ea = h_s$, $ap = h_{as}$ and the sum of h_s and h_{as} is greater than the maximum permissible vacuum inside the cylinder *i.e.* absolute pressure is less than water vapour pressure

Fig 12-6 Indicator diagram showing separation or cavitation

by an amount represented by *np* The water vapour pressure is normally about 2·5 metres of water head. Thus, separation will start and continue till the piston reaches *y*, the pressure then is equal to vapour pressure inside the cylinder.

This pressure inside the cylinder remains constant at vapour pressure upto the point *z* reached by the piston when the separated air again dissolves and water vapour again condenses and the cavities suddenly disappear and a higher vacuum is created in the cylinder. This process causes water to rush immediately inside the cylinder at a rate faster than the rate of movement of piston, thus resulting in the sudden rise of pressure inside the cylinder, the extent of it is shown by point *g*. The pressure wave generated travels to and fro inside the cylinder. This process continues for a short while till the piston moves upto the position *k*, where again the pressure reaches the normal value. The pulsation of the pressure wave causes knocking inside the cylinder. This knocking

With the help of eqns. (i) and (ii) the following table is computed.

Head, m of water	Beginning of stroke, $\theta = 0°$	Middle of stroke, $\theta = 90°$	End of stroke $\theta = 180°$
Suction lift, h_s	2·5	2·5	2·5
Acceleration head h_{as}	5·5	0·0	− 5·5
Friction head, h_{fs}	0·0	0.653	0·0
Total vacuum head, $h_s + h_{as} + h_{fs}$	8·0	3·153	− 3·0
Absolute pressure, H	10 − 8 = 2·0	10−3·153 = 6·847	10−(−3·0) = 13·3
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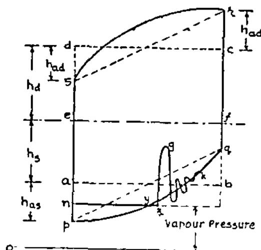


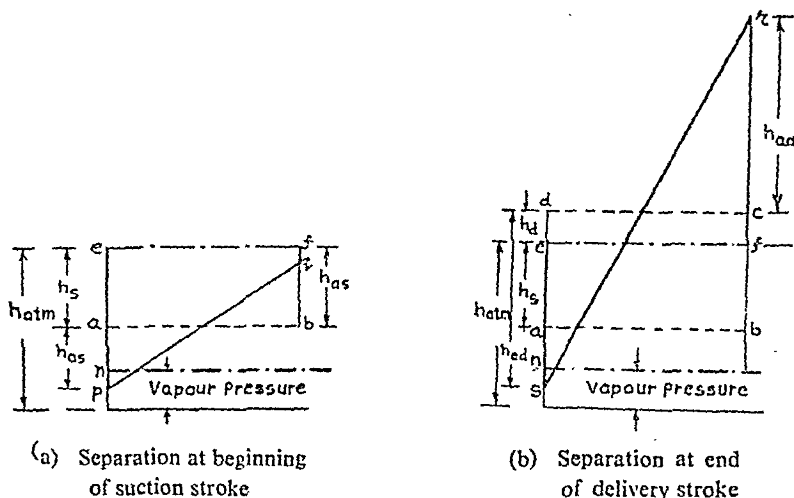
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will cause sudden increase in force on the piston and its mechanism. This becomes, at times, serious and it must be eliminated.

Maximum speed of rotation for satisfactory working of a reciprocating pump: For the satisfactory working of a reciprocating pump, the pressure inside the cylinder at any instant must not be less than the vapour pressure of the fluid, otherwise separation will occur. There are two positions of the piston where this can take place easily. One is at the beginning of the suction stroke and the other at the end of the delivery stroke. These positions are shown in figure 12-7.



(a) Separation at beginning of suction stroke

(b) Separation at end of delivery stroke

Fig. 12-7 Separation in reciprocating pump

From it, one can see that, to avoid separation at the beginning of suction stroke, fig. 12-7 (a),

$$\text{fluid vapour pressure (absolute)} < h_{atm} - h_s - h_{as}$$

Therefore, maximum speed for given length of suction pipe should be such that

$$h_s + h_{as} = h_{atm} - \text{fluid vapour pressure}$$

Similarly, from fig. 12-7(b) to avoid separation at the end of the delivery stroke, the speed (and hence h_{ad}) should be limited such that

$$\text{fluid vapour pressure} = h_{atm} + h_d - h_{ad}$$

The permissible maximum pressure without causing separation is sometimes known as the effective height of the barometer or effective atmospheric pressure head. The vapour pressure is usually taken as 3 metres head for the temperatures of fluid commonly observed. For water this pressure is known from steam tables for its given temperature. Let this pressure be p kg/cm². The vapour pressure in m of water is obtained from the relation

$$\frac{10{,}000p}{w} = \text{fluid vapour pressure in m.}$$

Hence, $\dfrac{10{,}000p}{w} = h_{atm} - h_s - h_{as}$

Now, $h_{as} = h_{atm} - h_s$ – vapour pressure

But, $h_{as} = \dfrac{l_s}{g} \dfrac{A}{A_s} \omega^2 r$ (at beginning of suction stroke)

$$\therefore \quad \frac{l_s}{g} \frac{A}{A_s} \omega^2 r = h_{atm} - h_s - \text{vapour pressure}$$

$$\therefore \ \omega^2 = \frac{g}{l_s} \frac{A_s}{A} \frac{1}{r} (h_{atm} - h_s - \text{vapour pressure}) \quad ..(12\cdot18)$$

This will give the maximum value of ω from which N can be determined. Thus, the pump cannot be run at a speed exceeding the speed determined from the above eqn (12·18) If we want to increase the speed N, we must decrease suction lift h_s, and vice-versa. Similarly for the position at end of the delivery stroke,

$$h_d + h_{atm} - h_{ad} = \text{vapour pressure}$$

$$\therefore \quad h_{ad} = h_d + h_{atm} - \text{vapour pressure}$$

$$i.e. \ \frac{l_d}{g} \frac{A}{A_d} \omega^2 r = h_d + h_{atm} - \text{vapour pressure}$$

$$\therefore \quad \omega^2 = \frac{g}{l_d} \cdot \frac{A_d}{A} \cdot \frac{1}{r} (h_d + h_{atm} - \text{vapour pressure}) \quad ..(12\cdot19)$$

This will give the maximum value of ω and hence, the speed. N can be determined. Thus, the speed should be found from both relations and the smaller of the two is the operating speed for satisfactory working. Usually h_{as} is small, the speed is determined from considerations of accelerating head on suction side *only*.

Problem-8 : *The plunger of a single-acting, reciprocating pump moves with S. H. M. The bore and stroke of the pump are 15 cm and 22·5 cm respectively. The length and diameter of the suction pipe is 3 m and 7·5 cm respectively. Water is drawn by the pump from a sump in which water level is 2 m below the centre line of the pump. Separation takes place when pressure inside the cylinder falls below 2·0 m of water (absolute). If the barometer reads 10 m, determine the maximum speed at which the pump can be run. Assume that there is no air vessel fitted on the pump.*

In order to avoid separation,

$$h_s + h_{a_s} = 10 - 2{\cdot}0 = 8{\cdot}0 \text{ m of water}$$

$$\therefore \quad h_{a_s} = 8{\cdot}0 - 2 = 6{\cdot}0 \text{ m of water}$$

But, $h_{a_s} = \dfrac{l_s}{g} \times \dfrac{A}{A_s}\,\omega^2 r$

i. e. $$6 = \frac{3}{9{\cdot}81} \times \left(\frac{15}{7{\cdot}5}\right)^2 \times \omega^2 \times \frac{22{\cdot}5}{100 \times 2}$$

$$\therefore \quad \omega^2 = 43{\cdot}6$$

$$\therefore \quad \omega = 6{\cdot}6 \text{ rad./sec.}$$

But, $\omega = \dfrac{2\pi N}{60}$

$$\therefore \quad 6{\cdot}6 = \frac{2\pi \times N}{60}$$

$$\therefore \quad N = 63 \text{ rpm.}$$

Problem-9 : *A single-acting reciprocating pump has a plunger of 15 cm diameter and a stroke of 22·5 cm. The diameter and length of the suction pipe is 10 cm and 5 m. The pump is placed at a height of 3 m above the water level in the sump. The diameter and length of the delivery pipe is 10 cm and 60 m respectively. The delivery head is 25 m. Separation takes place when the pressure falls below 0·28 kg/cm². absolute. Assuming S. H. M. of plunger and the barometric height as 10·3 m of water; determine the maximum theoretical discharge of the pump at its correct maximum* [illegible]

Maximum permissible vacuum $= 10{\cdot}3 - \frac{0{\cdot}28 \times 10{,}000}{1{,}000}$

$= 10{\cdot}3 - 2{\cdot}8 = 7{\cdot}5$ m of water

For the suction stroke,

$$7{\cdot}5 = h_s + h_{as} = 3 + h_{as}$$

$\therefore \quad h_{as} = 4{\cdot}5$ m of water

But, $h_{as} = \frac{l_s}{g} \times \frac{A}{A_s}\, \omega^2 r$

i.e. $4{\cdot}5 = \frac{5}{9{\cdot}81} \times \left(\frac{15}{10}\right)^2 \times \omega^2 \times \frac{22{\cdot}5}{100 \times 2}$

$\therefore \quad \omega^2 = 35$ and $\omega = 5{\cdot}92$ rad/sec.

$$N = \frac{30\omega}{\pi} = \frac{30 \times 5{\cdot}92}{\pi} = 56{\cdot}5 \text{ rpm.}$$

Now, at the end of the delivery stroke,

$$7{\cdot}5 = h_{ad} - h_d = h_{ad} - 25$$

$\therefore \quad h_{ad} = 32{\cdot}5$ m of water

But, $h_{ad} = \frac{l_d}{g} \times \frac{A}{A_d}\, \omega^2 r$

i.e. $32{\cdot}5 = \frac{60}{9{\cdot}81} \times \left(\frac{15}{10}\right)^2 \times \frac{22{\cdot}5}{100 \times 2} \cdot \omega^2$

$\therefore \quad \omega^2 = 20{\cdot}8$

$\therefore \quad \omega = 4{\cdot}57$ rad/sec

$$N = \frac{30\omega}{\pi} = \frac{30 \times 4{\cdot}57}{\pi} = 43{\cdot}7 \text{ rpm.}$$

Thus, the speed of rotation to avoid separation is lower at the end of delivery stroke than that at the begining of suction stroke. Therefore, the correct speed of rotation is 43·7 rpm.

Discharge, Q in litres/min. $= ALN \times 1{,}000$

$$= \frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times \frac{22{\cdot}5}{100} \times 43{\cdot}7 \times 1{,}000$$

[illegible]

Problem – 10 : *The following particulars relate to a single-acting reciprocating pump whose piston moves with S.H.M.*

Length of suction pipe	.. *5 m*
Suction lift	.. *3 m*
Diameter of suction pipe	.. *75 mm*
Stroke to bore ratio	.. *1·5:1*
Barometeric height (of water)	.. *10 m*
Vapour pressure of the liquid	.. *0·28 kg/cm²*
Theoretical discharge	.. *250 litres/min.*

Evaluate its correct speed of rotation and the diameter and stroke of the plunger.

Equating maximum vacuum head at beginning of suction to maximum permissible vacuum head,

$$h_s + h_{as} = 10 - \frac{0{\cdot}28 \times 100^2}{1{,}000}$$

i.e. $3 + h_{as} = 10 - 2{\cdot}8$

$\therefore \quad h_{as} = 4{\cdot}2$ m of water

But, $h_{as} = \frac{l_s}{g} \times \frac{A}{A_s}\, \omega^2 r$

i.e. $4{\cdot}2 = \frac{5}{9{\cdot}81} \times \left\{\frac{D}{0{\cdot}075}\right\}^2 \times \left\{\frac{2\pi N}{60}\right\}^2 \times \frac{1{\cdot}5D}{2}$

$\therefore \quad D^3N^2 = 5{\cdot}65$ .. (i)

Now, discharge per min., $Q = ALN$

i.e. $\frac{250}{1{,}000} = \frac{\pi}{4} D^2 \times 1{\cdot}5D \times N$

$\therefore \quad D^3N = 0{\cdot}212$ .. (iii)

Dividing eqn. (i) by eqn. (ii),

$$\frac{D^3N^2}{D^3N} = \frac{5{\cdot}65}{0{\cdot}212} = 26{\cdot}65$$

$\therefore \quad N = 26{\cdot}65$ rpm.

Hence, from eqn. (ii), $D_1 = \frac{0.212}{26.65} = 0.00795$

i.e. plunger diameter, $D = 0.1995$ m i.e. 199.5 mm

and stroke, $L = 1.5 \times 199.5 = 290$ mm

Problem – 11 : *A plunger is fitted in a vertical pipe which is full of water and whose lower end is submerged in a vessel containing water at a temperature of 60°C. The plunger is moved upward with maximum acceleration equal to 1.22 m/sec/sec. If the barometric height is 10 m of water, what is the maximum height above the water level in the vessel at which the plunger can operate without cavitation ?*

Let h_s be the maximum possible suction lift in metres without occurance of cavitation. Then h_s is also the length of the suction pipe.

Now, force required to accelerate water in suction pipe is

$$\frac{wA_s h_s}{g} \times \text{acceleration}$$

$$\therefore \quad \text{accelerating head, } h_{as} = \frac{\frac{wA_s h_s}{g} \times \text{acceleration}}{A_s \times w}$$

$$= \frac{h_s}{g} \times \text{acceleration}$$

$$\therefore \quad \text{maximum vacuum} = h_s + \frac{h_s}{g} \times 1.22 \text{ m of water}$$

Now, from steam tables, vapour pressure corresponding to 60°C is

$$0.203 \text{ kg/cm}^2; \text{ i.e. } \frac{0.203 \times 100^2}{1,000} = 2.03 \text{ m of water}$$

($\because$ 1 m³ of water weighs 1,000 kg)

$\therefore$ maximum permissible vacuum head = 10 – 2.03 = 7.97 m of water

$$\therefore \quad 7.97 = h_s + \frac{h_s}{9.81} \times 1.22 = h_s (1 + 0.124)$$

Problem – 12 : *A double acting, reciprocating pump has a piston 15 cm diameter and stroke of 22·5 cm. The diameter of the suction pipe as well as the delivery pipe is 10 cm. The lengths of the suction and delivery pipes are 5 m and 30 m respectively. Water is taken from a sump below the centre line of the pump and delivered to a height of 27·5 m above the centre line of the pump. Accelerating head in the suction pipe is $\frac{3}{4}$th of the suction lift. Determine the speed of rotation and the H.P. required to drive the pump. Assume that separation takes place when the pressure falls below 2·5 m of water. Take g = 9·81 m/sec² and coefficient of friction, f = 0·01.*

At the beginning of the suction stroke,

vacuum head = suction head + acceleration head

$$= h_s + h_{as} = h_s + \tfrac{3}{4}h_s = 1{\cdot}75\, h_s$$

In order to avoid separation, the vacuum at the beginning of suction stroke (which is maximum) should not be greater than 10 – 2·5 = 7·5 m of water.

$$1{\cdot}75 h_s = 7{\cdot}5 \quad \therefore \quad h_s = 4{\cdot}28 \text{ m of water}$$

∴ accelerating head in the suction pipe.

$$h_{as} = \tfrac{3}{4}\, h_s = 0{\cdot}75 \times 4{\cdot}28 = 3{\cdot}21 \text{ m of water}$$

$$\text{Now,} \quad h_{as} = \frac{l_s}{g} \times \frac{A}{A_s}\, \omega^2 r$$

$$\text{i.e.} \quad 3{\cdot}21 = \frac{5}{9{\cdot}81} \times \left(\frac{15}{10}\right)^2 \times \omega^2 \times \frac{22{\cdot}5}{2 \times 100}$$

$$\therefore \quad \omega^2 = 25$$

$$\therefore \quad \omega = 5 \text{ rad./sec.}$$

$$\text{But.} \quad \omega = \frac{2\pi N}{60} \quad \text{i.e.} \ 5 = \frac{2\pi N}{60}$$

$$\therefore \quad N = \underline{48 \text{ rpm.}}$$

Now, max. friction-head during suction

$$h_{fs} = \frac{4fl_s}{2gd_s}\left(\frac{A}{A_s}\omega r\right)^2$$

$$= \frac{4 \times 0{\cdot}01 \times 5 \times 100}{2 \times 9{\cdot}81 \times 10}\left\{\left(\frac{15}{10}\right)^2 \times 5 \times \frac{22{\cdot}5}{2 \times 100}\right\}^2$$

$$= 0{\cdot}165 \text{ m water}$$

∴ max. friction-head during delivery, $h_{fd} = h_{fs} \times \frac{l_d}{l_s}$

$$= 0.165 \times \frac{30}{5} = 0.98 \text{ m of water}$$

and max. accelerating head during suction $h_{as} = 3.21$ m of water

∴ max. accelerating head during delivery $h_{ad} = h_{as} \times \frac{l_d}{l_s}$

$$= 3.21 \times \frac{30}{5} = 19.3 \text{ m of water}$$

$$\therefore \quad \text{mean head, } H_m = h_s + h_d + \tfrac{2}{3} h_{fs} + \tfrac{2}{3} h_{fd}$$

$$= 4.28 + 27.5 + \tfrac{2}{3} \times 0.165 + \tfrac{2}{3} \times 0.98$$

$$= 32.54 \text{ m of water}$$

$$\text{Discharge, } W = \frac{wALN}{60} \times 2 = \frac{1{,}000}{60} \times \frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times \frac{22.5}{100} \times 48 \times 2$$

$$= 6.4 \text{ kg/sec.}$$

$$\therefore \quad \text{H.P. required} = \frac{W \times H_m}{75} = \frac{6.4 \times 32.54}{75}$$

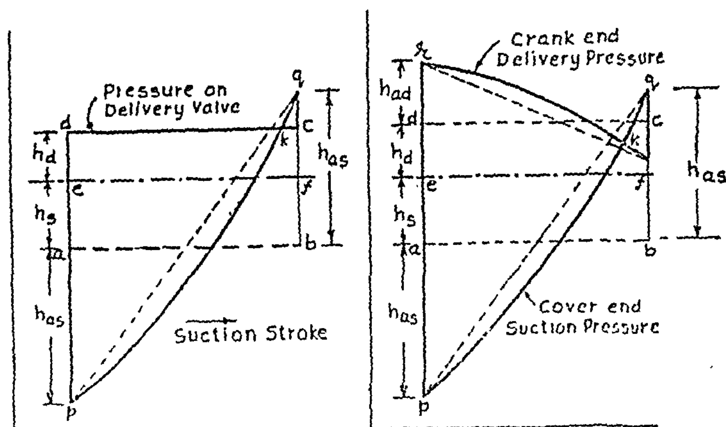
$$= 2.8$$

Negative Slip

When the actual discharge from the pump is greater than the volume swept by the plunger (theoretical discharge), slip is negative. This occurs when suction and delivery take place simultaneously on the same side of the plunger. This can happen on account of the following

(i) If the pressure inside the cylinder towards the end of the suction stroke becomes greater than the pressure on the delivery valve, this leads to opening of the delivery valve before the end of suction stroke and water passes directly from the suction pipe into the delivery pipe, both valves at that time remaining open simultaneously. This phenomena is represented in fig. 12-8. For a single acting pump, the pressure on the delivery valve is on account of the static head. h_d. It remains constant during the suction stroke and is represented by horizontal line dc as shown in figure.

If the speed of pump is higher, the acceleration head in the suction pipe will be of high order and it might so happen that towards the end of the suction stroke pressure inside the cylinder (represented by line pq) might exceed the delivery head h_d. At



(a) single-acting pump (b) double-acting pump

Fig. 12-8 Negative slip during suction stroke

point k the pressure inside the cylinder becomes equal to the delivery head. Further movement of the piston leads to increase in the cylinder pressure which exceeds h_d and in turn opens the delivery valve. Thus, for the remaining part of the suction stroke, both valves remain open simultaneously leading to increased discharge.

For a double-acting pump, fig. 12-9, when on one side of the plunger suction takes place, during that same stroke, on the other side of the plunger, simultaneously delivery also takes place. The pressure on the delivery valve varies due to acceleration head in the delivery pipe and as indicated by line rs in fig. 12-8 (b). The pressure inside the cylinder during the suction stroke is represented by line pq. It might so happen, that when the suction stroke is about to complete, the pressure inside the cylinder might become greater than the pressure on the delivery valve on that side. This

will lead to opening of the delivery valve when the suction valve is already open and consequently water will pass directly from the suction pipe into the delivery pipe causing enhanced discharge and hence the pump will have negative slip.

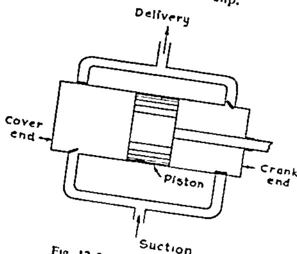


Fig. 12-9 Double-acting pump

(ii) Negative slip may occur towards the end of the delivery stroke also. When delivery stroke is about to be over, the pressure inside the cylinder falls below the pressure on the suction valve from the sump side. This will result in the opening of the suction valve which will permit some water to pass from the suction pipe to the delivery pipe through the cylinder.

Problem – 13 : *A double-acting, reciprocating pump, not fitted with air vessel, has plunger diameter of 15 cm and stroke of 22·5 cm and it moves with S H.M. The length of the suction pipe is 6 m. The vacuum at the beginning of the suction stroke is 9 m of water. The diameter of the suction and delivery pipes is 10 cm. The length of the delivery pipe is 12 m. The delivery head is 4 times the suction head. Negative slip is just to occur at the end of the suction stroke, inside the cylinder. Determine, (a) H.P. required to drive the pump, (b) pressure head at the middle of the delivery stroke. Take g = 9 81 m/sec² and coefficient friction, f = 0·01.*

As negative slip is just to occur at the end of the suction stroke, we have, acceleration head, h_{as} = suction lift + delivery head

$$= h_s + h_d = h_s + 4h_s = 5h_s$$

Since, $h_s + h_{as} = 9$ m, $\qquad h_s + 5h_s = 9$ m

$\therefore \quad h_s = 1{\cdot}5$ m and $\qquad h_d = 6$ m and $h_{as} = 9 - 1{\cdot}5 = 7{\cdot}5$ m

Further, we have accelerating head in the suction pipe,

$$h_{as} = \frac{l_s}{g}\,\frac{A}{A_s}\,\omega^2 r$$

$$i.e.\ 7{\cdot}5 = \frac{6}{9{\cdot}81} \times \left(\frac{15}{10}\right)^2 \times \omega^2 \times \frac{22{\cdot}5}{2 \times 100}$$

$$\therefore \quad \omega^2 = 47{\cdot}5$$

$$\therefore \quad \omega = 6{\cdot}9 \text{ rad./sec.}$$

$$\text{and} \quad N = \frac{60\omega}{2\pi} = \frac{60 \times 6{\cdot}9}{2\pi} = 66 \text{ rpm.}$$

Further, the friction-head in the suction pipe is

$$h_{fs} = \frac{4fl_s}{2gd_s}\left[\frac{A}{A_s}\,\omega r\right]^2$$

$$= \frac{4 \times 0{\cdot}01 \times 6}{2 \times 9{\cdot}81 \times 10/100} \times \left[\left(\frac{15}{10}\right)^2 \times 6{\cdot}9 \times \frac{22{\cdot}5}{100 \times 2}\right]^2$$

$$= 0{\cdot}375 \text{ m of water}$$

Similarly, friction-head in the delivery pipe is

$$h_{fd} = h_{fs}\,\frac{l_d}{l_s} \quad \text{(as diameters of the pipes are same)}$$

$$= 0{\cdot}375 \times \frac{12}{6} = 0{\cdot}75 \text{ m of water}$$

Hence, the manometric head is

$$H_m = h_s + h_d + \tfrac{2}{3}\,(h_{fs} + h_{fd})$$

$$= 1{\cdot}5 + 6 + \tfrac{2}{3}\,(0{\cdot}375 + 0{\cdot}75) = 8{\cdot}25 \text{ m of water}$$

Weight of water lifted is,

$$W = \frac{2wALN}{60}$$

$$= 2 \times 1{,}000 \times \frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times \frac{22{\cdot}5}{100} \times \frac{66}{60} = 87{\cdot}5 \text{ kg/sec}$$

Therefore, H. P. $= \dfrac{W \times H_m}{75} = \dfrac{8{\cdot}75 \times 8{\cdot}25}{75} = \underline{0{\cdot}975}$

Pressure head at the middle of the delivery stroke is

$h_{atm.} + h_d + h_{fd} = 10 + 6 + 0{\cdot}75 = \underline{16{\cdot}75 \text{ m of water}}$

Air Vessels

An air vessel is a closed cylindrical cast iron chamber communicating with delivery pipe or suction pipe at a point as close to the pump as possible. It contains water in the lower portion and air in the upper portion as shown in fig. 12-10. It serves as a temperory reservoir and damps down the fluctuation in the flow

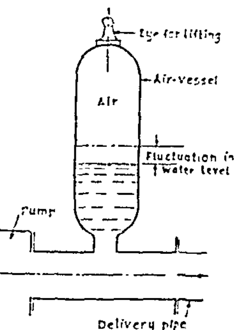


Fig. 12-10 Air vessel

The function of the air vessel fitted on the delivery side *is to accumulate the excess discharge of water* when the discharge from the pump is more than the mean discharge and supply it to the delivery pipe beyond the air vessel, when the discharge from the pump is less than the mean discharge. Thus, velocity and hence the rate of discharge *is averaged out in the delivery pipe beyond the air vessel.* The velocity of water in the delivery pipe with and without the air

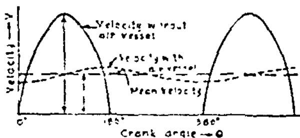


Fig. 12-11 Velocity in delivery pipe

vessel is shown in fig. 12-11. Due to inflow and outflow from air vessel, level of water in the air vessel changes and this, in turn,

changes the pressure of air in the vessel. However, it is desired to have constant air pressure in the vessel, the pressure fluctuations are minimized by using air vessels of large capacities. It usually suffices to give the air vessel a volume 6 to 9 times the pump displacement volume. The broken line in fig. 12-10 indicates the improvement as regards uniformity of delivery by fitting the air vessel.

Since air gets slowly dissolved in water under pressure, the air in the air vessel should periodically be renewed.

An air vessel fitted on the suction pipe performs exactly the same function that the air vessel does on the delivery side. The air vessel on the suction pipe is sometimes termed as *vacuum vessel.*

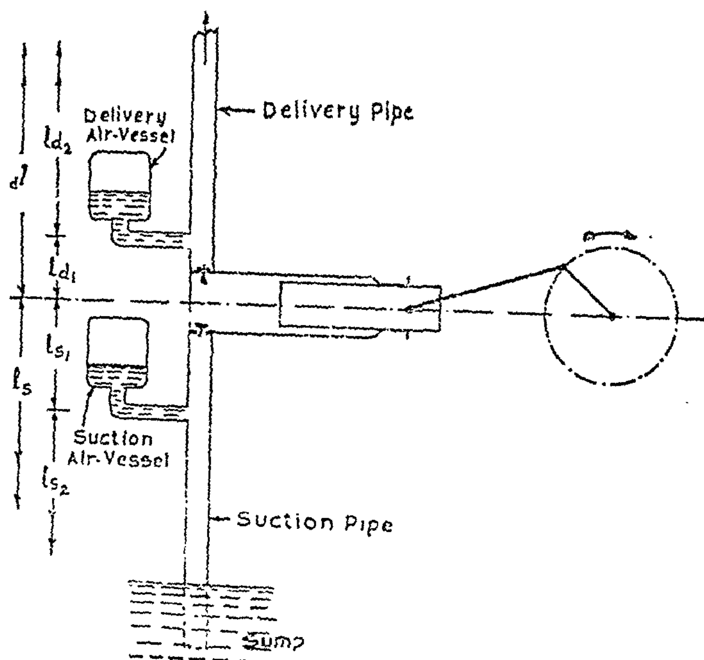


Fig. 12-12 Pump with air vessels

When air vessels are fitted on both delivery and suction sides of the pump (fig. 12-12), the velocity of water is made constant in

the pipe lines beyond air vessels and hence there is no acceleration of water in these pipe lengths This results in damping out the fluctuation of pressure in the pipe lines. The advantages of the air vessels can briefly be summarized as under :

(i) As velocity of water is constant in the pipe line, the discharge is constant and not fluctuating as in the case of pump without air vessel.

(ii) The velocity being constant, there is no acceleration of water, and hence no accelerating head is present. This can permit us to install the pump at a higher level or to run the pump at higher speed without separation occuring inside the cylinder.

(iii) The head lost in friction in the pipe depends on the velocity of water in the pipe. Now, water moves with constant average velocity which is less in magnitude than the velocity of water in the pipe when there is no air vessel on the pump. Thus by fitting air vessel the head lost in friction is reduced and consequently there is saving in H P. consumed by the pump.

Rate of flow of water into and from the air vessel : Consider the case of single-acting pump fitted with air vessels on both suction and delivery sides as shown in fig. 12-12. Water in the pipes beyond the air vessels will have a constant velocity during whole cycle, while water enters or leaves the cylinder during one stroke only. Let,

Q - rate of flow of water into or from the air vessel,
Q_1 - rate of flow of water into or from the cylinder,
Q_2 - average rate of flow of water in the pipes,
V_1 - velocity of water between air vessel and cylinder,
V_2 - velocity of water in the pipe beyond the air vessel.

Now, instantaneous rate of discharge from or into the cylinder is

$$Q_1 = A \omega r \sin \theta \qquad \ldots(12\text{·}20)$$

$\therefore$ velocity of flow in pipe between air vessel and cylinder is

$$V_1 = \frac{A}{A_{PIPE}} \omega r \sin\theta$$

changes the pressure of air in the vessel. However, it is desired to have constant air pressure in the vessel, the pressure fluctuations are minimized by using air vessels of large capacities. It usually suffices to give the air vessel a volume 6 to 9 times the pump displacement volume. The broken line in fig. 12–10 indicates the improvement as regards uniformity of delivery by fitting the air vessel.

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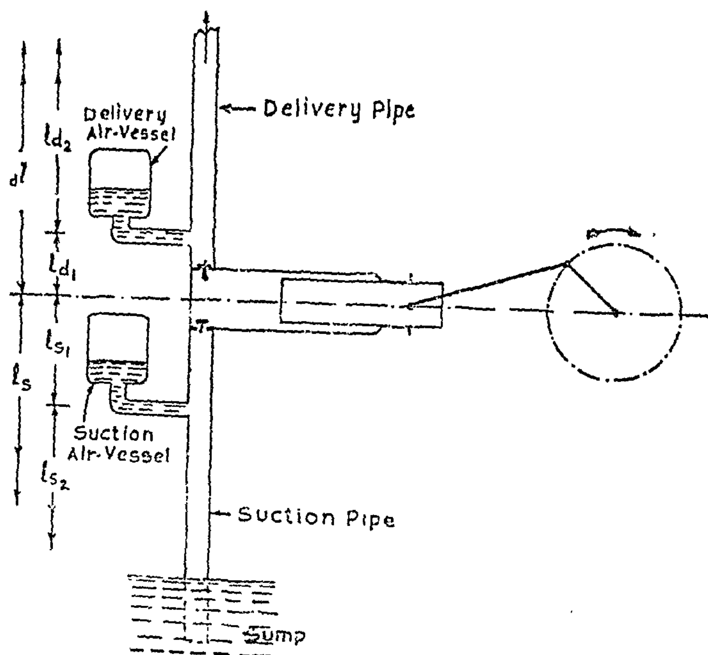


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V_1 – velocity of water between air vessel and cylinder,

V_2 – velocity of water in the pipe beyond the air vessel.

Now, instantaneous rate of discharge from or into the cylinder is

$$Q_1 = A\,\omega r \sin\theta \quad .. \quad .. \quad ..(12\cdot20)$$

$\therefore$ velocity of flow in pipe between air vessel and cylinder is

$$V_1 = \frac{A}{A_{PIPE}}\,\omega r \sin\theta$$

$$\text{and} \quad V_2 = \frac{ALN}{60} \times \frac{1}{A_{\text{PIPE}}} = \frac{A}{A_{\text{PIPE}}} \times \frac{2rN}{60}$$

$$\text{Since} \quad \omega = \frac{2\pi N}{60}, \quad \frac{2N}{60} = \frac{\omega}{\pi}$$

$$\therefore \quad V_2 = \frac{A}{A_{\text{PIPE}}} \times \frac{\omega r}{\pi} \qquad \text{.. (12·21)}$$

As the rate of flow = area × velocity of flow,

rate of flow of beyond air vessel, $Q_2 = A_{\text{PIPE}} \times V_2$

$$= \frac{A\omega r}{\pi} \qquad \text{.. (12·22)}$$

∴ rate of flow of water *from* the air vessel,

$$Q = Q_2 - Q_1$$

$$= \frac{A\omega r}{\pi} - A\omega r \sin\theta$$

$$= A\omega r \left(\frac{1}{\pi} - \sin\theta \right) \qquad \text{.. (12·23)}$$

The *negative* value of Q indicates that water is flowing *into* the air vessel. As $\sin\theta = \frac{1}{\pi}$, no flow occurs from or into the air vessel; this gives two values of θ (18° 34′ or 161° 26′) for which discharge from the cylinder is equal to mean discharge (*i e.* $Q_1 = Q_2$) These two points have been marked as A and B in fig. 12–13.

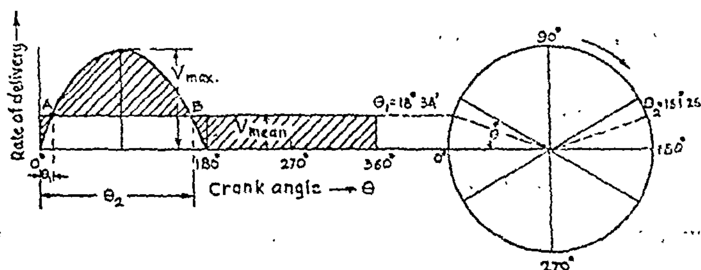


Fig. 12–13 Rate of discharge

Next, consider the pump to be double-acting.

Then, velocity of flow beyond air vessel, $V_2 = \frac{2A\omega r}{\pi A_{\text{PIPE}}}$ .. (12·24)

and rate of flow beyond air vessel, $Q_s = \frac{2A\omega r}{\pi}$ ... (12·25)

$\therefore$ rate of flow from air vessel, $= Q_s - Q_l$

$$= A\omega r\left(\frac{2}{\pi} - \sin\theta\right) \quad \text{.. (12·26)}$$

Indicator diagram for a reciprocating pump with air vessels: The following additional notations should be adopted for the purpose (ref. fig. 12–12).

V_{s_1} – velocity of water in the suction pipe beyond air vessel,
V_{d_1} – velocity of water in the delivery pipe beyond air vessel,
l_{s_1} – length of suction pipe between cylinder and air vessel,
l_{d_1} – length of delivery pipe between cylinder and air vessel,
h_{fs_1} – *head lost in friction in the suction pipe when air vessel,* is fitted.
h_{fd_1} – head lost in friction in the delivery pipe when air vessel is fitted,

As is established earlier,

$$V_{s_1} = \frac{A\omega r}{\pi A_s} \text{ and } V_{d_1} = \frac{A\omega r}{\pi A_d}$$

Hence, frictional head, $h_{fs_1} = \frac{4fl_{s_1}}{d_s}\frac{V_{s_1}^2}{2g} = \frac{4f(l_s - l_{s_1})}{2gd_s}\left\{\frac{A\omega r}{\pi A_s}\right\}^2$

If we neglect the length of the pipe between the air vessel and cylinder, considering it to be very small and assuming that air vessel is fitted very near to the cylinder, we have,

$$h_{fs_1} = \frac{4fl_s}{d_s}\left\{\frac{A\omega r}{\pi A_s}\right\}^2$$

This head remains constant for all positions of piston.

Similarly, $h_{fd_1} = \frac{4fl_d}{d_d}\left\{\frac{A\omega r}{\pi A_d}\right\}^2$

T [illegible]

The gross or manometric head now becomes

$$H_m = \frac{(h_s + h_d + h_{fs_2} + h_{fd_2})L}{L} = h_s + h_d + h_{fs_2} + h_{fd_2} \qquad ..(12\cdot27)$$

and the H.P. of the pump becomes $\frac{WH_m}{75}$.

Let us consider the accelerating head and frictional head in the pipe length between the air vessel and cylinder.

Let, h_{fs_1} – head lost in friction in the suction pipe length, l_{s_1},
h_{fd_1} – head lost in friction in the delivery pipe length, l_{d_1},
h_{as_1} – accelerating head in the suction pipe length, l_{s_1},
h_{ad_1} – accelerating head in the delivery pipe length, l_{d_1},

Then, $h_{fs_1} = \frac{4fl_{s_1}}{2gd_s}\left\{\frac{A}{A_s}\omega r\sin\theta\right\}^2$; $h_{fd_1} = \frac{4fl_{d_1}}{2gd_d}\left\{\frac{A}{A_d}\omega r\sin\theta\right\}^2$

and $h_{as_1} = \frac{l_{s_1}}{g}\left\{\frac{A}{A_s}\omega^2 r\cos\theta\right\}$; $h_{ad_1} = \frac{l_{d_1}}{g}\left\{\frac{A}{A_d}\omega^2 r\cos\theta\right\}$

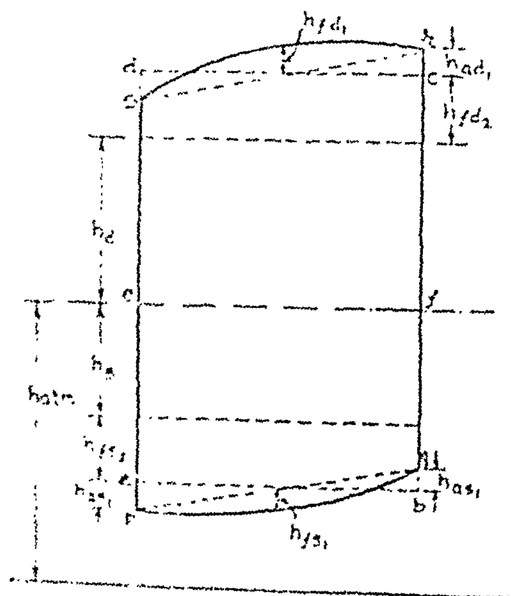


Fig. 12-14 Modified indicator diagram

The indicator diagram which has included all these effects is shown in figure 12-14.

Hence, the manometric head, H_m now becomes

$$H_m = \frac{(h_s + h_d)\,L + (h_{fs_2} + h_{fd_2})\,L + (\tfrac{2}{3}\,h_{fs_1} + \tfrac{2}{3}\,h_{fd_1})\,L}{L}$$

or $$H_m = (h_s + h_d) + (h_{fs_2} + h_{fd_2}) + \tfrac{2}{3}\,(h_{fs_1} + h_{fd_1}) \qquad \text{.. (12·28)}$$

If W is the discharge of fluid per sec.,

then, H.P. of pump when air vessels are fitted $= \dfrac{W H_m}{75}$ (W in kg/sec, H_m in m)

Pressure head during suction and delivery strokes : When air vessels are fitted on the suction and delivery pipes the total (gross) heads during suction and delivery strokes are as shown in the tabular form below, in terms of pressure head of water (gauge).

Point during the stroke	Beginning	Middle	End
Suction	$h_s + h_{as_1} + h_{fs_2}$	$h_s + h_{fs_2} + h_{fs_1}$	$h_s - h_{as_1} + h_{fs_2}$
Delivery	$h_d + h_{ad_1} + h_{fd_2}$	$h_d + h_{fd_2} + h_{fd_1}$	$h_d - h_{ad_1} + h_{fd_2}$

H P. saved by fitting air vessels : To find out the saving in the energy to be spent on the pump, consider the pump of fig. 12–12.

If $(W.D.)_{s_1}$ – work done/sec during the suction stroke when there is *no* air vessel, and

$(W.D.)_{s_2}$ – work done/sec. during the suction stroke when air vessel is fitted, then

$$(W.D.)_{s_1} = W\left(h_s + \tfrac{2}{3}\,h_{fs}\right) = W\left[\,h_s + \frac{2}{3} \times \frac{4f\,l_{s_1}}{2gd_s}\left\{\frac{A\omega r}{A_s}\right\}^2\right] \qquad \text{.. (12·29)}$$

and $$(W.D.)_{s_2} = W\,(h_s + h_{fs_2} + \tfrac{2}{3}\,h_{fs_1})$$

$$= W\left[\,h_s + \frac{4f\,l_s}{2gd_s}\left\{\frac{A\omega r}{\pi A_s}\right\}^2 + \frac{2}{3}\;\frac{4f\,l_{s_1}}{2gd_{s_2}}\left\{\frac{A\omega r}{A_s}\right\}^2\right] \qquad \text{.. (12·30)}$$

Last term in eqn. (12–30) can be neglected, it being very small.

Hence, work saved during the suction stroke by fitting an air vessel on the suction pipe, is

W.D. saved = $(W.D.)_{s1} - (W.D.)_{s2}$

% of work saved during the suction stroke is

$$\frac{(W.D.)_{s1} - (W.D.)_{s2}}{(W.D.)_{s1}} \times 100$$

If only friction work is considered,

$$\% \text{ of work saved in friction} = \frac{\frac{2}{3} l_s - l_{s2} \times \frac{1}{\pi^2} - \frac{2}{3} l_{s1}}{\frac{2}{3} l_s} \times 100$$

$$= \frac{\frac{2}{3} - \frac{l_{s2}}{l_s} \times \frac{1}{\pi^2} - \frac{2}{3} \frac{l_{s1}}{l_s}}{\frac{2}{3}} \times 100$$

If l_{s1} is small compared with l_s, *i.e.* $l_{s2} = l_s$, then

$$\% \text{ of work saved in friction} = \frac{\frac{2}{3} - \frac{1}{\pi^2}}{\frac{2}{3}} \times 100 = 84 \cdot 8$$

If the pump is double acting,

$$\% \text{ of work saved in friction} = \frac{\frac{2}{3} - \frac{4}{\pi^2}}{\frac{2}{3}} \times 100 = 39 \cdot 2$$

Further, if $(W.D.)_{d1}$ – work done during the delivery stroke when there is no air vessel fitted, and

$(W.D.)_{d2}$ – work done during delivery stroke when air vessel is fitted.

Then, $(W.D.)_{d1} = W(h_d + \frac{2}{3} h_{fd})$

$$= W\left[h_d + \frac{2}{3} \frac{4 f l_d}{2 g d_d} \left\{ \frac{A}{A_d} \omega r \right\}^2 \right] \quad .. \; (12 \cdot 31)$$

and $(W.D.)_{d2} = W (h_d + h_{fd2} + \frac{2}{3} h_{fd1})$

$$= W\left[h_d + \frac{4 f l_{d2}}{2g} \left\{ \frac{A \omega r}{\pi A_d} \right\}^2 + \frac{2}{3} \frac{4 f l_{d1}}{2 g d_d} \times \left\{ \frac{A \omega r}{A_d} \right\}^2 \right] \quad .. \; (12 \cdot 32)$$

Last term in eqn. (12·32) can be neglected, it being very small. Work saved during the delivery stroke by fitting an air vessel on the delivery pipe is,

$$\text{W.D. saved} = (\text{W.D.})_{d1} - (\text{W.D.})_{d2}$$
$$= \frac{(\text{W.D.})_{d1} - (\text{W.D.})_{d2}}{(\text{W.D.})_{d1}} \times 100\%$$

Hence, the total work saved $= [(\text{W.D.})_{s1} - (\text{W.D.})_{s2}] + [(\text{W.D.})_{d1} - (\text{W.D.})_{d2}]$

Problem-14 : *The following particulars refer to a double-acting reciprocating pump .*

Bore – 200 mm; stroke – 400 mm speed – 120 rpm.

Suction pipe diameter – 100 mm

Find the rate of flow from or to air vessel when crank makes angles of 30°, 90° and 120° from inner dead centre. Determine also the crank angles at which there is no flow from or to air vessel. Assume S H M. for the piston.

Since only suction pipe diameter is given, it means that the air vessel is fitted on the suction pipe only. Let positive sign indicate flow to air vessel.

The rate of flow to or from air vessel is

$$A\omega r\left(\frac{2}{\pi} - \sin\theta\right)$$
$$= \frac{\pi}{4}\left(\frac{200}{1{,}000}\right)^2 \times \frac{2\pi \times 120}{60} \times \frac{400}{2 \times 1{,}000}\left(\frac{2}{\pi} - \sin\theta\right)$$
$$= 0{\cdot}079\left(\frac{2}{\pi} - \sin\theta\right)$$

When $\theta = 30°$,

$$\text{rate of flow} = 0{\cdot}079\left(\frac{2}{\pi} - 0{\cdot}5\right)$$
$$= \underline{0{\cdot}0108 \text{ m}^3/\text{sec.}}$$

This flow is from suction pipe into the air vessel.

When $\theta = 90°$,

$$\text{rate of flow} = 0.079\left(\frac{2}{\pi} - 1\right)$$

$$= \underline{-\ 0.0292\ \text{m}^3/\text{sec.}}$$

This flow takes place from air vessel to cylinder.

When $\theta = 120°$,

$$\text{rate of flow} = 0.079\left(\frac{2}{\pi} - 0.866\right)$$

$$= \underline{-\ 0.0182\ \text{m}^3/\text{sec.}}$$

This flow is also from air vessel to cylinder.

The flow from or to the air vessel will be zero when

$$\sin\theta = \frac{2}{\pi}$$

$$\text{or}\ \ \theta = \underline{39.5°}\ \text{or}\ \underline{140.5°}$$

Problem-15: *A single acting reciprocating pump is provided with large air vessels on the suction and delivery sides close to the pump. The pump is placed at a height of 3.5 m above the water level in the pump. The plunger has diameter of 30 m and stroke of 45 cm. The diameter and length of the suction pipe is 15 cm and 6 m respectively. The delivery pipe has length of 45 m and diometer of 15 cm. The pump runs with S.H.M. such that separation takes place at the beginning of suction stroke. The effective height of the barometer is 8 m. Take coefficient of friction, f = 0.01. Determine the H.P. saved in overcoming friction in the suction and delivery pipes by fitting the air vessels. The speed of the pump is the same as the speed of a pump without an air vessel.*

We have, H_{atm} – vacuum pressure = 8 m. of water

and also, effective head is made up of static suction head and acceleration head in suction pipe, *i. e.*

$$8 = h_s + h_{as} = 3.5 + h_{as}$$

$$\therefore\ \ h_{as} = 8 - 3.5 = 4.5\ \text{m}$$

$$\text{and} \quad 4{\cdot}5 = \frac{l_s}{g}\frac{A}{A_s}\,\omega^2 r$$

$$= \frac{6}{9{\cdot}81} \times \left(\tfrac{30}{15}\right)^2 \omega^2 \times \frac{45}{2 \times 100}$$

$$\therefore \quad \omega^2 = 8{\cdot}2$$

$$\therefore \quad \omega = 2{\cdot}86 \text{ rad./sec. and hence, } N = 27{\cdot}5 \text{ rpm}$$

To estimate the saving in H.P. the frictional heads in suction and delivery pipes should be calculated.

$$h_{fs} = \frac{4\,fl_s}{d_s}\frac{\left(\frac{A}{A_s}\,\omega r\right)^2}{2g} = \frac{4 \times 0{\cdot}01 \times 6}{15/100} \times \frac{\left\{\left(\tfrac{30}{15}\right)^2 \times 2{\cdot}86 \times \frac{45}{2 \times 100}\right\}^2}{2 \times 9{\cdot}81}$$

$$= 0{\cdot}55 \text{ m}$$

As diameters of pipes are equal,

$$h_{fd} = h_{fs} \times \frac{l_d}{l_s} = 0{\cdot}55 \times \frac{45}{6} = 4{\cdot}13 \text{ m}$$

When air vessels are fitted, the velocity in the pipes becomes

$$V' = \frac{A}{A_{pipe}} \times \frac{\omega r}{\pi}$$

and the frictional heads become,

$$h_{fs_2} = 0{\cdot}55 \times \frac{\left(\frac{\omega r}{\pi}\right)^2}{(\omega r)^2} = 0{\cdot}55 \times \left(\frac{1}{\pi}\right)^2 = 0{\cdot}055 \text{ m}$$

$$\text{and } h_{fd_2} = 4{\cdot}13 \times \left(\frac{1}{\pi}\right)^2 = 0{\cdot}415 \text{ m}$$

When no air vessels are fitted, the gross or manometric head is

$$H_m = h_s + h_d + \tfrac{2}{3}\,(h_{fs} + h_{fd})$$

and when air vessels are fitted, it becomes

$$H_{m_2} = h_s + h_d + h_{fs_2} + h_{fd_2}$$

$\therefore$ head saved, $H_m' = H_m - H_{m1}$

$= \frac{2}{3}(h_{fs} + h_{fd}) - (h_{fs2} + h_{fd2})$

$= \frac{2}{3}(0.55 + 4.13) - (0.055 + 0.415)$

$= 2.65$ m

weight of water lifted, $W = \frac{wALN}{60} = 1{,}000 \times \frac{\pi}{4}\left(\frac{30}{100}\right)^2 \times \frac{45}{100} \times \frac{27.5}{60}$

$= 14.6$ kg/sec.

$\therefore$ H.P. saved $= \frac{W \times H_m'}{550} = \frac{14.6 \times 2.65}{75} = 0.515$

Duty of a Reciprocating Pump

The overall efficiency of the pump includes the efficiency of the prime mover. The practical way of expressing the overall efficiency of the pump is in terms of duty of the pump.

When the pump is driven by a steam engine, for example, duty of the pump is expressed in terms of work done in kg-m. for every 2.5×10^5 kcal. of heat energy supplied to the engine by a boiler. Thus, the duty takes into account the efficiency of the pump and the steam engine.

Let, p be the pressure at which the steam is supplied by a boiler to the engine driving the pump, and W be the weight of steam used per second. Then H is total heat of one kg of steam, under given supply conditions at boiler stop valve.

Total kcal supplied per second $= WH$

$$\text{Duty of the pump} = \frac{\text{work done is kg·m/sec.}}{WH} \times 2.5 \times 10^5$$

If the pump is driven by an internal combustion engine, using W kg of fuel per second having calorific value of 'C' kcal/kg,

$$\text{duty of the pump} = \frac{\text{work done in kg·m/sec.}}{W \times C} \times 2.5 \times 10^5$$

Similarly, it can be determined for an electric drive also.

Multi-cylinder Pumps

Continuous discharge with improved uniformity can also be obtained by using pumps having more than one cylinders. It is obvious that the inertia head in the suction and delivery pipes of a two cylinder pump will only be half as great an inertia head as in the pipes of the single cylinder pump. The various arrangements by which this improvement may be attained are shown schematically in fig. 12-15. The single cylinder pump is shown in fig. 12-15 (a) and a two-throw pump with plungers driven from cranks set at 180° out of phase is shown is 12-15 (b). A double acting single cylinder pump and single acting three-throw pump having three cranks set at 120° are shown in fig. 12-15 (c) and fig. 12-15 (d) respectively. Pumps of types (a), (b) & (c) have same gross discharge provided area of plunger of pump of type (a) is double the area of plungers of pumps of type (b) & (c) and strokes of all of them are equal) as that of (a) when driven at the same speed, but they deliver

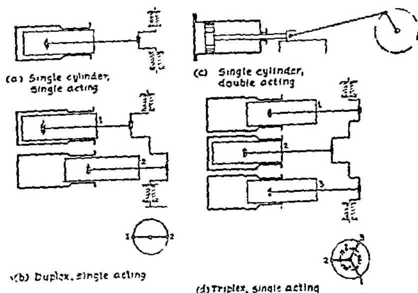


Fig. 12-15 one, two and three cylinder pumps

water with greater uniformity. This is evident from fig. 12-16, where the instantaneous discharge from each cylinder is compared

with the mean discharge of the pump sets. The three-throw pump yields still further improvement regarding inertia pressures. It may be observed that maximum inertia pressure in such a case is one-sixth of that in a single cylinder, single acting pump. This is

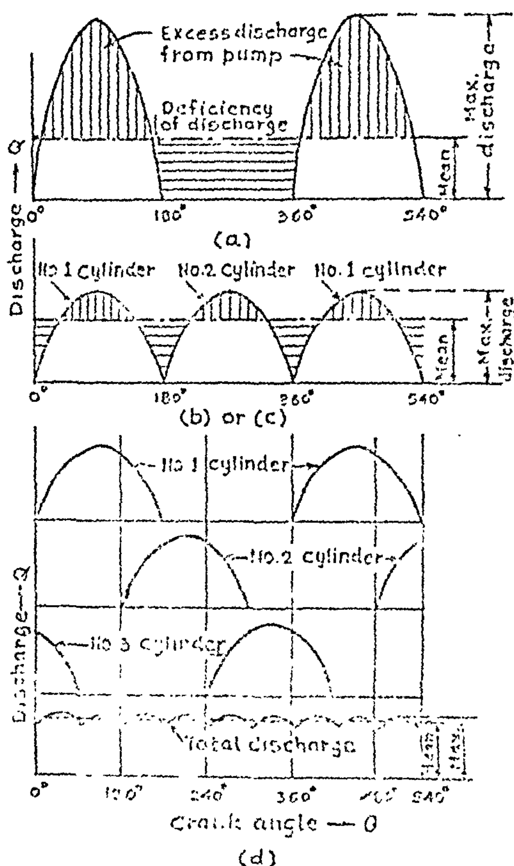


Fig. 12-16 Discharge curves for one, two and three cylinder pumps without air vessels

on account of the fact that for several portions of the cycle, two of the three cylinders are supplying water simultaneously. Thus on account of uniformity of discharge and turning moment, three-throw pumps, are preferable for both small and large installations.

A two cylinder (*duplex*) double acting pump having cranks at 90° is equivalent to a fourthrow (*i.e.* four-cylinder single acting with cranks set at 90°) pump. A pump with five cylinders can also be built up and it is named as *quintuplex* pump. In order to have the comparative idea of uniformity of discharge and thus reduction in inertia pressure, the ratio of maximum inertia pressure in multi-cylinder pumps to that in single cylinder single acting pump is presented in tabular form below :

No. of cylinders	Phase angle of cranks. degrees	Ratio of maximum inertia pressure
1	360	1.000
2	180	0.500
3	120	0.167
4	90	0.250
5	72	0.100
6	60	0.167

Careful examination of this table suggests the advantages of three-throw or five-throw pump; or, in general, the superiority of an *odd* number of cylinders over an *even* number.

Problem 18 *A three-throw pump has cylinders, 300 mm diameter, and 600 mm stroke. The pump is required to lift 100 litres/sec of water to a height of 100 m. Friction losses are estimated at 4 m in suction pipe and 15 m in delivery pipe. Velocity of water in pipes is 1 m/sec. If the pump efficiency is 90% and slip 2%, find the speed at which the pump should run and the H P required by it.*

Theoretical discharge/sec. $= 3 \times \frac{ALN}{60}$

Actual discharge/sec $= \frac{3 \times ALN}{60} \times (1 - \text{slip})$

i.e. $\frac{100}{1,000} = 3 \times \frac{\pi}{4} \times \left(\frac{300}{1,000}\right)^2 \times \frac{600}{1,000} \times \frac{N}{60} \times (1 - 0.02)$

$\therefore \quad N = 48.1$ rpm.

total head generated by pump is

$$H_m = 100 + 4 + 15 + \frac{V_d^2}{2g} = 119 + \frac{1}{2 \times 9 \cdot 81} = 119 \cdot 06 \text{ m}$$

$\therefore$ H.P. input to the pump $= \dfrac{WH_m}{75 \times \eta_o}$

$$= \frac{100 \times 119 \cdot 06}{75 \times 0 \cdot 9}$$

$$= \underline{176}$$

Problem–19 : *Pistons of a three-throw, single-acting pump have simple harmonic motion and are driven from cranks spaced 120 degrees apart and running at 75 rpm. The cylinder diameters and strokes are 6·5 cm and 15 cm respectively. The cylinders discharge to a common main 7·5 cm diameter and 75 m long, which rises 9 m above the pump.*

Plot graphs of the velocity and acceleration of water in the pipe on the base of crank angles for one complete turn. Calculate the head at the pump discharge flanges, when one plunger is at the beginning of its discharge stroke. No air vessel is fitted. Take coefficient of friction for mains as 0·01.

When No. 1 piston is at beginning of its delivery stroke,

No. 2 must have moved through 120° (delivery stroke), and

No. 3 must have moved through 240° (suction stroke).

Thus, No. 1 and No. 2 are delivering water to the discharge main.

Angular velocity, $\omega = \dfrac{2\pi \times 75}{60}$ 7·85 rad./sec.

Velocity in delivery main, $V_d = \dfrac{A}{A_d} \omega r \{ \sin\theta + \sin(\theta + 120°)\}$

$$= \frac{A}{A_d} \omega r (\sin 0 + \sin 120°)$$

$$= \left(\frac{6 \cdot 5}{7 \cdot 5}\right)^2 \times 7 \cdot 85 \times \frac{15}{2 \times 100} (0 + 0 \cdot 866)$$

$$= 0 \cdot 385 \text{ m/sec.}$$

Acceleration in the pipe $= \frac{A}{A_d}\, \omega^2 r\,(\cos 0 + \cos 120^\circ)$

$$= \left(\frac{6\cdot5}{7\cdot5}\right)^2 \times (7\cdot85)^2 \times \frac{15}{2\times100}(1-0\cdot5)$$

$$= 1\cdot74 \text{ m/sec}^2$$

Now, total head at the flange of the delivery mains is

$$h_d + h_{fd} + h_{ad} + \frac{V_d^2}{2g}$$

$$= 9 + \frac{4flV_d^2}{2gd} + \frac{l_d}{g}\times a_d + \frac{V_d^2}{2g} \quad (a_d = \text{acceleration in pipe})$$

$$= 9 + \frac{V_d^2}{2g}\left(1+\frac{4fl}{d}\right) + \frac{l_d}{g}\times 1\cdot74$$

$$= 9 + \frac{0\cdot385^2}{2\times9\cdot81}\left\{1+\frac{4\times0\cdot01\times75\times100}{7\cdot5}\right\} + \frac{75}{9\cdot81}\times1\cdot74$$

$$= 9 + 0\cdot0326 + 13\cdot3$$

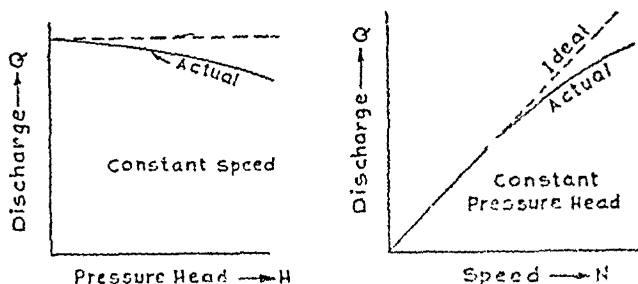
$$= 22\cdot3326 \text{ m}$$

Performance Characteristics

The fundamental theory of positive displacement pumps established earlier, can be used in studying the performance characteristics.

Discharge-head characteristics : It can be seen from eqn. (12·5) that discharge, Q is proportional to the speed of the given pump. Thus, at constant speed, the discharge is independent of pressure head under which it operates. This is shown in fig. 12·17(a). As soon as the effect of slip and leakage is considered, the discharge-head curve begins to drop, which is indicated by the full line of the figure. The difference between the ideal and actual discharge curves is termed as slip and the ratio of actual to ideal discharge is termed as coefficient of discharge or the volumetric efficiency of the pump. For well maintained pumps the volumetric efficiency is about 0·98. The usual range of this value is 0·85 to 0·98. The slip increases at faster rate with increase in head beyond 700 m due to compressibility effects.

Discharge-speed characteristics: As discussed above, for a given pump, ideally speaking, discharge is directly proportional to the speed, indicated by the dotted line of fig. 12-17 (b). It may, however, be noted that the speed cannot be increased beyond



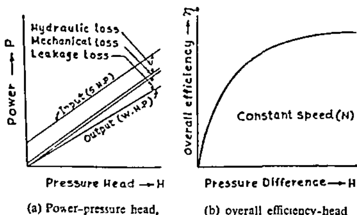
(a) Discharge-head (b) Discharge-speed

Fig. 12-17 Performance characteristics

certain limit on account of detrimental effects of separation, followed by cavitation. In practice, the inability of water to closely follow the piston and lag in valve closures with increased speed result in reduced discharge. Thus, the actual discharge curve at constant head bends down at higher speed.

Power-pressure head characteristics : Examining eqn. 12·2, the theoretical horse power *i.e.* water horse power (W.H.P.) is directly proportional to the pressure head when speed, and hence, discharge is kept constant. Actually speaking, with increase in head, the discharge slightly decreases (ref.: fig. 12·17a) at constant speed. Hence, the actual output power (W.H.P.) curve has less slope. On account of (i) mechanical losses, (ii) leakage losses, and (iii) hydraulic losses the input power (S.H.P.) required for driving the pump is greater than W.H.P. It is not possible to estimate these losses individually, yet we can form some impression of their general trends. At constant speed, and varying pressure head and nearly constant discharge, it is expected that hydraulic losses will remain unaltering; on the other hand, the mechanical and leakage losses increase as the pressure head rises. This is all depicted in fig. 12-18 (a). The losses can collectively be estimated by drawing

output and input curves and measuring the vertical intercept between them for a given pressure head.



(a) Power-pressure head, (b) overall efficiency-head

Fig. 12-18 Performance characteristics

It may be observed that total losses increase with increase of head. The overall efficiency which is the ratio of output (W.H.P) to input (S.H.P) of the pump is shown in fig. 12-18 (b).

Applications of Reciprocating Pumps

Reciprocating pumps have wide range of application in fields of industry, agriculture and steam and internal combustion engineering A few specific instances are feed pumps of boilers, pumps installed in dairies, village well pumps, hydraulic jacks, hydraulic presses, lubricating pumps of I.C engines, fuel pumps etc. For reason of their being of positive displacement type, they are preferred for the above-named applications Since, not only their capital cost, but maintenance cost also is higher, they are not adopted for industrial applications and their place is taken over by centrifugal pumps

Comparison of Reciprocating and Centrifugal Pumps

The relative advantages and disadvantages of the centrifugal pump over the reciprocating pump are stated as follows:

Comparison (item)	Reciprocating pump	Centrifugal pump
Starting torque	less	more
Higher speed	not possible- (reduction gearing essential)	possible- direct coupling with prime mover possible
Closing of sluice valve on delivery side	dangerous	not dangerous
Suction and delivery valves	necessary	not necessary
Size	too bulky	moderate
Uniformity of discharge	fluctuating-without air vessel uniform-with air vessel	Uniform – no air vessel required
Head	Very high heads with less discharge	small head with huge discharge – (head can be increased by adopting multi-stage pumps)
Efficiency	Very high – falls rapidly under low heads	Less
Purity of liquid pumped	Should be very clean	can deal with dirt and even mud without seriously imparing the working and efficiency
Balancing	not proper	proper
Torque	varying	uniform
Priming	not necessary	necessary

Other Types of Positive Displacement Pumps

Bubket pump : This is a vertical piston pump (fig. 12 – 19). The piston is provided with a bucket valve and the combined unit is termed as bucket. This is essentially a low speed pump and needs gear head with large speed reduction when driven by an electric motor. When operated by hand it is popularly known as a *hand pump.*

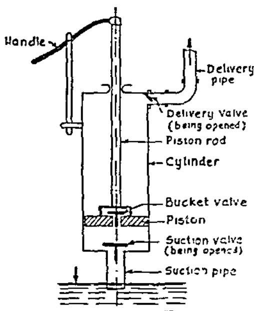


Fig. 12–19 Bucket pump or hand pump

When the bucket is raised by lowering the handle, piston valve remains closed as the pressure under it falls. This opens the suction valve. This is the suction cum delivery stroke and during this stroke water is sucked into the cylinder and water above the bucket is forced into the delivery pipe. When bucket is lowered neither delivery nor suction takes place. As the piston valve is open, water flows across the piston from the suction to delivery side of the cylinder.

Archimedeam screw : This consists of a helicoid of welded sheet-steel construction fitted within a cylinder, whose axis is inclined so that its lower end dips beneath the liquid to be lifted The appearance is very similar to a screw. It may be remembered that helicoid is the surface generated by a line perpendicular to an axis rotating about the axis and simultaneously advancing along it. The cylinder and the helicoid together form a series of cells in which liquid is entraped and lifted when the screw is revolved.

The discharge of the screw at a given speed falls rapidly as the inclination of the axis increases. Further, the discharge at a given speed and inclination increases as the lift diminishes. The overall efficiency may be about 55% or more.

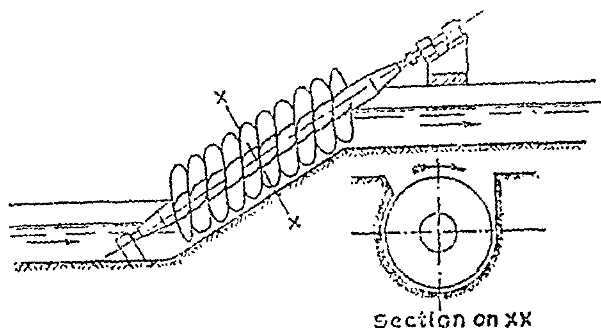


Fig. 12–20 Archimedean screw in canal

The Archimedean screw may be used for lifting small amounts or irrigation water, and lifting corrosive liquids that would be difficult to be handled by ordinary pumps. If the screw is worked against a fixed concrete semi-cylinderical housing, then the apparatus may be on such a scale that it can compete in cost and efficiency with low lift pumping plants.

Rotary pumps : There are many varieties of positive rotary pumps. Following are the chief points of difference between this class of pumps and reciprocating pumps :

(i) The chambers, whose successive filling and emptying produces flow, continuously revolve ;

(ii) The rotation of the chamber automatically connects to either suction pipe or delivery pipe, and thus valves are not needed.

(iii) The number of working chambers per revolution of the pump is so great that the inertia shocks are reduced to negligible limits. Thus, air vessels are not needed.

(iv) The working chamber is not necessarily cylindrical.

For these reasons the speed of rotary pumps can be much higher than that of reciprocating pumps. Thus, they are lighter,

cheaper, and more compact as compared to reciprocating pumps As the chamber is revolving, it is not possible to make it leak-proof; thus rotary pumps are more suited for handling viscous liquids such as oil rather than water or spirit. Small units are capable of developing pressures up to 350 kg/cm². They are very much used for hydraulic transmission systems, fuel injection systems of I. C. Engines, and lubrication systems. The best known pumps of this class are : vane pumps and gear-wheel pumps.

Vane pump : This pump consists of the rotor which carries flat radial vanes. This rotor along with its shaft is eccentrically mounted in cylindrical casing (fig. 12-21). In the same casing is also fitted a concentric fixed core on the outer periphery of which slide the vanes. The vanes are also free to slide in radial slots formed in the rotor. Thus, working chamber is formed by pair of adjacent vanes, the side cover plates of the casing, the inner casing surface, and outer rotor surface. As the rotor rotates the volume of the

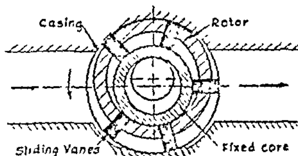


Fig. 12-21 Vane pump

working chamber increases as they travel along the suction side and subsequently reduces along the delivery side Thus liquid is drawn in from the suction side and then forced out on the delivery side.

Gear-wheel pump : This pump is a modified version of the vane pump. Fig 12-22 illustrates the principle of working of this pump. It consists of two identical intermeshing spur wheels on shafts supported in bearings provided in the casing of the pump. The wheels rotate in the casing, in opposite directions, and the

casing is so shaped as to give very fine clearance between the top land of the teeth and the inner surface of the casing. For small

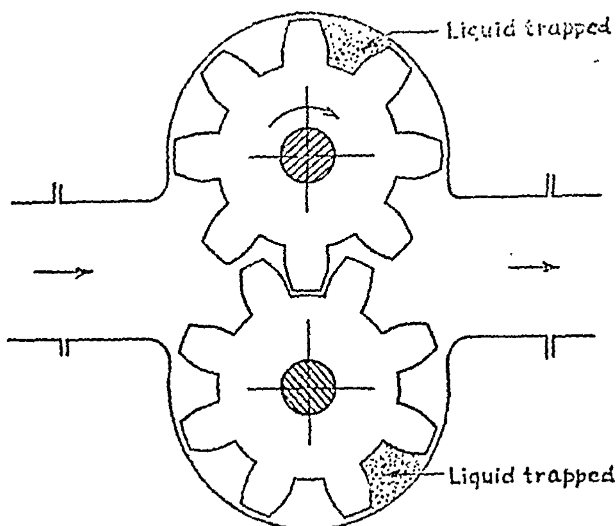


Fig. 12-22 Gear wheel pump

capacity pumps usually one of the pinions is driven from external source and other usually revolves idly. For heavy duty pumps, both the pinions are driven by connecting external timing gears. It may be noted that both the pinions are simultaneously pumping the liquid, hence for the same size of pinion as that of the vane pump, this pump delivers double the quantity of liquid. These types of pumps have been constructed to develop pressures upto 140 kg/cm^2. and discharge upto 450 litres/min.

Bore-hole pump : For water works in places where the level of water in wells and bore-holes is deeper than 8 m, reciprocating type of bore-hole pumps, as shown in fig. 12-23, are usually installed. These are slow-speed pumps run by engine or motor located at the ground level. Power and motion are transmitted to the bucket through the mechanism consisting of crank, connecting rod, bell crank lever, and vertical pump rod. The bucket is fixed at the end of the pump rod and remains very near to the water level. The bucket valve reciprocates in the working barrel.

The delivery valve or bucket valve has its seating on the piston or the bucket. The foot valve (suction valve) is fitted at the end of the working barrel or pump barrel.

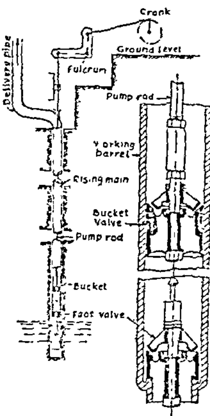


Fig. 12-23 Bore-hole pump

When the pump rod, and hence the bucket is lowered, the delivery valve opens and water in the barrel passes from the lower to the upper side of the bucket, the foot valve remaining closed. When the bucket is raised, water over the delivery valve is lifted bodily and thus below the bucket in the barrel vacuum is created which induces fresh water to enter the barrel through the foot valve. The cycle is continuously repeated till the pump is operated.

In some instances, bore-hole pumps are installed for water works in conjuction with force pumps. In that case, bore-hole pumps lift water to the ground level and the force pumps further force it up into the overhead tank or pressure main Bore-hole pumps are also employed on oil fields.

TUTORIAL - 12

Construction and working

1 Explain, giving neat line diagram, the construction and working of a reciprocating pump. Why it is called a positive displacement pump ?

2 Define the following with reference to a reciprocating pump : (a) overall efficiency, (b) coefficient of discharge and [illegible]

Indicator diagram

3 Draw a simple indicator diagram for single-cylinder single-acting reciprocating pump. Indicate thereon the effects of acceleration and friction in the pipe lines.

4 Water is lifted to a total height of 30 m by a single-acting reciprocating pump having bore of 22·5 cm and stroke of 37·5 cm. If the speed of the pump is 30 rpm., and the actual quantity of water lifted is 430 L.P.M., find the percentage slip, coefficient of discharge, and ideal theoretical H.P.

[8·2; 0·91; 3·14]

5 Water is raised to a height of 25 m by a single-acting, single-cylinder reciprocating pump. The pump runs at 30 rpm. The diameter of the cylinder is 15 cm and the stroke is 25 cm. Find the theoretical discharge in litres/sec., ideal H.P., percentage slip, and the coefficient of discharge, if the actual discharge is 2·1 litres/sec.

[2·21; 0·735; 5; 0·95]

6 A single-acting reciprocating pump has bore of 15 cm and stroke of 26 cm. The diameter of the suction as well as delivery pipe is 10 cm. The lengths of the suction and delivery pipes are 9 m and 45 m respectively. The head lost due to friction in the delivery pipe when the piston is at mid-stroke is 5 m of water. If coefficient of friction is 0·01, determine the maximum inertia head and maximum frictional head in the suction pipe. Also determine the speed of the pump in rpm.

[1·725 m; 1 m; 24·2]

7 In a pumping installation a force pump is required to deliver 475×10^3 litres per hour of water against a static head of 40 m (measured vertically from the water level in the sump upto the delivery level). The delivery pipe is 45 cm in diameter having a total length of 5,650 m. Find the H. P. required to drive the pump, if overall efficiency is 70%. Take $f = 0{\cdot}01$. The suction lift is negligible.

[139]

8 A double-acting reciprocating pump running at 30 rpm. supplies water through a main of 45 cm diameter and 1·6 km long. The bore is 60 cm and stroke 90 cm. The pump supplies water through a static head of 15 m. Mechanical efficiency of the pump is 80% If the slip of the pump is positive equal to 5%, determine the H.P. required to drive the pump and the quantity of water delivered in L.P.M. Take $f = 0{\cdot}005$.

[99; 15,300]

9 A single-acting reciprocating pump has plunger diameter of 25 cm and stroke of 45 cm. The delivery pipe is 12 cm diameter and 5 m long. The suction pipe is 15 cm in diameter and 4 m long. The piston moves with S.H.M. When the pump runs at 45 rpm, determine, (i) maximum accelerating heads in the suction and delivery pipes, and (ii) maximum friction head in the suction and delivery pipes. There is no air vessel on the pump. Take $f = 0{\cdot}01$ for pipes.

[(i) 5·17 m, 11·2 m; (iii) 0·48 m, 1·82 m]

10 A double-acting reciprocating pump has piston diameter of 15 cm and a stroke of 30 cm. The pump makes 60 double strokes per minute. The centre line of the pump is 0·5 m above the water level in the sump. The length of the suction pipe which is 7·5 cm in diameter is 3 m. Determine the pressure in kg/cm^2 absolute inside the cylinder, (i) at the beginning (ii) middle, and (iii) at the end of the suction stroke. Coefficient of friction for the pipe may be taken as 0·008. Neglect the velocity head in the suction pipe.

[(i) 0·225; (ii) 0·8574; (iii) 1·675]

11 A single-acting reciprocating pump has cylinder diameter of 10 cm and stroke of 20 cm. The pump makes 100 strokes per minute. It draws water from a sump, the surface of which is 1·8 m below the centre line of the pump cylinder. The total length of the suction pipe is 4·5 m and its diameter is 5 cm. Determine the absolute pressure in m of water inside the cylinder, (i) at the beginning, (ii) at the middle, and (iii) at the

end of the suction stroke. Take $f = 0{\cdot}01$ and barometer reads 10·4 m of water. State whether separation occurs or not.

[(i) 3·35, (iii) 7·7, (iii) 13·8; No.]

12 The piston of double-acting reciprocating pump moves with simple harmonic motion and makes 30 double strokes per minute. The diameter and the stroke of the piston are 30 cm and 45 cm respectively. The centre line of the cylinder is 3 m above the water level in the sump. The length of the suction pipe is 6 m. There is no air vessel on the suction side. Find the diameter of the suction pipe so that no separation takes place at the beginning of the suction stroke, assuming it to occur at a pressure of 0·7 kg/cm^2 below atmospheric pressure. If the air vessel is provided just near the cylinder on the delivery side only and if the suction pipe is 120 m long and of the same diameter as that of the suction pipe; find the H. P. required to drive the pump. The delivery head is 75 m. Neglect accelerating head on the delivery side and take $f = 0{\cdot}008$ for the pipes. Assume S.H.M. for the piston.

[21 cm; 45·9]

13 A single-acting reciprocating pump has cylinder diameter of 10 cm and stroke of 20 cm. The pump maker 120 strokes per minute. It draws water from a sump, the surface of which is 1·5 m below the centre line of the pump cylinder. The total length of the suction pipe is 4·5 m and its diameter is 5 cm. Determine the absolute pressure inside the cylinder, (i) at the beginning, (ii) at the end of the suction stroke. The coefficient of friction for the pipe is 0·01 and barometer reads 10 m of water.

[(i) 1·25 m, (ii) 7·344 m (ii) 15·75 m of water]

14 A reciprocating plunger pump is used to pump oil of sp. gr. 0·8. The suction pipe is 10 cm in diameter and 7·5 m long. The bore and stroke of the cylinder is 15 cm and 22·5 cm respectively. The pump is situated at a height of 1·8 m above the suction tank. Find the maximum speed at which the pump can be run, if the pressure inside the cylinder is not to be

21 A reciprocating plunger pump is used to pump oil of sp. gr. 0·8. The suction pipe is 10 cm in diameter and 8 m long. The bore and stroke of the cylinder are 15 cm and 22 cm respectively. The pump is situated at a height of 2 m above the suction tank. Find the maximum speed at which the pump can be run, if the minimum pressure inside the cylinder is not to be less than 0·2 kg/cm^2 absolute. Assume S.H.M. for the plunger.

[19·1 rpm.]

22 The lengths of suction pipe and delivery pipe of a double acting reciprocating pump are 6 m and 300 m respectively. There is an air vessel just near the cylinder of a pump on the delivery side. The centre line of the pump is 2·5 m above the water level in the sump. The plunger moves with S.H.M. The plunger is 22 cm in diamer and has 40 cm stroke. The pump runs at 30 rpm. The diameter of the suction and delivery pipes is the same. If separation occurs at the beginning of the suction stroke when the pressure falls below 2 m of water, determine the diameter of the suction pipe.

If water is to be lifted to a total static head of 60 m, find the H. P. of the pump, neglecting all losses except that due to friction in the delivery pipe. Assume coefficient of friction in pipe as 0·01.

[10 cm; 14·75]

23 The lengths of suction and delivery pipes of a double-acting reciprocating pump are 6 m and 35 m respectively. There is an air vessel just near to the cylinder of the pump on the delivery side. The centre line of the pump is 3 m above the water level in the sump. The plunger moves with S.H.M. The plunger is 20 cm in diameter and has 30 cm stroke. The pump runs at 30 rpm. The diameter of the suction and delivery pipes is the same. If separation occurs at the beginning of the suction stroke when the absolute pressure falls below 2 m of water, determine the diameter of the suction pipe. If water is to be lifted to a total static head of 60 m, find the H.P. of the pump neglecting all losses except that due to friction in the delivery pipe. Assume coefficient of friction as 0·01.

[8·5 cm; 76·3]

24 The piston of a double-acting reciprocating pump has stroke of 25 cm and the diameter of *the cylinder is 15 cm.* The pump is placed at a height of 5 m above the water level in the sump. The delivery head is 25 m. The lengths of the suction and delivery pipes are 6 m and 35 m respectively. The diameter of the suction as well as delivery pipe is 10 cm. The separation takes place when the absolute pressure inside the cylinder falls below 3 m of water. Determine, (a) *the maximum speed of rotation, and (b) the load on the piston when* the crank has moved 30° from the I D.C. position. Take coefficient of friction as 0·01. Barometer reads 10 m of water. Assume S.H.M. for the plunger and there is no air vessel on the pump.

[(a) 32·5 rpm.; (b) 740 kg]

25 The following particulars relate to a single-acting reciprocating pump whose piston is moving with S.H M.

Length of the suction pipe	.. 4·5 m
Suction lift	.. 2·5 m
Diameter of the suction pipe ..	.. 7·5 cm
Stroke to bore ratio	.. 1·5 : 1
Barometric height of water ..	.. 10·4 m
Vapour pressure of the liquid ..	.. *0·28 kg/cm²*
Theoretical discharge	.. 250 L.P.M.

Evaluate its correct speed of rotation, the diameter and the stroke of the plunger.

[*37 rpm.*; 18 cm; 26·5 cm]

26 The piston of a double-acting reciprocating pump has stroke *of 22 cm and the diameter of the cylinder is 15 cm.* The pump is placed at a height of 4·5 m above the water level in sump. The delivery head is 26 m. The lengths of the suction and delivery pipes are 6 m and 30 m respectively. The diameter of the suction as well as delivery pipe is 10 cm. The separation takes place when the absolute pressure inside the cylinder falls below 2·4 m of water. Determine, (a) the

maximum speed of rotation, and (b) the load on the piston when the crank has moved 30° from the I.D.C. position. Take coefficient of friction as 0·01. Barometer reads 10·4 m of water. Assume S.H.M. and there is no air vessel on the pump.

[(a) 43 rpm.; (b) 870 kg]

27 A double-acting reciprocating pump has piston 15 cm in diameter and stroke of 20 cm. The diameter of the suction pipe as well as the delivery pipe is 10 cm. The length of the suction and delivery pipes are 5 m and 30 m respectively. Water is taken from a sump below the centre line of the pump and delivered to a height of 27 m above centre line of the pump. Accelerating head in the suction pipe is $\frac{3}{4}$th suction lift. Determine the speed of rotation and theoretical H.P. required to drive the pump. Assume that separtion takes place when the absolute pressure falls below 2·5 m of water. Coefficient of friction in the pipe is 0·01.

[50·4 rpm.; 2·53]

28 A double-acting single cylinder reciprocating pump has cylinder of 10 cm diameter and stroke of 15 cm. The ratio of length of crank to connecting rod is 1:5. The static suction lift is 2 m and the length of the suction pipe which is 6·2 cm diameter, is 4·3 m. Resistance of foot valve and poppet valve are together equivalent to an extra 2 m of 6·2 cm diameter pipe. Calculate the maximum speed at which separation will take place if it occurs when the pressure falls below 1·2 m of water absolute. Calculate also the maximum frictional head at this speed. Take coefficient of friction in the pipes as 0·01.

[80 rpm.; 0·54 m]

29 A double-acting single-cylinder reciprocating pump has cylinder of 10 cm diameter and stroke of 15 cm. The ratio of length of crank to connecting rod is 1:5. The static suction lift is 2 m and the length of the suction pipe, which is 5 cm diameter is 5 m. Resistances of foot valve and poppet valve are together equivalent to an extra 2 m of 5 cm diameter pipe.

Calculate the maximnm speed at which separation will take place if it is to occur when the pressure falls below 1·5 m of absolute. Calculate also the maximum frictional head at this speed. Take cofficient of friction as 0 01 for pipes.

[17·9 rpm; 0·096 m of water]

Negative slip

30 What is negative slip ? Explain with the help of indicator diagram when negative slip occurs during suction stroke of (i) single-acting pump, and (ii) double acting pump.

31 A double-acting reciprocating pump without air vessel has plunger diameter of 15 cm and stroke of 20 cm and it moves with S.H M. The length of the suction pipes is 6 m. The pressure at the beginning of the suction stroke is 8 m of water (vacuum) The diameter of the suction as well as delivery pipe is 20 cm. The length of the delivery pipe is 12 m. The delivery head is 4 times the suction head. The negative slip is just to occur at the end of the suction stroke, inside the cylinder Determine, (a) H P. required to drive the pump, and (b) pressure head at *the middle of the delivery stroke.* ($g = 9{\cdot}81$ m/sec^2 and $f = 0 01$).

[(a) 0 755, (b) 7·246 m of water]

32 A single-acting reciprocating pump draws water from a sump and delivers to a height of 5 m above its own level. The length of the stroke of the pump is twice its diameter. The measured discharge of the pump is 215 L.P.M. The coefficient of discharge of the pump is 0·96. The length of the suction pipe is 3 m and its diameter is 7·5 cm. Vapour pressure of the water is 2·45 m of water absolute. The pump runs at such a speed that separation is just to occur at the beginning of the suction stroke and the negative slip is to oocur just before the end of the suction stroke. Determine, (a) the speed of the pump (b) cylinder diameter and the stroke length, and (c) pressure at the middle of the suction stroke. Take coeffi-cient friction for the pipe as 0 01.

[(a) 76 rpm.; (b) 12 cm, 24 cm; (c) 8·3

maximum speed of rotation, and (b) the load on the piston when the crank has moved 30° from the I.D.C. position. Take coefficient of friction as 0·01. Barometer reads 10·4 m of water. Assume S.H.M. and there is no air vessel on the pump.

[(a) 43 rpm.; (b) 870 kg]

27 A double-acting reciprocating pump has piston 15 cm in diameter and stroke of 20 cm. The diameter of the suction pipe as well as the delivery pipe is 10 cm. The length of the suction and delivery pipes are 5 m and 30 m respectively. Water is taken from a sump below the centre line of the pump and delivered to a height of 27 m above centre line of the pump. Accelerating head in the suction pipe is $\frac{3}{4}$th suction lift. Determine the speed of rotation and theoretical H.P. required to drive the pump. Assume that separtion takes place when the absolute pressure falls below 2·5 m of water. Coefficient of friction in the pipe is 0·01.

[50·4 rpm.; 2·53]

28 A double-acting single cylinder reciprocating pump has cylinder of 10 cm diameter and stroke of 15 cm. The ratio of length of crank to connecting rod is 1:5. The static suction lift is 2 m and the length of the suction pipe which is 6·2 cm diameter, is 4·3 m. Resistance of foot valve and poppet valve are together equivalent to an extra 2 m of 6·2 cm diameter pipe. Calculate the maximum speed at which separation will take place if it occurs when the pressure falls below 1·2 m of water absolute. Calculate also the maximum frictional head at this speed. Take coefficient of friction in the pipes as 0·01.

[80 rpm.; 0·54 m]

29 A double-acting single-cylinder reciprocating pump has cylinder of 10 cm diameter and stroke of 15 cm. The ratio of length of crank to connecting rod is 1:5. The static suction lift is 2 m and the length of the suction pipe, which is 5 cm diameter is 5 m. Resistances of foot valve and poppet valve are together equivalent to an extra 2 m of 5 cm diameter pipe.

Calculate the maximnm speed at which separation will take place if it is to occur when the pressure falls below 1·5 m of absolute. Calculate also the maximum frictional head at this speed. Take cofficient of friction as 0 01 for pipes.

[17·9 rpm; 0 096 m of water]

Negative slip

30 What is negative slip ? Explain with the help of indicator diagram when negative slip occurs during suction stroke of (i) single-acting pump, and (ii) double acting pump.

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[(a) 0 755; (b) 7·246 m of water]

32 A single-acting reciprocating pump draws water from a sump and delivers to a height of 5 m above its own level The length of the stroke of the pump is twice its diameter. The measured discharge of the pump is 215 L.P.M The coefficient of discharge of the pump is 0·96. The length of the suction pipe is 3 m and its diameter is 7·5 cm. Vapour pressure of the water is 2·45 m of water absolute. The pump runs at such a speed that separation is just to occur at the beginning of the suction stroke and the negative slip is to oocur just before the end of the suction stroke. Determine, (a) the speed of the pump (b) cylinder diameter and the stroke length, and (c) pressure at the middle of the suction stroke. Take coefficient friction for the pipe as 0·01.

[(a) 76 rpm.; (b) 12 cm, 24 cm; (c) 8·3 m water]

Air-vessels

33 Draw discharge-crank angle diagram for a single-acting single cylinder reciprocating pump. Derive an expression for the mean rate of discharge in terms of cylinder dimensions and speed of rotation.

34 What is an air-vessel ? Explain the advantage secured by fitting air-vessels on delivery and suction pipes. What should be the size of the air-vessel ?

35 Establish expression for rate of discharge from and into the air vessel fitted on the delivery pipe very close to cylinder of a (i) single-acting pump, and (ii) double-acting pump.

36 Draw the indicator diagram showing effect of fitting air-vessels on the reciprocating pump. What is the percentage saving in work done against friction for single-acting and double-acting pump due to the provision of large air vessels ?

37 A double-acting reciprocating pump has bore of 22 cm and stroke of 45 cm. The suction pipe has diameter of 15 cm. There is an air vessel fitted on it just near to the cylinder. The pump runs with S. H. M. at a speed of 90 rpm. Determine the position of the crank at which there is no flow into or from the air vessel when the crank makes angles of 30°, 90° and 120° with the inner dead centre.

[39·5° and 140·5°; + 12·1; – 32·3; – 20·4 litres/sec.]

38 Determine the minimum speed at which a double-acting reciprocating pump could be run under the following condition:

(a) No air vessels on suction side, and

(b) A very large air vessel on the suction side close to the pump. The suction lift = 3 m; length of the suction pipe = 5·5 m; diameter of the suction pipe = 7·5 cm;diameter of plunger of pump = 15 cm; stroke length = 30 cm; head lost in friction in the suction pipe is given by the expression $0{\cdot}025 \frac{l}{d} \cdot \frac{V^2}{2g}$. The separation takes place at the beginning of the suction stroke when pressure falls below 2·45 m of water absolute. Assume S.H.M. for the piston.

[(a) 36 rpm. (b) 355 rpm.]

39 A double-acting reciprocating pump running at 40 rpm. has stroke of 30 cm and diameter of 20 cm. The suction head and delivery head are 3 m and 30 m respectively. The suction and delivery pipes are each 8 cm in diameter, and their lengths are 5 m and 50 m respectively. Large air vessels are provided 2 m away on the suction side and 3 m away on the delivery side, both distances being measured along the pipe lines. Neglecting the entrance and exit losses for the pipe and taking $f = 0{\cdot}008$, calculate for the beginning of the strokes (a) pressure heads at the two ends of the cylinder, and (b) load on the piston.

Neglect the effect of the piston rod. Assume S.H.M. for the piston.

[3·28 ; 9·96 ; 50·97 m of water ; 975 kg]

40 Find the work saved by fitting air vessels to a double acting reciprocating pump having the following specifications :

Suction pipe diameter = 10 cm; delivery pipe diameter = 7·5 cm; length of the suction pipe = 5·5 m; length of the delivery pipe = 45 m; plunger diameter = 15 cm; length of the stroke = 30 cm.

The plunger makes 60 double strokes per minute with S.H.M. The air vessels are fitted at a distance of 1·5 m and 3 m from the cylinder on the suction and delivery side respectively. Take coefficient of friction as 0 01 for pipes.

[108 kg-m/sec]

41 A single-acting reciprocating pump is provided with large air vessels on the suction and delivery sides just near the pump. The pump is placed at a height of 4 m above the water level in the sump. The plunger has diameter of 30 cm and a stroke of 45 cm. The diameter and length of the suction pipe are 15 cm and 6 m respectively. The delivery pipe has length of 50 m and diameter of 15 cm. The pump runs with simple harmonic motion such that the separation takes place

at the beginning of the suction stroke. The effective height of the barometer is 8 m. Take $f = 0{\cdot}01$. Determine the H.P. saved in overcoming friction in the suction and delivery pipes as compared to a pump without air vessels. The speed of the pump is the same as the maximum speed of a pump without air vessels.

[0·473 H.P.]

42 The following data refers to a double-acting reciprocating pump :

Suction head = 3·65 m ; diameter of the suction pipe = 10 cm ; length of the suction pipe = 4·6 m ; diameter of the plunger = 20 cm; length of the stroke = 45 cm ; coefficient of friction = 0·008.

Separation occurs at an absolute pressure of 2·45 m of water. Barometer reads 10·4 m of water.

Determine the maximum speed at which the above pump must run (a) if no air vessel is fitted on the suction side, and (b) if a large air vessel is fitted on the suction side very near to the pump. Assume S.H.M. for the punger.

[(a) 30 rpm ; (b) 128 rpm.]

Multi-stage pumps

43 Draw discharge-crank angle diagrams for, (a) single-acting single-cylinder, (b) single-cylinder, double-acting, (c) Duplex single-acting with phase difference of 180°, and (d) triplex single-acting with phase difference of 120° reciprocating pumps. Thus show that multi-cylinder pumps have continuous discharge with improved uniformity. Prove the superiority of an odd number of cylinders over even number of cylinders.

44 A three-throw pump has cylinders 30 cm diameter and 60 cm stroke. The pump is required to lift 5,500 L.P.M. of water to a height of 91·5 m. Frictional losses are estimated at 4·5 m in suction pipe and 14 m in delivery pipe. Velocity of water in pipes is 1·2 m/sec. If the pump efficiency is 98% and slip 2·5%, find the speed at which the pump should run and the H.P. required to drive it.

[42 rpm.; 134·5]

45 Pistons of a three-throw, single-acting pump have simple harmonic motion and are driven from cranks spaced 120° apart and running at 75 rpm. The cylinder diameter and stroke are 6 cm and 15 cm respectively. The cylinders discharge to a common main 10 cm diameter and 75 m long which rises 9 m above the pump.

Plot graphs of the velocity and acceleration of water in the pipe on a base of crank angle for one complete turn. Calculate the head at the pump discharge flanges, when one plunger is at the beginning of its discharge stroke. No air vessel is fitted. Take coefficient of friction for mains as 0·01.

[15·43 m of water]

Performance characteristics

46 Draw the theoretical and actual discharge-head at constant speed and discharge-speed at constant head characteristic curves of a reciprocating pump. Explain the causes of their being not coincident.

47 Draw the power-head curve of a reciprocating pump running at constant speed Indicate thereon variation of various losses and their relative magnitudes Deduce therefrom the overall efficiency-head curve

48 Indicate the field of application of reciprocating pumps.

Other types of positive displacement pumps

49 Explain with neat sketches, the construction and working of the following pumps :

(a) lift pump, (b) vane pump, (c) gear-wheel pump,

(d) bore-hole pump, and (e) Archimedean screw.

Give in each case their field of application.

13 HYDRAULIC MACHINERY

Introduction

We know of machines which can be run by electric power or by mechanical power. The power in such case is transmitted from one point to another point through a mechanism in mechanical power transmission and through conductors in case of electric power transmission. Power can also be transmitted from one place to another by means of liquids. This arrangement is, in general, referred to as *hydraulic power transmission*. There are certain machines which can directly be run by hydraulic energy. Such machines are known as hydraulically operated machines. This type (hydraulic) of power is used in machines which are required to move slowly and great effort is required for their operation. In these machines water is admitted, under pressure, to the cylinder which exerts a very high pressure on the ram or the piston in doing mechanical work. This type of working is employed in the following type of machines :

(a) Hydraulic lift,
(b) Hydraulic crane,
(c) Hydraulic shaper,
(d) Hydraulic riveter,
(e) Hydraulic jack,
(f) Hydraulic press, and
(g) Hydraulic testing machine.

These machines are supplied hydraulic power from power station in which are installed electric motor driven or engine driven

centrifugal pumps which supply the liquid or water continuously. The power station also requires an accumulator to store the power during the idle period when the machines do not work. These machines take their liquid supply from their delivery mains which are fed from a bigger main carrying the liquid under pressure from the power station to the industrial premises as shown in fig. 13-1.

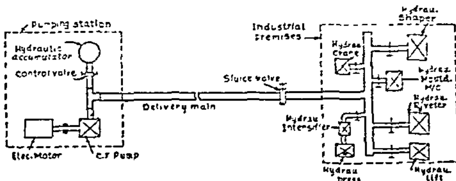


Fig. 13-1 Hydraulic power transmission

The industrial premise house has various machines depending upon the type of premise and work to be carried out in it. In a typical case, an industrial premise consists of a hydraulic crane, a hydraulic press, a hydraulic lift, a hydraulic jack, a hydraulic shaper etc. Sometimes we have to supply one machine with liquid whose pressure is higher than the pressure of liquid supplied to other machines. This can be achieved by using an intensifier, which raises the pressure of the liquid. This is shown in the case of the hydraulic press of fig. 13-1. It may be noted that in a majority of cases water is the liquid that is generally used; however, for specific applications, mineral oils are also employed as in servo-motor control mechanism, hydraulic jack, etc.

Hydraulic Accumulator

Analogus to the electric accumulator or storage battery, the hydraulic accumulator is an apparatus for temporarily storing the generated energy of water. It works as an accessory to other

machines such as lifts or cranes which are required to put out a great amount of work during a small interval of time which is followed by an idle period. Like the flywheel of the reciprocating engine, the accumulator collects energy from the pump which works continuously and at lower rate of discharge. During the idle period of the driven machine (crane or lift) the accumulator goes on storing the liquid at constant pressure and during the working period it supplies the accumulated water along with the direct discharge from the pump. Thus, the capacity of the pump, and hence the cost is greatly reduced. Moreover, the power consumption is also lower, thus effecting saving in the running cost. The typical arrangement is shown in fig. 13-2.

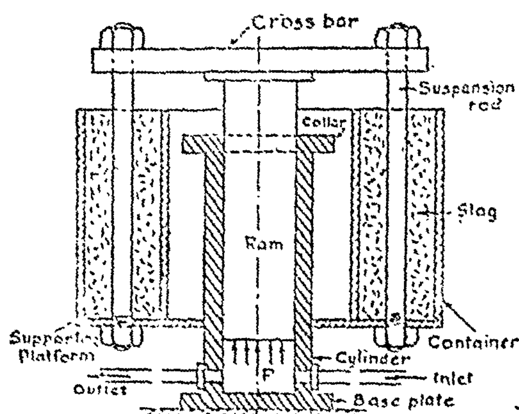


Fig. 13-2 Simple accumulator

Construction : It consists of a cylinder in which a ram is sliding. The ram carries at its top a cross bar from which a supporting platform is suspended by means of screwed suspension rods. The container is placed over the supporting platform and it is filled with heavy materials like slag, lead, etc.

To the bottom of the cylinder is connected a pipe line carrying water from the main to the cylinder, when not required by the machine it is operating. The cylinder has another pipe connection

to discharge water from the cylinder during the fall of the ram. The accumulator is fixed on the ground by means of foundation bolts or it is simply placed on a consolidated base.

Working: Pressure, p of water from the main pipe when admitted inside the cylinder, acts on the lower end of the ram and forces it out. This will continue till the supporting platform touches the collar of the cylinder. The accumulator has then stored the maximum amount of energy which it can hold. The upward force must be such that it must overcome the dead weight on the ram and its packing friction. During the downward stroke, water is forced out through the pipe and this will continue till the cross bar touches the collar of the cylinder.

Theory : Let, p – pressure of water,

a – area of cross-section,

W – dead weight on the ram, and

h – stroke or height of fall of the ram,

Then, volume of the accumulator $= ah$

and, weight on the ram, $W = pa$

Now, work done in lifting $= pah$

This equals the maximum energy stored, which is termed as the *capacity* of the accumulator.

$$\text{Thus capacity of accumulator} \left.\begin{array}{l} = pah \\ = p \times \text{cylinder volume} \end{array}\right\} \quad \ldots(13.1)$$

As the pressure of water becomes higher, the dead weight on the ram of simple accumulator becomes excessively big and the size and cost become prohibitive and hence, as a remedy to this a new version in the form of the differential accumulator is developed.

Differential accumulator: It consists of a fixed ram whose lower portion is enlarged by fixing a brass bush. The fixed ram has a collar at the top which works as an upper stop. The ram is fixed on a base plate which carries also the lower stops. The fixed ram is surrounded with a sliding cylinder and it is loaded with cast iron ring weights shown in fig. 13-3. The ring weights are resting on a collar of the cylinder at its lower end. Passages for water to enter and leave the cylinder are provided in the ram, and connected to inlet and outlet pipes.

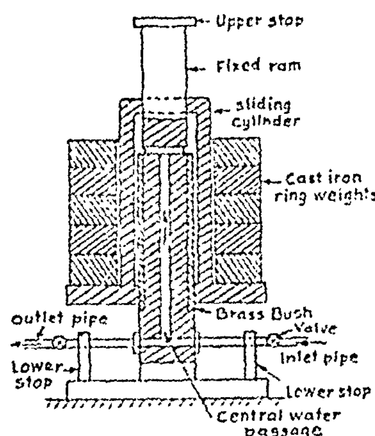


Fig. 13-3 Differential accumulator

Working: Water enters the cylinder through the passage by opening the valve on the inlet pipe line. This water creates force on the cylinder in the upward direction and hence the cylinder ascends with its ring weights till it reaches the end of its stroke, when the top of the sliding cylinder touches the collar. Care should be taken to close the inlet valve before the cylinder reaches its end of the stroke; otherwise collor of upper stop will be knocked off and the whole cylinder will come down resulting in the brakedown of the accumulator.

When water is drawn from the accumulator through the outlet pipe, the cylinder slides downward till the bottom of the cylinder touches the lower stop. The weight on the cylinder must be sufficient to overcome the pressure of water.

Theory: Let D – external diameter of the brass bush, cm
d – diameter of the fixed ram, cm
p – pressure of water kg/cm^2
h – lift of the accumulator, m
t – time taken in seconds to lift the weight, and
W – total load on the cylinder, kg.

Then, total load on cylinder, $W = p \times \frac{\pi}{4} (D^2 - d^2)$ kg

and capacity of the accumulator $= Wh = p \times \frac{\pi}{4} (D^2 - d^2) \times h$ kg-m

Further, work done/sec $= \dfrac{p \times \frac{\pi}{4} (D^2 - d^2)\, h}{t} = \dfrac{Wh}{t}$

∴ H. P. supplied by the accumulator is

$$\frac{Wh}{t \times 75} = \frac{p \times \frac{\pi}{4} (D^2 - d^2)\, h}{t \times 75} \qquad \text{.. (13·2)}$$

Uses : Accumulators are the devices which temporarily store the energy of water, and work in association with lifts, cranes, etc. The advantage of the differential accumulator over the simple accumulator is that it stores energy with comparatively less dead weight.

Problem 1 : *An accumulator has a ram of 250 mm diameter and a lift of 3 m. Water is supplied at a pressure of 500 m of water. Find the necessary load on the ram and capacity in horse power-hour.*

Pressure on the ram, $p = \dfrac{1{,}000 \times 500}{10{,}000} = 50$ kg/cm²

Area of the ram, $a = \frac{\pi}{4} D^2 = \frac{\pi}{4}\, 25^2 = 490$ cm²

Load on ram, $W = a \times p = 490 \times 50 = 24{,}500$ kg

Capacity of accumlator $= p \times a \times h = W \times h$

$= 24{,}500 \times 3 = 73{,}500$ kg-m

∴ Capacity in H. P. - hour $= \dfrac{73{,}500}{4{,}500 \times 60} = 0{\cdot}273$

Problem-2 : *The ram of a hydraulic accumulator is loaded with a weight of W tonnes. The packing friction of the ram is equivalent to 5% of the load on the ram. The diameter of the ram is 200 mm. When the ram is falling steadily, the pressure inside the accumulator is 75 kg/cm². Determine the pressure of water required when the*

ram is ascending with uniform velocity, and what will then be the load on the ram?

Load required to overcome packing friction $= 0{\cdot}05W$ tonnes.

When the ram is descending, the frictional force is acting vertically upwards and hence the net load on water is the difference of the dead weight, W on the ram and the frictional force. Hence,

$$\text{pressure of water} = \frac{\text{net load}}{\text{area}} = \frac{(1-0{\cdot}05)\,W \times 1{,}000}{\frac{\pi}{4}\,20^2}$$

$$= 3{\cdot}02W \text{ kg/cm}^2$$

$$\text{or} \quad 75 = 3{\cdot}02W$$

$$\therefore \quad W = 24{\cdot}8 \text{ tonnes}$$

Further, when the ram is ascending, the total load is

$$W' = \text{load on the ram} + \text{load due to packing friction}$$
$$= W + 0{\cdot}05W = 1{\cdot}05W \text{ tonnes}$$

$$\text{Hence pressure of water} = \frac{1{\cdot}05 \times 24{\cdot}8 \times 1{,}000}{\frac{\pi}{4}20^2}$$

$$= 82{\cdot}8 \text{ kg/cm}^2.$$

The gross load on water $= 1{\cdot}05 \times 24{\cdot}8 =$ 25 tonnes

Problem-3 : *An accumulator has ram of diameter 230 mm and it has lift of 4 m. The packing friction is equivalent to 3% of the load on the ram. The ram falls steadily through its full height in 2 minutes. The accumulator is loaded to the extent of 60 tonnes. If the pump is delivering 30 litres/sec. through the accumulator when the ram is falling, determine the total H. P. delivered during the working period.*

$$\text{Effective weight on the ram, } W' = W_{\text{dead}} \times \frac{100-3}{100}$$

$$= 60 \times 0{\cdot}97 = 58{\cdot}2 \text{ tonnes}$$

$$\text{Pressure of water, } p = \frac{\text{effective load}}{\text{area of ram}}$$

$$= \frac{58{\cdot}2 \times 1{,}000}{\frac{\pi}{4}\left(\frac{230}{10}\right)^2} = 140 \text{ kg/cm}^2$$

Hence, pressure head, $H = \frac{p}{w} = \frac{140 \times 100^2}{1{,}000} = 1{,}400$ m of water

Since the pump supplies 30 liters/sec. under the head of 1,400 m, and one litre of water weighs one kilogramme, so the work supplied by the pump $= WH = wQH = 1 \times 30 \times 1{,}400$

$$= 42{,}000 \text{ kg.m/sec.}$$

Again, work supplied by the accumulator is

effective load × distance moved/sec.

$$= 58{\cdot}2 \times 1{,}000 \times \frac{4}{2 \times 60} = 1{,}940 \text{ kg.m/sec.}$$

Hence, total work supplied is $42{,}000 + 1{,}940 = 43{,}940$ kg.m/sec.

$\therefore$ total H.P. delivered $= \frac{43{,}940}{75} = \underline{585}$

Problem – 4 : *A differential hydraulic accumulator is supplied with water under a pressure head of 500 m for water. The diameters of the smaller and larger part of the ram are 100 mm and 125 mm respectively. If the packing friction amounts to 4% of the load on the moving cylinder, determine the total load on the moving cylinder.*

Let W kg be the total weight of the moving cylinder.

As the cylinder is moving up and packing friction is 4%, the total load on the cylinder = W + packing friction load

$$= W + 0{\cdot}04W = 1{\cdot}04W$$

Hence, pressure of water $= \frac{\text{load}}{\text{area}} = \frac{1{\cdot}04W}{\frac{\pi}{4}(12{\cdot}5^2 - 10^2)}$

$$= 0{\cdot}026W \text{ kg/cm}^2$$

On the other hand pressure of water equivalent to the head is

$$\frac{wH}{10{,}000} = \frac{1{,}000 \times 500}{1{,}0000} = 50 \text{ kg/cm}^2$$

$$\therefore \quad 50 = 0{\cdot}026W$$

$$\therefore \quad W = 1{,}925 \text{ kg}$$

or weight on the moving cylinder $= \frac{1{,}925}{1{,}000} = \underline{1{\cdot}925 \text{ tonnes}}$

Problem – 5: *An accumulator supplies water at a pressure of 80 kg/cm² to a 10 cm main. A hydraulic lift, situated at a distance of 900 m from the accumulator, is supplied with water under pressure from this main. The ram of the lift is 20 cm in diameter and the load on it, inclusive of its own weight, is 12 tonnes. The friction of the ram cage etc. is equivalent to an addition of 5% of the gross load. Determine the speed with which the lift will ascend if the value of the coefficient of friction in the supply main is 0·008. Neglect other losses.*

Head lost due to friction in the pipe is

$$= \frac{4flV^2}{2gd} = \frac{4 \times 0{\cdot}008 \times 900 \times V^2}{2 \times 9{\cdot}81 \times 0{\cdot}1}$$

$$= 14{\cdot}7V^2 \text{ m of water} \quad \text{.. (i)}$$

Gross load on the ram, taking friction into account is

$$W' = (1 + 0{\cdot}05) \times 12 \times 1{,}000 = 12{,}600 \text{ kg}$$

$$\text{Pressure of water on the ram} = \frac{12{,}600}{\frac{\pi}{4}(20)^2} = 40 \text{ kg/cm}^2$$

Pressure lost due to friction = 80 – 40 = 40 kg/cm²

Equivalent head lost in friction is

$$\frac{10{,}000p}{w} = \frac{10{,}000 \times 40}{1{,}000}$$

$$= 400 \text{ m of water} \quad \text{.. (ii)}$$

From equations (i) and (ii) we have

$$14{\cdot}7V^2 = 400$$

$$\therefore \quad V^2 = 27{\cdot}2$$

$$\therefore \quad V = 5{\cdot}2 \text{ m/sec}$$

Let V' be the velocity of water in the cylinder of the lift.

Since, the rate of volume of water flowing inside the cylinder of the lift is the same as rate of volume of water flowing in the main,

$$5{\cdot}2 \times \frac{\pi}{4}(10)^2 = V' \times \frac{\pi}{4}(20)^2$$

$$\therefore \quad V' = \frac{5{\cdot}2 \times 100}{400} = 1{\cdot}3 \text{ m/sec}$$

i.e. the lift ascends with the speed of 1·3 m/sec.

Problem - 6 : *35 H.P. is to be transmitted from an accumulator to a hydraulic crane situated 2·5 km away and connected by a 150 mm diameter pipe line. If the losses in the pipe line are equivalent to 2%, find the diameter of the ram which is loaded with 130 tonnes. Take coefficient of friction for the pipe line as 0·01.*

The frictional head in the pipe line is

$$h_f = \frac{4flV^2}{2gd}$$

$$= \frac{4 \times 0{\cdot}01 \times 2{,}500V^2}{2 \times 9{\cdot}81 \times \frac{150}{1{,}000}} = 34V^2$$

H.P. lost in friction = 0·02 × 35 = 0·7

Now, discharge of water through the pipe line is

$$W = \frac{\pi}{4}\left(\frac{150}{1{,}000}\right)^2 \times V \times 1{,}000 = 17{\cdot}7V \text{ kg/sec.}$$

Since, friction H.P. is

$$\frac{Wh_f}{75} = \frac{17{\cdot}7V \times 34V^2}{75} = 8V^3$$

$\therefore \quad 0{\cdot}7 = 8V^3$

$\therefore \quad V^3 = 0{\cdot}0875$

$\therefore \quad V = 0{\cdot}442$ m/sec.

$$\text{Now, pressure of water} = \frac{\text{load}}{\text{area of the ram}}$$

$$= \frac{130 \times 1{,}000}{\frac{\pi}{4}D^2}$$

$$= \frac{1{,}65{,}900}{D^2} \text{ kg/cm}^2$$

Hence, pressure head is,

$$H = \frac{p}{w} = \frac{1{,}65{,}900}{D^2} \times \frac{10{,}000}{1{,}000} = \frac{16{,}59{,}000}{D^2} \text{ m of water}$$

$W = 17{\cdot}7V = 17{\cdot}7 \times 0{\cdot}442 = 7{\cdot}82$ kg/sec.

$$\text{i.e. } 35 - 0{\cdot}3 = \frac{7{\cdot}82 \times \frac{16{,}59{,}000}{D^2}}{75}$$

$\therefore \quad D^2 = 5{,}050$

$\therefore \quad D = 71$ cm

Intensifier

Hydraulic intensifier is a device for stepping up the pressure of water for use in machines requiring very high pressure during their working period. The working of such machines is intermittent. The intensifier is analogus to an electric transformer which steps up the voltage, the difference being that whereas the transformer supplies current at stepped up (higher) voltage continuously, the intensifier supplies the liquid at higher pressure intermittently.

A question may arise as to why a multi-stage pump developing the required high pressure is not employed in place of an intensifier. The reason lies in the fact that machines working with high pressure liquid generally require intermittent supply of liquid for their operation and hence installation of pumps which supply the liquid continuously is not justified.

Construction : It consists of one fixed ram and, two cylinders, one fixed and the other movable. The smaller cylinder slides over the fixed ram and at the same time this sliding occurs inside the fixed bigger cylinder.

There is an opening at the top of the fixed cylinder and it is connected to a low pressure water connection through a valve. It is also connected to the waste water system through another valve. The functions of these two valves, can be combined in a single piston valve as used in case of servo-motor of the turbine governing.

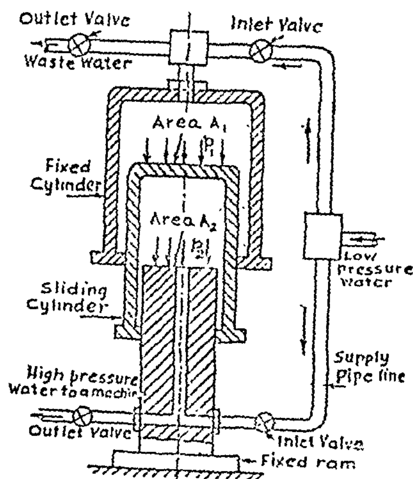


Fig. 13-4 Hydraulic intensifier

There is a passage provided for water through the fixed ram. This passage is connected to the low pressure water supply main through one valve and to the high pressure delivery main through another valve. These two

valves may also be replaced by a single piston valve. The sliding cylinder has a collar which works as stoppage device when the cylinder is at the end of its upward stroke. The cylinder is hollow from inside.

Working: Let the sliding cylinder be at the bottom of its stroke. The discharge valve of the fixed cylinder and the inlet valve of the fixed ram are opened, and the low pressure water inlet valve of the fixed cylinder and the high pressure water outlet valve of the fixed ram are kept closed. This enables the low pressure supply water to enter into the sliding cylinder forcing it to rise up till the collar of the sliding cylinder comes in contact with the collar of the fixed ram and thus the sliding cylinder reaches the end of its upward stroke.

Now the discharge valve of the fixed cylinder is closed and the low pressure inlet valve of the fixed cylinder is opened. The low pressure inlet valve of the fixed ram is closed and the high pressure outlet valve of the fixed ram is opened. This enables the low pressure water to enter inside the fixed cylinder from the top, which in turn forces the sliding cylinder downward The downward movement forces the high pressure water from the sliding cylinder, through the passage of the fixed ram and the outlet valve to the hydraulic machines This is continued so long as the sliding ram does not reach the end of its stroke.

The above process is repeated in turn and the high pressure water is supplied intermittently It supplies high pressure water during the downward stroke only at the expense of a large quantity of low pressure water. This intensifier is known as a single acting one. If continuous supply is required, two intensifier should work in parallel and out of phase.

Theory: Let A_1 – area of cross-section of the external end of the sliding cylinder, (sliding ram in fig. 13-4), cm^2

A_2 – area of cross-section (with outside diameter) of the fixed ram, cm^2

p_1 – pressure of water from supply main, kg/cm^2

and p_2 – pressure of water delivered by the intensifier, kg/cm^2

Considering the equilibrium of the sliding cylinder, for any position,

$$p_1 \times A_1 = p_2 \times A_2$$

$$\therefore \quad p_2 = p_1 \times \frac{A_1}{A_2} \quad .. \quad .. \quad .. \quad ..(13 \cdot 3)$$

The above machine can also be used for reducing the pressure of water by interchanging the pipe connection for the waste water and for the supply water, *i.e.* waste water pipe connection should be done on the fixed ram and the pipe supplying water to a machine should be connected on to the fixed cylinder.

Application : This is used where high pressure hydraulic machines are to be worked with the supply of low pressure water. It is used to operate hydraulic press, crane, lift, etc.

Problem-7 *Water is supplied to a hydraulic intensifier at pressure of 2 kg/cm². The diameters of the sliding cylinder and the fixed ram of the intensifier are 150 mm and 75 mm respectively. This intensifier supplies water to a hydraulic crane. The diameter of the ram of the crane cylinder is 300 mm. If the efficiency of the crane is 80% determine the load lifted. The load on the crane moves 2 m when the ram of the crane moves 300 mm.*

For equlibrium of the sliding cylinder, we have

$$p_1 \times A_1 = p_2 \times A_2$$

$$\therefore \quad p_2 = p_1 \times \frac{A_1}{A_2} = 2 \times \frac{\frac{\pi}{4} D^2}{\frac{\pi}{4} d^2} = 2\left(\frac{15}{7 \cdot 5}\right)^2 = 8 \text{ kg/cm}^2$$

Load on the crane ram (effort) $= 8 \times \frac{\pi}{4}(30)^2 = 5{,}650$ kg

Now, efficiency, $\eta = \dfrac{\text{load}}{\text{effort} \times \text{velocity ratio}}$

$\therefore$ load to be lifted by the crane $= 5{,}650 \times 0 \cdot 8 \times \dfrac{300}{2{,}000}$

$= \underline{678 \text{ kg}}$

Problem-8 : *A hydraulic intensifier is supplying water to a hydraulic press at pressure of 200 kg/cm². The stroke of the intensifier is 1 m and its capacity 15 litres. If water is supplied at pressure of 40 kg/cm² from a low pressure main, determine diameter of the sliding cylinder and ram of the intensifier.*

Using eqn. (13·3),

$$p_1 A_1 = p_2 A_2$$

$$i.e.\ p_1 \times \frac{\pi}{4} D^2 = p_2 \times \frac{\pi}{4} d^2$$

$$\therefore \quad \frac{D}{d} = \sqrt{\frac{p_2}{p_1}} = \sqrt{\frac{200}{40}}$$

$$\therefore \quad D = (\sqrt{5})\, d$$

Capacity of the intensifier is

$$Q = A_2 \times L, \text{ where } L = \text{stroke length.}$$

$$= \frac{\pi}{4}\left(\frac{d}{100}\right)^2 \times 1 \text{ m}^3 \text{ (where } d \text{ is in cm)}$$

$$\text{Also,} \quad Q = \frac{15 \times 1}{1{,}000} \text{ m}^3$$

$$\therefore \frac{\pi}{4}\left(\frac{d}{100}\right)^2 = \frac{15}{1{,}000}$$

$$\therefore \quad d^2 = \frac{15 \times 4 \times 10{,}000}{1{,}000 \times \pi}$$

$$\therefore \quad d = 13{\cdot}8 \text{ cm}$$

$$\therefore \quad D = \sqrt{5} \times 13{\cdot}8 = 30{\cdot}9 \text{ cm}$$

Problem-9 : *A hydraulic intensifier is supplying water to a direct acting hydraulic lift, having a ram diameter of 15 cm. The diameters of the sliding cylinder and the fixed ram of the intensifier are 15 cm and 10 cm respectively. The intensifier is suppled with water from an overhead tank whose level is 30 m above the level of the intensifier, through a pipe line 5 cm in diameter and 700 m long. If efficiency of the lift is 80%, determine the load lifted by it. Assume packing friction equal to 3% of the load on sliding cylinder. The efficiency of transmission of the pipe line is 70%. Take coefficient of friction as 0 01.*

As, the efficiency of transmission $= \dfrac{H - H_f}{H}$

$$i.e.\quad 0{\cdot}7 = \frac{30 - H_f}{30}$$

$$\therefore \quad H_f = 9 \text{ m of water}$$

Pressure head near the intensifier $= H - H_f$

$= 30 - 9 = 21$ m of water

$$\text{Pressure of water supplied} = \frac{wH}{10{,}000} = \frac{1{,}000 \times 21}{10{,}000}$$

$$= 2{\cdot}1 \text{ kg/cm}^2$$

Effective load on the cylinder $= \dfrac{\pi}{4}\,(15)^2 \times 2{\cdot}1\,(1 - 0{\cdot}03)$

Equating the load on the cylinder and the ram

$$\frac{\pi}{4}\,(15)^2 \times 2{\cdot}1\,(1 - 0{\cdot}03) = \frac{\pi}{4}\,(10)^2 \times p_2$$

$\therefore \quad p_2 = 4{\cdot}58$ kg/cm²

$\therefore$ Pressure of water supplied to the cylinder of the lift

$= 4{\cdot}58$ kg/cm²

$\therefore$ load on the ram of the lift $= \dfrac{\pi}{4}\,(15)^2 \times 4{\cdot}58 = 810$ kg

$\therefore$ load lifted by the lift $= 810 \times 0{\cdot}8$

$= 648$ kg

Hydraulic Press

Hydraulic presses are employed for preparing cotton bales, cloth bales, for plastic moulding, for metal pressing work (stamping), for punching and shearing steel plates, etc. The principle of working is that of the simple Brahma's press. It utilizes the principle inunciated by Pascal *viz.* liquid exerts equal pressure in all directions. A very high mechanical advantage is achieved by providing the very small ratio of plunger and ram. A still higher mechanical advantage can be obtained by use

of leverage to operate the plunger. Hydraulic presses are constructed in many divergent forms depending upon the purpose for which they are employed. The range of the capacity varies from few kilograms to 5,000 tonnes.

Construction : The press consists of two fixed cylinders the smaller cylinder with plunger in it is fixed on the top of a water tank. The plunger is operated by means of a lever pivoted on this cylinder. The leverage being l_1/l_2. The cylinders are connected by a pipe line at their bottom with a non-return valve in the pipe line as shown in the fig. 13-5. A return pipe is connected at one

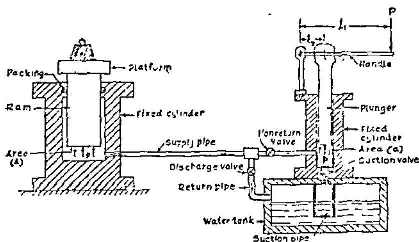


Fig 13-5 Hydraulic press

end of the above supply pipe line with a discharge valve discharging return water in the tank. The small cylinder is provided with a suction valve and a small suction pipe at its bottom.

The large fixed cylinder has a ram sliding in it; and a platform is fitted on the top of the ram, on which the load is placed. Packing is provided to prevent leakage of the high pressure water from the larger cylinder.

Working : Let the plunger be moved up The upward movement of the plunger creates vacuum inside the cylinder and hence water is sucked in it through the suction pipe and suction valve.

When the plunger is moved downward by applying force P at the end of the lever, the suction valve will be closed and water will be forced out from the cylinder through the pipe line to the large cylinder. This water will create an upward force on the sliding ram, which helps in lifting the load. The upward movement of the ram is continued till the job is completed. After this the discharge valve is opened and water is made to discharge through the return pipe to the water tank. Thus the ram is lowered.

Theory : Let W – load on the ram, kg

p – pressure of water, kg/cm²

a – area of cross-section of the plunger, cm²

A – area of cross-section of the sliding ram, cm²

c – percentage packing friction for each of the ram and plunger,

P – force applied at the end of lever, kg,

F – force on the plunger when the load is raised, kg

l – leverage of the lever $= \left(\frac{l_1}{l_2}\right)$,

L_1 – length of the stroke of the plunger, m,

n – number of strokes required to lift the load,

L_1 – lift or travel of the sliding ram, m

Taking the moments, about the fulcrum of the lever,

$$P \times l_1 = F \times l_2$$

$$\therefore \quad F = P \times \frac{l_1}{l_2} = Pl$$

The effective force transmitted to create pressure on water is reduced by the amount lost in overcoming packing friction.

$$i.e. \quad p \times a = F\left(1 - \frac{c}{100}\right) = Pl\left(1 - \frac{c}{100}\right)$$

$$\therefore \; p = \frac{Pl}{a}\left(1 - \frac{c}{100}\right) \quad \text{.. (13·4)}$$

As liquid transmits pressure equally in all directions, the pressure of water below the sliding ram is also p.

Hence, force on the ram $= p \times A$

The effective force responsible for lifting the load

$$= p \times A\left(1 - \frac{c}{100}\right)$$

$$\therefore \quad W = p \times A\left(1 - \frac{c}{100}\right) \qquad \text{.. (13·5)}$$

Substituting the value of p from eqn. (13·4) in eqn. (13·5) we have,

$$W = \frac{Pl}{a}\left(1 - \frac{c}{100}\right) \times A\left(1 - \frac{c}{100}\right)^2$$

$$= \frac{PlA}{a}\left(1 - \frac{c}{100}\right)^2$$

$$\therefore \quad F = Pl = \frac{Wa}{A\left(1 - \frac{c}{100}\right)^2} \qquad \text{.. (13·6)}$$

If there is no loss due to packing friction,

$$W = \frac{PlA}{a}$$

Now, volume of water displaced by plunger per stroke $= a \times L_1$

and volume of water required to raise the load $= A \times L_2$

$\therefore$ number of strokes of the plunger required to lift the load $\Big\}\ n = \dfrac{AL_2}{al_1}$ ..(13·7)

If t is the time in second, taken to raise the load

number of strokes per second $= \dfrac{n}{t}$

Further, work supplied to the plunger per sec. is

$$\text{W.D.} = F \times L_1 \times \frac{n}{t}$$

$$= \frac{WaL_1 n}{A\left(1 - \frac{c}{100}\right)^2 \times t} \qquad \text{.. (from eqn. 13·6)}$$

$\therefore$ H.P. required to drive the plunger

$$= \frac{WaL_1n}{A\left(1-\frac{c}{100}\right)^2 \times t \times 75} \quad ..\ (13{\cdot}8)$$

Problem-10 : *In a hydraulic press, the diameter of the ram and plunger are 300 mm and 40 mm respectively. The leverage of the handle is 10 to 1. The packing friction of the ram as well as the plunger is 3% of load. The lift of the load is 1 m and the stroke of the plunger is 200 mm. A load of 100 tonnes is to be raised in 2 minutes. Determine the force applied at the end of the lever, H.P. required to drive the plunger, and the number of strokes to be performed per second.*

Using eqn. (13·6), force on the plunger,

$$F = \frac{W \times a}{A\left(1-\frac{c}{100}\right)^2} = \frac{15 \times 1{,}000 \times \frac{\pi}{4}\left(\frac{120}{1000}\right)^2}{\frac{\pi}{4}\left(\frac{300}{1{,}000}\right)^2 \times \left(1-\frac{3}{100}\right)^2} = 248 \text{ kg}$$

But, effort to be applied at the end of lever,

$$P = \frac{F}{l} = \frac{248}{10} = 28{\cdot}4 \text{ kg}$$

Now, number of strokes $= \dfrac{AL_2}{aL_1} = \dfrac{\frac{\pi}{4}(30)^2}{\frac{\pi}{4}(40)^2} \times \dfrac{1{,}000}{200} = \underline{281 \text{ strokes}}$

Using eqn. (13·8), H.P. required to drive the plunger

$$= \frac{\frac{200}{1{,}000} \times 15 \times 1{,}000 \times \frac{\pi}{4}\left(\frac{40}{1{,}000}\right)^2 \times 281}{120 \times \frac{\pi}{4}\left(\frac{300}{1{,}000}\right)^2 \times (0{\cdot}97)^2 \times 75}$$

$= \underline{1{\cdot}77 \text{ H.P.}}$

Problem-11 : *A load of 20 tonnes is to be lifted by means of a hydraulic press. The maximum effort applied at the end of a lever is 25 kg. If the diameter of ram is 150 mm, determine the diameters*

of plunger. The leverage of the lever is 30:1. Determine also the number of strokes to be performed in order to lift the load through 500 mm, if the stroke of the plunger is 250 mm. Disregard the effect of friction.

Force on the plunger, F = effort × leverage = 25 × 30 = 750 kg

Force on the plunger is given by the eqn. (13 6) as

$$F = \frac{Wa}{A\left(1 - \frac{c}{100}\right)^2}$$

or $F = \frac{Wa}{A}$ as $c = 0$

i.e. $$750 = \frac{20 \times 1{,}000 \times \frac{\pi}{4}\left(\frac{d}{1{,}000}\right)^2}{\frac{\pi}{4}\left(\frac{D}{1{,}000}\right)^2}$$ (where plunger diameter d, and ram diameter D are in mm)

or $$750 = 20{,}000\left(\frac{d}{D}\right)^2$$

∴ $$\frac{D}{d} = \sqrt{\frac{20{,}000}{750}} = 5{\cdot}15$$

∴ $$d = \frac{D}{5{\cdot}15} = \frac{150}{5{\cdot}15} = \underline{29{\cdot}2 \text{ mm}}$$

Using eqn. (13·7) number of strokes is given by,

$$n = \frac{AL_2}{aL_1} = \left(\frac{D}{d}\right)^2 \times \frac{L_2}{L_1}$$

∴ $$n = (5{\cdot}15)^2 \times \frac{0{\cdot}50}{0{\cdot}25} \approx \underline{53{\cdot}5 \text{ strokes}}$$

Problem-12: *The ram of a hydraulic press is 15 cm in diameter and is worked from an intensifier, of the piston and ram type, which receives its low pressure supply of water from a tank whose surface level is 30 m above the level of the intensifier piston, through a pipe 8 cm diameter and 200 m long. The friction coefficient for the low pressure supply pipe is 0·005. The press ram is exerting a force of 40 tonnes. The intensifier ram is 10 cm in diameter while the piston*

$\therefore \quad V^2 = 1{\cdot}57$

$\therefore \quad V = 1{\cdot}25$ m/sec

Let, v' = velocity of the piston of the intensifier,

v = velocity of the ram of the press.

The quantity of water flowing through the pipe line enters the press and is the same as that discharged by the intensifier.

$$\text{Then, } v' = \frac{\pi}{4}\left(\tfrac{8}{100}\right)^2 = v' \times \frac{\pi}{4}\left(\tfrac{100}{100}\right)^2$$

$$\therefore \quad v' = V \times \left(\tfrac{8}{100}\right)^2 \qquad ..(\text{i})$$

We also have,

$$V' \times \frac{\pi}{4}\left(\tfrac{10}{100}\right)^2 = v \times \frac{\pi}{4}\left(\tfrac{15}{100}\right)^2$$

$$\therefore \quad v' = v \times \left(\tfrac{15}{10}\right)^2 \qquad ..(\text{ii})$$

$$\therefore \quad V \times \left(\tfrac{8}{100}\right)^2 = v \times \left(\tfrac{15}{10}\right)^2$$

$$v = V\left(\tfrac{8}{100}\right)^2 \times \left(\tfrac{10}{15}\right)^2$$

$$= 1{\cdot}25 \times \frac{64}{10{,}000} \times \frac{100}{225} = 0{\cdot}00356 \text{ m/sec}$$

or $0\ 00356 \times 100 \times 60 = 21{\cdot}36$ cm/min.

Hydraulic Ram (Hydram)

Though it is very difficult and inconvenient to think of a pump which can lift the fluid without consuming any mechanical or electrical energy, it has to be conceded that there exists one such pump which is automatic in its action and works by utilizing a large quantity of water falling through a limited height and in turn raises smaller quantity of water to surprisingly great height. Rams have been constructed and are in use and they lift water to a height which is about 50 to 60 times the height of the available head. Hydraulic rams are specially suitable for country estates and farms in the vicinity of which large quantity of low-head water is readily available. The self-working of the ram is based on the inertia forces intermittently created and destroyed. This action is understood from the construction and working of the ram explained below.

is 100 cm in diameter. The friction of each of the three packings may be taken as 2% of the total pressure on the appropriate piston or ram. Neglecting all other minor losses, determine the velocity of the press ram.

As the packing friction is 2%, the gross load on the ram of the press while it is working is $= \frac{W}{0 \cdot 98} = \frac{40}{0 \cdot 98} = 40 \cdot 8$ tonnes

Hence, pressure of water on the ram $= \dfrac{40 \cdot 8 \times 1{,}000}{\frac{\pi}{4}(15)^2}$

$= 236 \text{ kg/cm}^2$

Water fed to this ram is supplied by the ram of the intensifier. Hence, pressure of water on the ram of the intensifier $= 236 \text{ kg/cm}^2$. If p is pressure of water on the piston of the intensifier, then force on the piston $= p \times A$

As there is packing friction of 2% for the piston and for the ram, the effective force transmitted to the ram is $p \times A \times 0 \cdot 98 \times 0 \cdot 98$.

Hence, pressure of water on the ram side is

$$= \frac{p \times A \times 0 \cdot 98 \times 0 \cdot 98}{a}$$

$$= p \times \left(\frac{D}{d}\right)^2 \times (0 \cdot 98)^2$$ where D and d are diameters of the piston and ram respectively.

$$\therefore \quad 236 = p \times \left(\frac{D}{d}\right)^2 \times (0 \cdot 98)^2 = p \times \left(\tfrac{100}{10}\right)^2 \times 0 \cdot 96$$

$$\therefore \quad p = 2 \cdot 46 \text{ kg/cm}^2$$

$$\therefore \quad \text{pressure head, } \frac{p}{w} = \frac{10{,}000 \times 2 \cdot 46}{1{,}000} = 24 \cdot 6 \text{ m of water}$$

$\therefore$ head lost in friction in the pipe line, $h_f = 30 - 24 \cdot 6 = 5 \cdot 4$ m of water

$$\text{Further, } h_f = \frac{4flV^2}{2gd}$$

$$\text{or } 4 = \frac{4 \times 0 \cdot 005 \times 200 \times V^2}{2 \times 9 \cdot 81 \times \frac{8}{100}}$$

$\therefore \quad V^2 = 1.57$

$\therefore \quad V = 1.25$ m/sec

Let, v' = velocity of the piston of the intensifier,

v = velocity of the ram of the press.

The quantity of water flowing through the pipe line enters the press and is the same as that discharged by the intensifier.

$$\text{Then, } v' = \frac{\pi}{4}\left(\frac{8}{100}\right)^2 = v' \times \frac{\pi}{4}\left(\frac{100}{100}\right)^2$$

$$\therefore \quad v' = V \times \left(\frac{8}{100}\right)^2 \qquad \text{.. (i)}$$

We also have,

$$V' \times \frac{\pi}{4}\left(\frac{10}{100}\right)^2 = v \times \frac{\pi}{4}\left(\frac{15}{100}\right)^2$$

$$\therefore \quad v' = v \times \left(\frac{15}{10}\right)^2 \qquad \text{.. (ii)}$$

$$\therefore \quad V \times \left(\frac{8}{100}\right)^2 = v \times \left(\frac{15}{10}\right)^2$$

$$v = V\left(\frac{8}{100}\right)^2 \times \left(\frac{10}{15}\right)^2$$

$$= 1.25 \times \frac{64}{10{,}000} \times \frac{100}{225} = 0.00356 \text{ m/sec}$$

or $0.00356 \times 100 \times 60 = 21.36$ *cm/min.*

Hydraulic Ram (Hydram)

Though it is very difficult and inconvenient to think of a pump which can lift the fluid without consuming any mechanical or electrical energy, it has to be conceded that there exists one such pump which is automatic in its action and works by utilizing a large quantity of water falling through a limited height and in turn raises smaller quantity of water to surprisingly great height. Rams have been constructed and are in use and they lift water to a height which is about 50 to 60 times the height of the available head. Hydraulic rams are specially suitable for country estates and farms in the vicinity of which large quantity of low-head water is readily available. The self-working of the ram is based on the inertia forces intermittently created and destroyed. This action is understood from the construction and working of the ram explained below.

Construction : The hydraulic ram consists of a supply tank constructed in the rivulet and connected to a valve box by means of a long drive pipe. The valve box has two valves on it. One is the waste water valve which opens inside the valve box (fig. 13-6). The other is the delivery valve which opens outward into the mouth of the air vessel which is fitted on the top of the valve box.

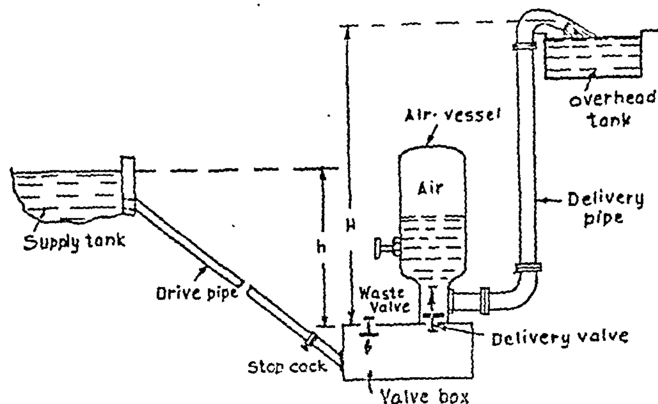


Fig. 13-6 Working of hydraulic ram

A delivery pipe connects the mouth of the air vessel to the overhead tank.

Working : Let the waste valve remain open, (because of its own weight or a spring) and the delivery valve then remains closed because of pressure from the air vessel. Water in the valve box and drive pipe is then at rest. Now, if we open the stop cock on the drive pipe, water starts rushing to valve box and flowing past the waste valve it goes back to the rivulet (fig. 13-7). By flowing in this manner, water gradually attains higher velocity and a stage comes when the velocity head and hence the corresponding pressure created near the waste valve is sufficient to lift the valve and ultimately close it. The water in the valve box comes immediately to rest, while water in the drive pipe line is in motion and it will take some time before it also comes to rest.

The moving column of water in the supply pipe causes rise of pressure inside the valve box due to hammer blow, or surge. This

rise of pressure opens the delivery valve and some quantity of water flows into the air vessel. The pressure of air in the air vessel increases and this forces water to flow into the delivery pipe and finally to the overhead tank.

The pressure wave generated in the valve box is reflected back into the supply pipe and water gets retarded and momentarily it starts flowing backward into the pipe. This reduces the pressure in the valve box, which in turn leads to opening of the waste valve and consequently closing of the delivery valve.

As the waste valve now opens, again water starts flowing from the valve box, which makes water in the drive pipe to flow down the pipe with an acceleration so that velocity of the water increases.

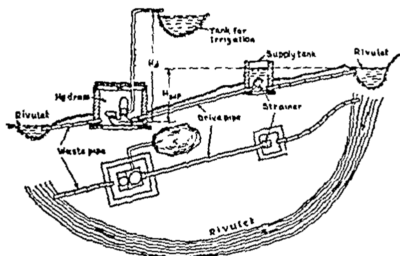


Fig. 13-7 Layout of hydraulic ram

Finally the pressure becomes sufficiently high to close the waste valve and water is delivered again. The process of acceleration and retardation of water column in pipe line works like a piston of a reciprocating pump; and hence this machine is termed as hydraulic ram. Compared to the quantity flowing out to the waste, a smaller quantity of water is lifted and delivered. Hence,

such a device is adaptable where ample supply of water is available. A typical layout of an installation of the hydraulic ram is presented in fig. 13-7.

Theory : Let, W_1 – quantity of water escaping through the waste valve,

W_2 – quantity of water lifted or useful water,

H – head through which the water is lifted,

h – head through which the water is falling, (available head).

D'Aubuisson's efficiency : Total quantity of water supplied from the supply tank = $W_1 + W_2$

Input energy = $(W_1 + W_2)\, h$

Output energy = $W_2 H$

$$\therefore \quad \text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{W_2 H}{(W_1 + W_2)\, h} \qquad .. (13{\cdot}9)$$

Rankine's efficiency : In this case it is assumed that weight W_2 of water is lifted through a height, $(H - h)$ as water had already an original height, h above the waste valve.

Hence, the actual output is $W_2 (H - h)$

$$\therefore \quad \text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{W_2 (H - h)}{(W_1 + W_2)\, h} \qquad .. (13{\cdot}10)$$

The overall efficiency of the hydraulic ram is as high as 80%.

Applications : This type of ram is used in pumping water from a rivulet, for irrigation purpose. It is also used to pump clean water from a well with the help of large quantity of dirty water. This is done by a special type of ram. This type of ram is used where time of pumping is not a question and quantity of useful water required is not large.

Advantages : The hydraulic ram has the following favourable aspects. There are no moving parts as in the case of centrifugal and reciprocating pumps hence the repairing cost is very low and does not require frequent oiling. It does not require an attendent to look after for all the time. The erection cost is very low, and requires limited space. There is almost no running cost.

Problem-13 : *A hydraulic ram is supplied with water under a head of 5 m and it delivers water to a head of 35 m. Quantity of waste water is 20 times that of useful water. Calculate efficiencies of the ram.*

Using eqn. (13·9),

$$\text{D' Aubuisson's eff.} = \frac{\text{output}}{\text{input}} = \frac{W_2 H}{(W_1 + W_2) h}$$

$$= \frac{W_2 \times 35}{(20 W_2 + W_2) 5}$$

$$= 0{\cdot}333 \text{ or } 30{\cdot}3\%$$

Using eqn. (13·10)

$$\text{Rankine's eff.} = \frac{W_2 (H - h)}{(W_1 + W_2) h}$$

$$= \frac{W_2 (35 - 5)}{21\ W_2 \times 5}$$

$$= 0{\cdot}286 \text{ or } 28{\cdot}6\%$$

Problem-14 : *A hydraulic ram has waste valve of 4 kg weight with a lift of 10 mm. The diameter of the waste valve is 25 cm. The valve makes 100 beats per minute. The ram is supplied with water through 10 m long 15 cm diameter pipe line. If the delivery head is 20 m, determine neglecting friction, the discharge in kg/min.*

Pressure required to close the waste valve is

$$= \frac{\text{weight}}{\text{area}} = \frac{4}{\frac{\pi}{4} (25)^2} = 0\ 0081 \text{ m of water}$$

$\therefore$ pressure head $= \dfrac{0{\cdot}0081 \times 10{,}000}{1{,}000} = 0{\cdot}081$ m of water

$\therefore$ velocity past the waste valve $= \sqrt{2gH} = \sqrt{2 \times 9{\cdot}81 \times 0{\cdot}081}$
$= 1{\cdot}26$ m/sec

Area of flow past the waste valve $= \pi d \times l$ (where l = lift of valve)
$= \pi \times 25 \times \frac{10}{10} = 25\pi$ cm^2

Area of supply pipe $= \dfrac{\pi}{4}\,(15)^2 = 56{\cdot}25\pi$ cm^2

$\therefore$ maximum velocity of water in the supply pipe,

$$V_s = 1{\cdot}26 \times \frac{25\pi}{56{\cdot}25\pi} = 0{\cdot}561 \text{ m/sec}$$

K.E. of water in the supply pipe

$$= \frac{W v_s^2}{2g} = \frac{1{,}000 \times \frac{\pi}{4}\left(\frac{15}{100}\right)^2 \times 10 \times (0{\cdot}561)^2}{2 \times 9{\cdot}81} = 2{\cdot}84 \text{ kg–m}$$

This K. E. is utilised in lifting the water to a height of 20 m

Hence, weight of water lifted $= \dfrac{2{\cdot}84}{20} = 0{\cdot}142$ kg/beat

Discharge $= 0{\cdot}142 \times 100 = 14{\cdot}2$ kg/min.

Problem-15 : *A hydraulic ram with a supply pipe 100 mm diameter and 7 m long has a waste valve 150 mm diameter and 2 kg weight. The lift of the waste valve is 15 mm. Determine the discharge through the delivery pipe against a head of 16 m, if the number of beats is 130 per minute.*

Pressure of water in the valve box to close it is given by

$$p = \frac{\text{weight of the valve}}{\text{area of the valve}}$$

$$\text{or } p = \frac{2}{\frac{\pi}{4}\left(\frac{150}{10}\right)^2} = 0{\cdot}0113 \text{ kg/cm}^2$$

Pressure head, $H = \frac{p}{w} = \frac{0{\cdot}0113\,(100)^2}{1{,}000} = 0{\cdot}113$ m of water

Velocity of efflux, *i.e.* velocity of the waste water past the valve just before closing, $V = \sqrt{2gH}$

$$= \sqrt{2 \times 9{\cdot}81 \times 0{\cdot}113}$$
$$= 1{\cdot}49 \text{ m/sec.}$$

Area of flow past the waste valve $= \pi dl$

$$= \pi \times \left(\frac{150}{10 \times 100}\right) \times \frac{15}{1{,}000}$$
$$= \frac{\pi \times 2{,}250}{10{,}00{,}000} = 0{\cdot}00707 \text{ m}^2$$

Discharge of waste water, $Q = 0{\cdot}00707 \times 1{\cdot}49 = 0\,01355 \text{ m}^3$

Maximum velocity of water in the supply pipe is

$$V_s = \frac{Q}{a} = \frac{0\,01055}{\frac{\pi}{4}\left(\frac{100}{1{,}000}\right)^2}$$

Retarding head is equal to the delivery head,

$$\text{Hence,} \quad 16 = \frac{l_s}{g} \times \frac{V_s}{t}$$

$$t = \frac{l}{g} \times \frac{V_s}{16} = \frac{7}{9{\cdot}81} \times \frac{1{\cdot}34}{16} = 0{\cdot}0598 \text{ sec.}$$

During the above period the waste valve remains closed and hence water is supplied to the delivery pipe.

Hence, discharge per beat

$$= \text{mean velocity} \times \text{area of pipe} \times \text{time}$$
$$= \frac{1{\cdot}34}{2} \times \frac{\pi}{4}\left(\frac{100}{1{,}000}\right)^2 \times 0{\cdot}0598$$
$$= 0\,000314 \text{ m}^3$$

$\therefore$ discharge per minute $= 0{\cdot}000314 \times 135 = \underline{0{\cdot}0407 \text{ m}^3}$

Hydraulic Jack

This is a very handy, useful portable type lifting device which is very widely used in road transport and railways for temporary lifting the cars or wagons for purpose of carrying out repair works.

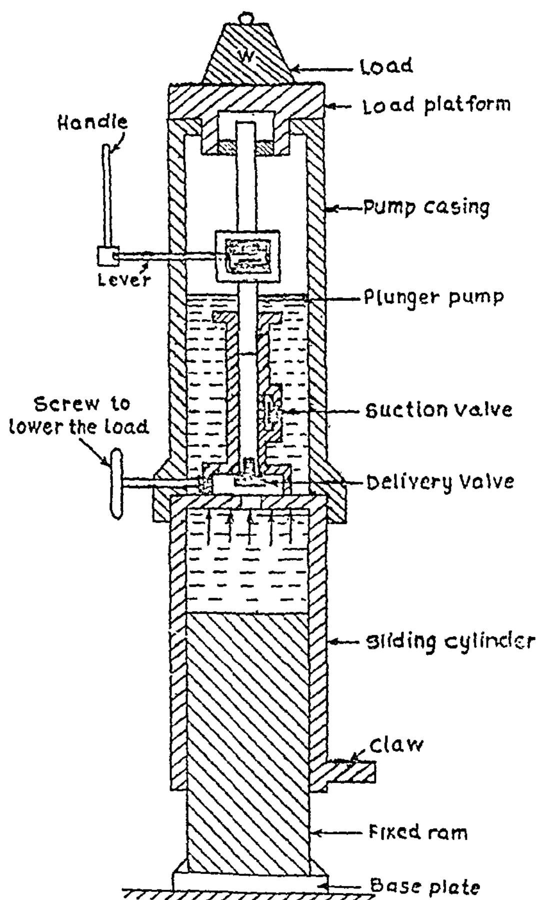


Fig. 13-8 Hydraulic jack

The jack consists in effect, of a portable short-stroke lift fed from a hand pump formed integral in the casting. It works on the same principle as the hydraulic press.

Construction : The jack consists of a fixed ram over which slides a sliding cylinder. On the top of the sliding cylinder the pump assembly is arranged which also gets lifted along with the load. The loading platform is provided on the top of the pump casting. In the pump casting, a plunger pump is located and is operated by a lever mechanism shown in fig. 13-8. The barrel for the plunger pump has suction valve at the middle and delivery valve fitted at its bottom The lower part of the pump barrel is fitted with a screw to lower the load. The sliding cylinder has got a claw which is additionally used for lifting the load. The fixed ram is founded on a base plate for fixing it on the ground.

Working : The load to be lifted is placed on the platform or the claw. Alternately the jack is stationed below the job to be raised. The lowering screw is kept in position to prevent liquid from returning from the sliding cylinder to the pump casing.

The plunger of the liquid pump is moved upward by applying force at the end of the handle of the lever This upward movement of the plunger creates vacuum inside the barrel, the suction valve opens and liquid enters the barrel from the pump casing. When the plunger moves downward, it is forced out from the plunger cylinder through the delivery valve into the sliding cylinder and during this operation the suction valve remains closed. Liquid that is forced under pressure inside the sliding cylinder, creates an upward force on the top of the sliding cylinder, from inside and consequently the sliding cylinder, with the pump casing and load on it, moves upward. This upward movement continues till the desired lift of the job is achieved When the load platform is to be lowered, the screw is opened; and high pressure liquid will be forced into the pump casing from the sliding cylinder through the hole of the screw. This results in the downward sliding of the cylinder over the fixed ram and the job is lowered.

Applications : This is used for lifting heavy load which cannot be easily lifted manually. This is also used for raising the railway wagons for replacement of wheels.

Hydraulic Crane

Hydraulic cranes are employed for raising and transferring heavy loads. In this the load velocity is greater compared with the effort velocity. This necessitates greater effort which is obtained by installing a jigger arrangement. Cranes are widely used on docks for loading and unloading ships, foundary work-shops, and heavy industries.

Construction : This consists of a hollow vertical post at the top of which one end of the rod is hinged (fig. 13-9). One end of the jib is hinged near the foot of the vertical post and the other end is

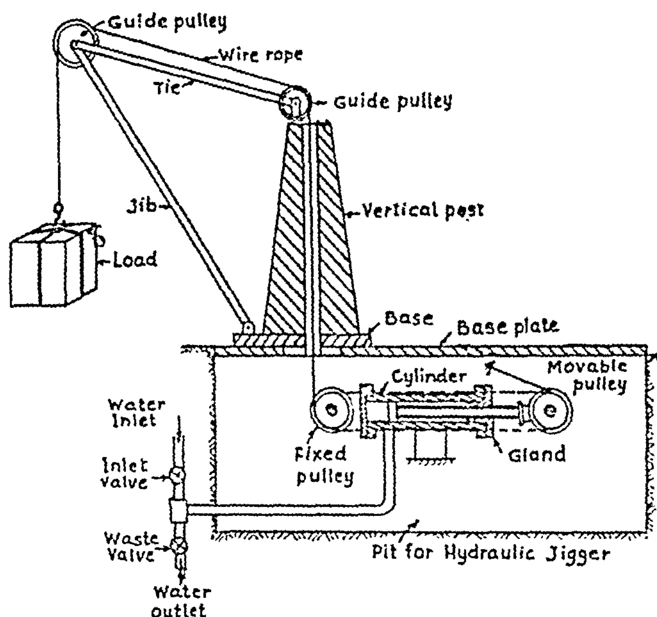


Fig. 13-9 Hydraulic crane

connected to the tie rod by means of a pin joint. A guide pulley is mounted at the junction of the tie rod and jib. The crane post is supported on the base plate. The jigger arrangement is provided in the pit for increasing load velocity. One end of the wire rope

is fixed to the body and the other end, after passing round the pulleys of the jigger, guide pulley at the top of the vertical post and the guide pulley at the junction, carries a load.

The jigger consists of a fixed cylinder inside which a piston is sliding. One end of the piston rod is connected to the piston and the other end carries a pulley block. There is another pulley block fixed on the head of the cylinder. There is a pipe connection for supplying water under pressure. The vertical post mounted on the base plate has an arrangement for turning the base, so that the load can be shifted.

Working : When the load is to be raised, the water is admitted under pressure inside the cylinder of the jigger through the pipe. This water forces the piston and piston rod along with the pulley-block to move outward; and the distance between two pulley blocks is increased. Thus, due to increased distance wire rope will be pulled and the load will be raised as the other end of the rope in fixed. If it is desired to shift the load the vertical post can be rotated through desired angle.

For lowering the load, the valve on the supply pipes is closed and the valve on the discharge side is opened. Hence, the water will be forced out from the cylinder and the piston will move inward, consequently decreasing the distance between the pulley blocks. Thus, the rope will be released and load will be lowered. This will continue till the piston reaches the end of the inward stroke.

Air Lift Pump

Air lift pump uses compressed air. It works on the principle that mixture of air and water is lighter than solid water *i e.* air when allowed to mix with water reduces the density Thus the pressure of given column of mixture is less than the pressure of a much shorter column of solid water.

This type of pump can lift more water through a bore hole than any other form of pump. It is used for lift (h_2) upto 160 m. The adventage of this pump is that it has no moving parts below the water level, and therefore cannot be damaged by solids in suspension in the water. On the other hand, its efficiency is low. Only 20 to 40% of the energy supplied to air can appear in the form of useful water horse-power.

Construction : It consists of a compressor and a tank to store the compressed air which is supplied to the pump through the air-

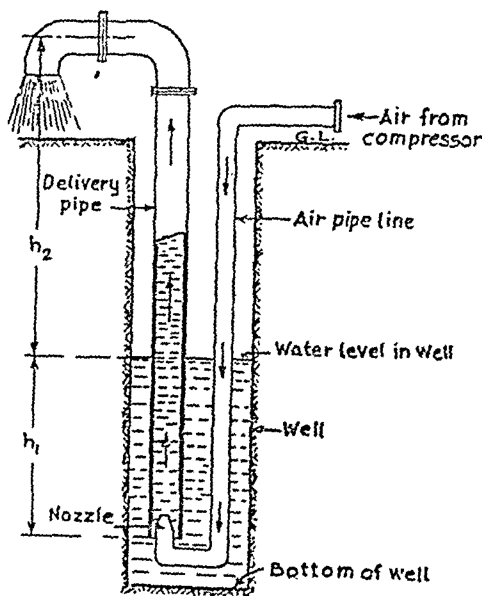


Fig. 13-10 Air lift pump

pipe line as shown in fig. 13-10. The lower end of the pipe carrying the compressed air is fitted with a nozzle. This nozzle is inserted inside the delivery pipe at its bottom. The other end of the delivery pipe is free to discharge water. There may be one or more nozzle inserted at the bottom of the delivery pipe depending upon the design of the pump.

Working : As a result of *introduction of compressed air in the delivery pipe, the water column becomes impregnated with air bubbles. As more and more air gets in the delivery pipe,* the density of water is reduced. *Thus, the force exerted by a water* column h_1 inside the *delivery pipe, at the bottom of the pipe is less* than the force exerted by a water column h_1 outside the delivery pipe. Hence the outside *column of water will support a higher* water column and water will be *raised inside the pipe and finally* will be discharged.

Best results are obtained if the useful lift h_2 is less than the submergence h_1, the ratio $\frac{h_2}{h_1}$ varies from ½ to 1.

For the efficient working and to keep down the frictional losses, the velocity of the mixture of water and air should not be more than 1·8 m/sec. The volume of air required to lift water depends upon the delivery head, and its efficiency.

TUTORIAL – 13

General

1 Enumerate the different machines which work by utilizing hydraulic energy. Draw a layout plan of arrangement of a hydraulic power transmission and utilization of a typical shop.

Accumulator

2 Sketch the arrangement of the simple accumulator and describe its working. State its electrical and mechanical analogies. Which are its popular fields of application ?

3 What is differential accumulator ? How does it differ from a simple accumulator ? Give a neat sketch, showing its construction.

4 The differential hydraulic accumulator is supplied with water under a pressure head of 550 m of water. The diameter of the smaller and larger parts of the ram are 10 and 12 cm respectively. If the packing friction amounts to 4% of the load on

the moving cylinder, determine the total weight of the loaded moving cylinder.

[1·83 Tonnes]

5 An accumulator has a ram of 20 cm diameter and a lift of 7 m. Water is supplied at a pressure of 450 m of water. Find the necessary load on the ram and capacity in H.P.–Hours.

[14·1 tonnes; 0·366]

6 The ram of a hydraulic accumulator is loaded with a weight of W tonnes. The packing friction of a ram is equivalent to 5% of the load on the ram. The diameter of the ram is 25 cm. When the ram is falling steadily the pressure inside the accumulator is 70 kg/cm^2. Determine the pressure of water required when the ram is ascending with uniform velocity and what will be the load on the ram ?

[74 kg/cm^2; 34·6 tonnes]

7 An accumulator has 20 cm ram and 6 m lift. The packing friction is equivalent to 3 per cent of the load on the ram. The ram falls through its full height in 2 minutes. The accumulator is loaded with 60 tonnes total weight. If the pump is delivering 0·02 m^3/sec. through the accumulator when the ram is falling, determine the H.P. delivered.

[534]

8 An accumulator is maintaining a pressure of 77 kg/cm^2 in a 10 cm. diameter main. A hydraulic lift situated at a distance of 900 m from the accumulator is supplied with water under pressure from the main. The ram of the lift is 20 cm in diameter and the load on it, inclusive of its own weight, is 12 tonnes. The friction of the ram, cage, etc. is equivalent to an additional 5% load of the gross load. Determine the speed at which the lift will ascend if the value of the coefficient of friction in the supply main is 0·008. Neglect other losses.

[1·28 m/sec.]

9 35 H.P. is transmitted from an accumulator through a 15 cm diameter pipe line 2·414 km. long. If the loss is to be 2%, find the diameter of the ram which is loaded with 130 tonnes. Take $f = 0{\cdot}01$ for the pipe.

[71 cm]

Intensifier

10 What is a hydraulic intensifier ? Explain its analogy with an electrical transformer. With the help of a neat sketch, describe its working. Name the machines in conjunction with which it is usually found.

11 A hydraulic intensifier is supplying water to a hydraulic press at a pressure of 200 kg/cm². The stroke of the intensifier is 1 m and its capacity is 20 litres. If water is supplied at a pressure of 40 kg/cm² from a low pressure main, determine diameters of two rams of the intensifier.

[16 cm, 35·8 cm]

12 Water is supplied to a hydraulic intensifier at a pressure of 2·1 kg/cm². The diameter of the sliding and fixed rams of the intensifier are 7·5 and 15 cm respectively. This intensifier supplies water to hydraulic crane. The diameter of the ram of the crane cylinder is 30 cm. If the efficiency of the crane is 80%, determine the load lifted. The load on the crane moves 2 m when the ram of the crane moves 40 cm.

[949 kg]

13 A hydraulic intensifier is supplying water to a direct acting hydraulic lift, having ram diameter of 15 cm. The diameters of the sliding cylinder and the fixed ram of the intensifier are 15 cm and 10 cm respectively. The intensifier is supplied with water, from an *overhead tank whose level is 24 m above* the level of the intensifier, through a pipe line 5 cm in diameter and 600 m long. If the efficiency of the lift is 80%, determine the load lifted by it. Assume packing friction equal to 3% of the load on sliding cylinder. The efficiency of transmission of the pipe line is 70%. Take coefficient of friction for the pipe line as 0·01.

[519·2 kg]

Hydraulic press

14 How did the conception of a hydraulic press evolve ? Give a working sketch to explain its construction and function. What constitutes the mechanical advantage of the press ? Mention a few applications of it.

15 In a hydraulic press the diameter of the ram and plunger are 30 and 3·75 cm respectively. The leverage of the handle is 10 to 1. The packing friction of the ram as well as the plunger is 3% of load. The lift of the load is 0·6 m and the stroke of the plunger is 10 cm. A load of 15 tonnes is to be raised in 2 minutes. Determine the force applied at the end of the lever, H.P. required to drive the plunger, and the number of strokes per second.

[24·78 kg; 1·06; 386]

16 A load of 20 metric tons is to be lifted by means of a hydraulic press. The maximum effort applied at the end of lever is 22 kg. If the diameter of the ram is 15 cm, determine neglecting friction, the diameter of the plunger. The leverage of the handle is 30:1. Determine also the number of strokes performed when the load rises by 45 cm, and the stroke of the plunger is 20 cm.

[2·73 cm, 68]

17 The ram of a hydraulic press is 15 cm in diameter, and is worked from an intensifier (of the piston and ram type) which receives its low pressure supply of water from a tank whose surface level is 27 m above the level of the intensifier piston, through a pipe 7·5 cm diameter and 200 m long. The friction coefficient for the low pressure supply pipe is 0·005. The press ram is exerting a force of 40 tonnes. The intensifier ram is 10 cm in diameter while the piston is 100 cm in diameter. The friction of each of the three packings may be taken as 2% of the total pressure on the approximate piston or ram. Neglecting all other minor losses, determine the velocity of the press ram.

[15·36 cm/min.]

Hydram

18 Can you envisage of a pump without a conventional prim mover? If you know of one such, describe it.

19 What are the advantages of a hydraulic ram over the othe pumps? Enunciate D'Aubuisson's and Rankine's efficiencie of the ram

20 A hydraulic ram is supplied with water under a head o 5 m and it delivers water to a head of 30 m The wast water is 20 times the useful water. Calculate the efficiencie of the ram.

[D' Aubuisson's η – 28 6%; Rankine's η – 23.8%.

21 A hydraulic ram with a supply pipe 10 cm diameter and 7 n long has a waste valve 150 mm diameter. The weight of th waste valve is 2 kg. The travel of the valve is 1 cm. Deter mine the discharge through the delivery pipe against a head o 16 m The number of beats is 130 per minute.

[0 0182 m^3/min.

Hydraulic Jack

22 What functions are discharged by a hydraulic jack? Why oi is usually preferred to water as working fluid? Describe a typical jack you are familiar with.

Hydraulic Crane

23 What do you understand by a jigger? What velocity ratio the *cranes usually* have? Draw a good sketch to illustrate the mechanism of the crane.

Air Lift Pump

24 Explain clearly the principles on which the air lift pump works. *Sketch* and describe the construction and working of the air lift pump.

APPENDIX

Constants and Equivalents

Density of water w	$= 1{,}000$ kg/m^3 or 1,000 gm/litre
	$= 62{\cdot}4$ lb/ft^3
Density of steel, ρ	$= 490$ lb/ft^3
	$= 2{\cdot}08$ kg/cm^3
1 atmosphere	$= 1{\cdot}033$ kg/cm^2
	$= 14{\cdot}7$ lb/in^2
	= 760 mm (30 in) of mercury or 34 feet of water
1 acre	= 43,560 sq.ft. = 4,840 sq. yd.
1 acre (metric)	= 100m × 100m = (10,000 sq.m.)
1 H.P.	= 746 watts or 550 ft.lb./sec.
	= 76 kg-m/sec
1 H.P. (metric)	= 75 kg-m/sec. or 4,500 kg-m/min.
1 kW	= 102 kg-m or 1·34 H.P.
1 in.	= 25·4 mm
1 mm	= 0·039 in.
1 metre	= 3·2803 ft.
1 foot	= 0·3048 m
1 mile	= 1·6093 km (kilometre)
1 km	= 0·6214 mile
1 ton	= 2,240 lb
1 ton (metric)	= 1,000 kg
1 quintal	= 100 kg
1 kg	= 2·2046 lb
1 lb.	= 0·4536 kg
1 in^2	= 6·451 cm^2
1 cm^2	= 0·155 in^2
1 sq. mile	= 640 acres = 258·9 Hectaces
1 lb./in.^{2}	= 0·07 kg/cm^2
1 kg./cm^2	= 14·22 lb/in^2
1 m^3	= 35·315 ft^3
	= 1,000 litres
1 ft^3	= 0·0283 m^3
1 litre	= 1,000 cm^3
1 G.P.M. (gallon/min.)	= 4·546 litres/min.

MATHEMATICAL TABLES

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	[illegible]
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	[illegible]
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						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
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15	1761	1790	1818	1847	1875						3	6	9	11	14	17	[illegible]	[illegible]	26
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16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
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						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
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						2900	2923	2945	2967	2989	2	4	6	9	11	13	15	17	19
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36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	[illegible]	[illegible]	[illegible]	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6096	6107	6117	1	2	3	4	5	6	8	9	[illegible]
41	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6222	1	2	3	4	5	6	7	8	9
42	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6325	1	2	3	4	5	6	7	8	[illegible]
43	6335	6345	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6425	1	2	3	4	5	6	7	8	[illegible]
44	[illegible]	[illegible]	[illegible]	6464	6474	[illegible]	[illegible]	[illegible]	[illegible]	6522	1	2	3	4	5	6	7	8	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6609	6618	1	2	3	4	5	6	7	8	[illegible]
46	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2	3	4	5	6	7	7	[illegible]
47	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	3	4	5	5	6	7	[illegible]
48	6812	6821	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2	3	4	4	5	6	7	[illegible]
49	6902	6911	6920	6928	6937	6946	[illegible]	[illegible]	[illegible]	[illegible]	1	2	3	4	4	5	6	[illegible]	[illegible]

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
76	8808	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
77	8865	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
78	8921	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
79	8976	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
80	9031	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
91	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
92	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
93	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
94	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
95	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															4	4	5	6	7
															4	5	5	6	7
															4	5	5	6	7
															4	5	5	6	7
															4	5	5	6	7
															4	5	6	7	7
															4	5	6	7	8
															4	5	6	7	8
															4	5	6	7	8
															5	5	6	7	8
															5	6	6	7	8
															5	6	7	8	9
															5	6	7	8	9
															5	6	7	8	9
															5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
															5	6	7	9	10
															5	7	8	9	10
															6	7	8	9	10
															6	7	8	9	10
															6	7	8	9	11
															6	7	8	10	11
															6	7	9	10	11
															6	8	9	10	11
															6	8	9	10	12
															7	8	9	10	12
															7	8	9	11	12
															7	8	10	11	12
															7	8	10	11	13
															7	[illegible]	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
															8	10	12	13	15
															8	10	12	13	15
															9	10	12	14	16
															9	11	12	14	16
															9	11	13	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17

NATURAL SINES

Degrees.	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences. 1′	2′	3′	4′	5′
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degrees	0 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1'	2'	3'	4'	5'
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	6	8	10
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	6	8	10
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	6	8	10
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	6	8	10
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	6	8	9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	6	7	9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	5	7	9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4	5	7	9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	5	7	9
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	5	7	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	5	7	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	5	6	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	5	6	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	5	6	8
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	4	6	7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	4	6	7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	4	6	7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	4	5	7
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	4	5	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3	4	5	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	4	5	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	3	5	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	3	4	6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	3	4	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	3	4	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	3	4	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	3	4	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	3	3	4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	2	3	4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2	2	3	4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2	3	4
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2	3	3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2	3	3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2	2	3
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	1	1	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	1	1	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	1	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	0	0	0
90	1·000														

NATURAL TANGENTS

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences. 1′	2′	3′	4′	5′
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5												6	9	12	15
6												6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1354	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·19											6	9	12	15
12	·21											6	9	12	15
13	·23											6	9	12	15
14	·24											6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867											6	9	13	16
17	·3057											6	10	13	16
18	·3249											6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919							7	10	13	17
22	·4040	4061	4081	4101	4122							7	10	14	17
23	·4245	4265	4286	4307	4327							7	10	14	17
24	·4452	4473	4494	4515	4536							7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877											7	11	15	18
27	·5095											7	11	15	18
28	·5317											8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31												8	12	16	20
32												8	12	16	20
33												8	13	17	21
34												9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841										9	14	19	24
39	·8098	8127										10	15	20	24
40												10	15	20	25
41												10	16	21	26
42												11	16	21	27
43												11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

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